The Variable

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In this issue

Cover Image

This month's cover image was created by Francine Champagne, a multi-media artist from Vancouver Island. This image, reminiscent of snowflakes, features the p6m symmetry group, a two-dimensional repetitive pattern that exhibits order six symmetry. A pattern with this symmetry can be considered a tessellation of the plane with equal triangular tiles, or equivalently, a tessellation of the plane with equal hexagonal tiles. See more of Francine's tessellations work at www.tessellations.ca.

SMTS Executive (2016-2018)

The Variable

Notice to Readers

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Notice to Contributors

The Variable welcomes a variety of submissions for consideration from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants, teacher educators, researchers, and students of all ages, although we encourage Saskatchewan teachers of mathematics as our main contributors. Submitted material is assessed for interest, presentation, and suitability for the target audience.

Please submit articles by email to thevariable@smts.ca in Microsoft Word format. Articles should be limited to 3000 words or less; authors should also include a photo and a short biographical statement of 75 words or less. Accepted contributions are subject to revision. Editors reserve the right to edit manuscripts for clarity, brevity, grammar, and style.

Call for Contributions

The Variable **is looking for contributions from all members of the mathematics education community**, including classroom teachers, consultants, and teacher educators. Consider sharing a favorite lesson, an essay, a book review, or any other work of interest to mathematics teachers in Saskatchewan. When accepted for publication, your article will be shared with a wide audience of mathematics educators in Saskatchewan and beyond.

We also welcome student contributions in the form of artwork, stories, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom, and for teachers to recognize students' efforts during their journey of learning mathematics.

All work is published under a Creative Commons license. **To submit or propose an article, please contact us at thevariable@smts.ca**. We look forward to hearing from you!

Ilona & Nat,
Editors

Message from the President

Welcome back! I hope you had a restful and rejuvenating break. While this likely happened in between stretches of planning and marking—my Facebook feed was full of photos of colleagues trying to do this in the most festive ways possible—I hope you took some real time off, too. As teaching responsibilities intensify, it's more important than ever to carve out dedicated time for yourself and your family.

While September (and February, for high school folks) marks the real time of new beginnings in our classrooms, January is when many of the other communities that we belong to make plans and set goals for the year ahead. It's a time of renewal, new hopes, and promise. And, while there might be nothing particularly new about your classroom in January, the turn of the year can still serve as an opportunity to review and reflect.

Last October, at SUM Conference, Steve Leinwand asked us to choose two things to intentionally change or try this school year. While I know that some of you could never limit yourself to that kind of reasonable and manageable change, you may wish to consider which changes have been most impactful for you and your students. Are you able to narrow that long list of things you wanted to try, and focus in on just two? If you chose two right from the start, how are they going? What changes might you make? Or, is the start of a new year a nice reminder that you need to get back to the goals you set for yourself earlier in the school year?

One of the most powerful things I have learned from yoga is the practice of checking in on myself without judging, and simply noticing. Am I accomplishing what I had planned? Why might that be? Should I make some changes? Maybe you had planned to practice number talks regularly in your classroom. Or to facilitate weekly math journaling. Or to switch up your homework format. Or to integrate Desmos into your lessons. Or to use random grouping more often. But, despite your best intentions, maybe the reality is that those plans went sideways sometime in November… or the third week of September. Or, maybe you're still on track! Either way, your math community isn't here to judge. We're here to support you in what you need once you've noticed where you are. So I invite you to take a few minutes to reflect and to notice. How are your original plans for this school year coming along? Where are your students excelling? Where are they struggling? Then, consider how you will move forward from here: Should you modify your plans to make them more sustainable? Should you try something different? Should you keep going exactly as is?

Before everything ramps back up into crazy mode, I wish you a few quiet moments to breathe in… breathe out… and to reflect on the school year so far, without judgment but with deliberation, and to renew or recalibrate your plans as needed. Knock it out of the park in 2018, everyone!

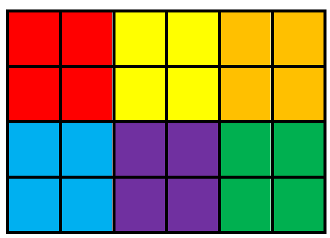
Michelle Naidu

Welcome to this month's edition of Problems to Ponder! Have an interesting solution? Send it to *thevariable@smts.ca* *for publication in a future issue of* The Variable!

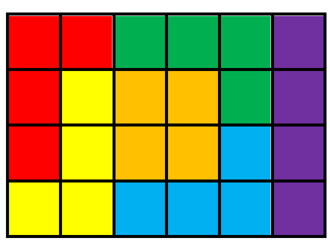
Primary Tasks (Kindergarten-Intermediate)

Colour-Coding Brownies[1]

Sam has brought a pan of brownies to a birthday party that has been cut into 24 equal pieces. He wants to share them equally among himself and his 5 friends at the party. Partition the pan of brownies and use colour coding to show how the brownies can be shared fairly. (Here is an example:



Here is another way in which Sam could share the brownies. Are there others?



Adaptations and extensions: What if there were 8 (or 12, or 9) kids at the party? What if the brownies had been divided into 30 (or 12, or 15) pieces?

[1] *Adapted from* Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching.* San Francisco, CA: Jossey-Bass.

Wooden Legs [2]
Wendy builds wooden dollhouse furniture. She uses the same kind of legs to make 3-legged stools and 4-legged tables. She has a supply of 31 legs. How many stools and tables might she make?

Extensions: How many stools and tables might she make if she must use all of the legs?

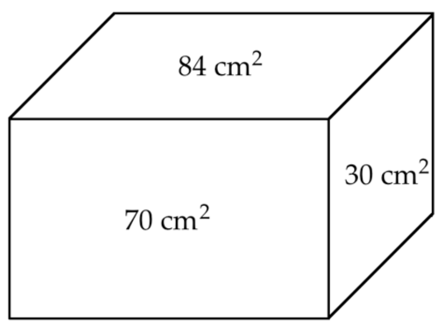
Frog Farming [3]
Farmer Mead would like to raise frogs. She wants to build a rectangular pen for them and has found 36 meters of fencing in her barn that she'd like to use.

a) Design at least four different rectangular pens that she could build. Each pen must use all 36 meters of fence. Give the length and with for each of the pens.

b) If each frog needs one square meter of area (1 m2), how many frogs will each of your four pens hold? Can you design a pen with the greatest frog capacity possible?

Intermediate and Secondary Tasks (Intermediate-Grade 12)

Mystery Box [4]
The areas of the faces of a rectangular box are 84 cm^2, 70 cm^2 and 30 cm^2. What is the volume of the box?



[2] *Adapted from* Ray, M. (2013). Powerful problem solving: Activities for sense making with the mathematical practices. Portsmouth, NH: Heinemann.

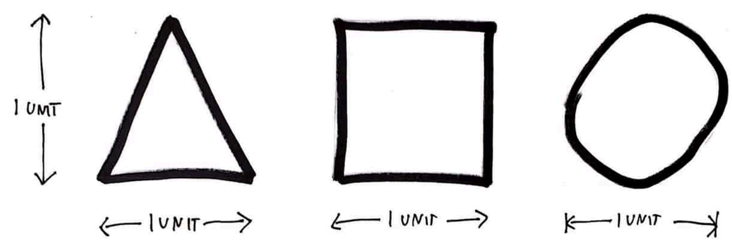
[3] Ray, M. (2013). Powerful problem solving: Activities for sense making with the mathematical practices. Portsmouth, NH: Heinemann.

[4] Bellos, A. (2016, December 5). Can you solve it? Are you smarter than a Singaporean 10-year-old? *The Guardian.* Retrieved from https://www.theguardian.com/science/2016/dec/05/did-you-solve-it-are-you-smarter-than-a-singaporean-ten-year-old

Marbles in a Bag[5]
A bag contains 16 marbles, some black and the remainder white. Two marbles are drawn at random at the same time. It is equally likely that the two marbles will be the same colour as different colours. How many marbles of each colour are there inside the bag?

Extension: Suppose the bag contains x black marbles and y white marbles. What are the possible values of the positive integers x and y for which it is equally likely that two marbles selected at random will be the same colour as different?

The Solid of Many Faces[6]
Draw a 3-dimensional picture of a solid shape that goes through each of the holes below, exactly touching every point on each of the sides as it passes through. (The object must be made of a solid that is not elastic and does not squash.)



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[5] *Adapted from* Barbeau, E. (1995). *After math: Puzzles and brainteasers*. Toronto, ON: Walls & Emerson, Inc.

[6] Can you solve it? Are you smarter than an architect? *The Guardian.* Retrieved from https://www.theguardian.com/science/2017/jul/17/can-you-solve-it-are-you-smarter-than-an-architect

Alternate Angles

Alternate Angles is a bimonthly column on problems from multiple perspectives: various methods that could be used to solve them, insights we get from their solution, the new paths that they can lead us to once they have been solved, and how they can be used in the classroom.

Telling Time[1]

Shawn Godin

Welcome back, problem solvers. Last time, I left you with the following problem:

> You have two hourglasses: one that measures 9 minutes and one that measures 13 minutes. Determine how to measure 30 minutes using these two hourglasses.

This problem is the "easy version" of question P6 from the book *The Inquisitive Problem Solver* by Paul Vaderlind, Richard Guy, and Loren Larson.

This problem may look familiar, and we will explore some of the related problems later on. First, we'll use a useful problem-solving technique and consider an easier problem. Instead of looking at 9-minute and 13-minute hourglasses, we will work with 3-minute and 5-minute hourglasses and look for some patterns.

With small numbers, we can attack the problem with pencil and paper, or we could use some manipulatives to help us out. For example, if you used linking cubes, you could make groups of threes and fives to see which numbers can be created. After some playing around, you should get: 3, 5, 6, 8, 9, 10, 11, 12, 13,… and every number after that. At this point, I am wondering which numbers can and cannot be represented by adding groups of 3's and 5's, or any two numbers.

We can start to see some patterns if we write all possible numbers down in the right way. For our simplified version, since we are using 3- and 5-minute hourglasses, I will record all

[1] This column was originally published as "What's the Problem? Telling Time," in the *Ontario Mathematics Gazette, 53*(4), 2015, pp. 12-13. Reprinted with permission.

the numbers in a table with three columns. In Figure 1, we can see that once we reach a number such as 3, all other numbers following it in the same column are reachable just by adding another 3. Thus, if we are searching for numbers that we can represent, we just need to fill all the columns. Since the 3 takes care of the last column (multiples of three), the 5 just needs to take care of the other (3 – 1) = 2 columns. Thus (3 – 1) x 5 = 10 is the last number to be filled, and (3 – 1) x 5 – 3 = 7 is the last number that we cannot represent.

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15

Figure 1

What we have actually solved is a version of the coin problem, which states: *Given unlimited coins of denominations d1, d2,…, dn, what is the largest amount of money that* ***cannot*** *be represented using these coins?* In our case, we found out that the largest amount of money that cannot be represented with 3¢ and 5¢ coins is 7¢. The solution to the two-coin version has been known for a long time, while for three or more coins, no explicit formula is known. If you have unlimited coins of denominations a¢ and b¢, where a and b have no common factors (why is this important?), the problem is solved in exactly the same way as we did with the 3¢ and 5¢ coins. If $a < b$ and we put all numbers into a table with a columns, then a takes care of the last column (the multiples of a), and each multiple of b takes care of a new column. The last column will be taken care of by $(a – 1) \times b$, and so the smallest number that *cannot* be represented is

$$(a-1)\times b - a = ab - a - b = (a-1)(b-1) - 1.$$

If our problem were dealing with 9¢ and 13¢ coins, the smallest amount not representable would be 95¢.

Unfortunately, this problem isn't equivalent to our problem. At this point, we will look at another related problem. Imagine that you have two unmarked jugs that hold 5 litres and 9 litres, respectively. You have a vat and you want to put exactly 7 litres into the vat. How do you do it?

Unlike the coin problem, with the jug problem, we can dump out previously poured water, so this problem involves addition and subtraction. You can obtain a solution by adding 9's and subtracting 5's, or vice versa. For example, I can add a 9, then continue subtracting 5's until I don't have a full 5 litres in the vat, at which point, I add another 9. The contents of the vat would be 9, 4, 13, 8, 3, 12, 7—and the problem is solved. You may want to try adding 5's and subtracting 9's, to show that the problem can be solved this way as well. It is also interesting to note that when this problem appears, there usually isn't a vat. Try your hand at measuring 7 litres if all you have are two unmarked jugs that hold 5 litres and 9 litres. It is possible.

Both the coin problem and the jug problem are examples of problems that can be modelled with *Diophantine equations*. Diophantine equations are those where we are interested in integer or whole-number solutions. The coin problem could have been modelled by the equation $3x + 5y = 30$, where the variables are non-negative integers. The jug problem can be modelled by the equation $5x + 9y = 7$, where the variables are integers. The method used in Figure 1 is closely related to *modular arithmetic*, sometimes called clock arithmetic. All numbers in the same column are equivalent modulo 3 (in this case). A surprising amount of interesting mathematics occurs when we consider not the numbers themselves, but the *residue* classes (columns) to which they belong.

Our problem lies somewhere in between the coin and the jug problem. All "solutions" to the coin problem work for the hourglass problem. On the other hand, only some of the new "solutions" to the jug problem work for the hourglass problem (the "hard" version at least).

In our problem, we can start both timers simultaneously and turn over the 9-minute timer when it is done. When the 13-minute timer is done, we know that 4 minutes have passed for the 9-minute timer, so we could turn the timer back over and measure 4 minutes. We could also have "started timing" when the 13-minute timer finished and the 9-minute timer would be a 5-minute timer (or a 4-minute timer, if we turned it over). Similarly, we can start both timers at the same time, turn them both over when they are done. Then, when the 9-minute timer is done for the second time, 5 minutes will have passed on the 13-minute timer. This means that we use this as our "start" point, and we can use the 13-minute timer to time 5 or 8 minutes.

If you don't mind waiting, you can use this process to turn the 9-minute timer into a 1/8-, 2/7-, 3/6-, or a 4/5- minute timer, while the 13-minute timer can be turned into a 1/12-, 2/11-, 3/10-, 4/9-, 5/8-, or 6/7-minute timer. Thus, we can solve the problem, with a head start, by doing 3 + 9 + 9 + 9 = 30 or 4 + 13 + 13 = 30 (or 8 + 9 + 13, or several other ways).

> "Sometimes, the questions you ask yourself yield more interesting mathematics than the question you were asked."

The original problem from *The Inquisitive Problem Solver* asked you to time 30 minutes without a head start, which makes it a little trickier, but possible. There are a couple of possible solutions. One possible solution is as follows: Turn both hourglasses over at the same time, and turn the 9-minute hourglass over as soon as it finishes. When the 13-minute hourglass is finished, 4 minutes will have passed in the 9-minute glass. At this point, turn it over and let it run out. When it runs out, 17 minutes will have passed. Now turn over the 13-minute hourglass. When it is finished, you will have passed 30 minutes.

This problem shows the power of considering an easier problem, and the fact that sometimes, the questions you ask yourself yield more interesting mathematics than the question you were asked.

And now for your homework:

In the expression below, six out of the seven boxes ($\square$) contain addition signs, and the remaining box contains a subtraction sign.

$$1 \square 2 \square 3 \square 4 \square 5 \square 6 \square 7 \square 8 = 30$$

Where should the subtraction sign go to make the equation true?

Until next time, happy problem solving!

Shawn Godin teaches and is a department head at Cairine Wilson Secondary School in Orleans, Ontario. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: leading workshops, writing articles, working on local projects and helping create mathematics contests.

Spotlight on the Profession

In conversation with Malke Rosenfeld

In this monthly column, we speak with a notable member of the mathematics education community about their work and their perspectives on the teaching and learning of mathematics. This month, we had the pleasure of speaking with Malke Rosenfeld.

Malke Rosenfeld is a percussive dance teaching artist, math explorer, math artist, TEDx presenter, author, and editor. Her interdisciplinary inquiry focuses on the intersection between percussive dance and mathematics and how to build meaningful learning experiences at this crossroads. Malke's interests also include embodied cognition in mathematics learning, task and activity design in a moving math classroom, elementary math education, and writing as a professional development tool. Her teaching and artistic endeavors focus on explorations of the relationship between number, rhythm, constraint, and shape in a variety of modalities. Malke delights in creating rich environments in which children and adults can explore, make, play, and talk math based on their own questions and inclinations.

First things first, thank you for taking the time for this conversation!

For the past decade or so, you have been exploring the relationship between mathematics learning and dance, which has included developing and facilitating a program called Math in Your Feet and, most recently, publishing a book entitled Math on the Move *(2016, Heinemann). In most people's eyes, mathematics and dance are an unlikely pair. So I would like to start by asking: How can the two disciplines inform and complement each other?*

Both math and dance are highly creative and expressive human endeavors. Not all of math is danceable and not all dance is mathematical, but there are some really nice overlaps between the two. In particular, the moving body is best positioned to explore and express the verbs of math, literally embodying the mathematical practices. There are also certain math ideas that can literally be put into action. In the early stages of writing my book, I did an experiment with dancers in three dance forms different from mine (hip hop, modern, and belly dance!) to see how/if certain action-oriented math ideas might resonate with the dancers. I also wondered if these ideas could function as choreographic prompts for what is called "movement invention," and it turned out I was onto something!

I created a list organized around three big-picture mathematical ideas: relationships, transformations, and rules. More specific concepts within the three categories included ideas such as dilate, algorithm, compose/decompose, scale, iteration, function, and many others; I also provided short definitions emphasizing the conceptual meaning of each term. In one exploration, a college dance professor had her students choose a mathematical concept from the list and explore it further as a way to create variations in a movement sequence. One of her students, a double major in dance and math, examined the definition of 'function' and felt that it could be used as a way to encourage changing the movement quality; specifically, she thought that converted 'values' could be manifested in movement as converted 'qualities'. (The full story can be found here. I'm also happy to share the original list.)

One tool that you have created to bridge the disciplines of art and mathematics is Jump Patterns, which you describe as "an integration of traditional percussive dance and elementary-level mathematics" (Rosenfeld, 2011, p. 78). What kinds of concepts and mathematical processes can students engage with through Jump Patterns and other "moving-scale" activities?

> "Exploring ideas at moving- or body-scale allows novices and experts alike to get a feel for what it means to "do math," think mathematically, and deepen one's intuition around a variety of concepts."

Wow! It's hard to believe that it's been six years since that article was published. Since then, many more math teachers and mathematicians have interacted with Jump Patterns and what we do with them in Math in Your Feet. Based on their experiences and perspectives, I have come to understand that the math in my math-and-dance classroom includes transformational geometry, working within mathematical constraints, group theory, pure mathematics, dynamic investigations of pattern units, patterning (creation of complex moving patterns), equivalence relations, and spatial reasoning. Exploring these ideas at moving- or body-scale allows anyone, novice and expert math learners alike, to get a feel for what it means to "do math," think mathematically, and deepen one's intuition around a variety of concepts.

Why explore these concepts through dance? What does this context afford that, for instance, paper does not?

This is a great question and I think the answer is simple. No math concept can be understood completely in just one representation or mode, including doing math at "body scale." The late Zoltan Dienes (creator of the Dienes blocks, which are also called base 10 blocks and inhabit many elementary math classrooms) proposed a theory of Multiple Embodiments, encouraging math educators to provide their learners with opportunities to explore a math idea in multiple, varied contexts and representations and to create generalizations based on these experiences. When I first looked into his work, I was tickled to find that he had written a book titled *Mathematics Through the Senses, Games, Dance, and Art* (1973, National Foundation for Educational Research), which is about giving children experiences with mathematical ideas through physical games and group dances.

In addition to the math-and-dance work I do, I have begun to investigate ideas for whole-body based math lessons that are outside a dance system. These are activities where the familiar mathematical objects we generally see on the page (such as polygons and polyhedra, hundred charts, or open number lines) are "scaled up" so that learners can interact with the idea or tool with their whole body in a meaningful way at a larger scale. I am enjoying the challenge of converting math that typically lives on the page into this new body-based modality. In collaboration with some amazing math educators, in addition to my book I've created four scaled-up, body-based math lesson plans for people to try out.

Overall, the whole-body modality for math learning is not a panacea for math education necessarily, but I'm hoping it is a reminder that learners at all levels need a chance to (quite literally) move away from the page now and again in an effort to explore more deeply "what math is." From what I've seen, mathematics is as much about the process of doing as it is in the final result. In his essay *A Mathematician's Lament*, Paul Lockhart speaks clearly about the need for learners to do the actual work within a creative discipline such as music, visual art, or mathematics. His example of learning to read music before playing an instrument illustrates the absurdity of thinking math is solely about notation or memorization. The whole, moving body, harnessed in pursuit of honest exploration of mathematical ideas (whether dance or non-dance) can provide learners with a sense of what it means to think and do in both disciplines.

"Learners at all levels need a chance to (quite literally) move away from the page now and again in an effort to explore more deeply 'what math is.'"

In your book Math on the Move, you acknowledge that the Math in Your Feet program is

> the perfect blend of the two most anxiety-inducing disciplines for most of us raised in American society […]. For one thing, both math and dance have a lot in common in their apparent ability to invoke fright and a flight response. They also share the deep-seated myth that we as a culture hold about learning and knowledge: that you are either good at math or dance or you're not. (Rosenfeld, 2016, p. xiii)

How do you alleviate the trepidation that students—and, for that matter, teachers—may have when trying whole body learning activities in the math classroom for the first time?

Honestly, my experience over many years has shown that children, even through the middle grades, are mostly excited and curious about what we're doing and what comes next, and are pretty much all-in from the get-go. This makes sense, because children's bodies and psyches are primed for movement; from birth, our physical exploration of the world around us is said to be the foundation of human intelligence, which is what is meant by the term "embodied cognition." Another benefit to using the moving body in math class is that it supports further developments in spatial reasoning, which is literally built through the body *and* is the foundation of mathematical thought. Moving is a natural activity for young people. Teachers who have begun to do this work with their own students have reported to me that the movement aspect focuses learners on the mathematics at hand and, even better, that the movement itself *becomes part of the reasoning*.

"Children's bodies and psyches are primed for movement; from birth, our physical exploration of the world around us is said to be the foundation of human intelligence."

In terms of adult anxiety, I imagine that a teacher's first thought about whole-body learning is that they have to be expert dancers or movers for this kind of approach to work in their classrooms, which is absolutely not true! I also imagine teachers are also wondering about what might happen if students get out of control while they're out of their seats. In reality, the expectations for a moving-math activity follow on from others you already have in place in your classroom.

To quell the qualms, it might be helpful for educators to think of the body as a "thinking tool" with an instructional focus similar to the work and discussion that goes on during an activity where hand-scale manipulatives are put to use. The added bonus of a whole-body approach (which I don't think is on most people's radar until it happens in front of them) is that when you get learners out of their seats for math, you will likely see surprising new strengths emerge in your students. There are always learners who struggle with math on the page, but in this body-based modality, some learners are more able to express their mathematical thinking and/or take the lead in problem solving in a way they haven't exhibited at their desks. Everyone benefits from changing the mode and scale of a math investigation, but for some learners, having the opportunity to realize, even for a moment, that they understand math can help keep that door open.

Also important to know is that moving math is a context for non-permanent problem solving, similar to what Peter Liljedahl has created with vertical non-permanent whiteboard surfaces (e.g., Liljedahl, 2016). This means that learners can easily change and adapt their their work and thinking and are not hindered by a final answer until they've decided they've got it the way they want it. In the movement context at least, this leads to a higher level of learner motivation and perseverance through the inevitable tangles.

> "Much of the adult anxiety around a whole-body approach to math learning can be overcome by knowing that the teacher's role is one of facilitator, not expert mover."

Overall, I think much of the adult anxiety around a whole-body approach to math learning can be overcome by knowing that the teacher's role is one of facilitator, not expert mover. I've thought deeply and long about the elements of a moving math classroom (whether in a dance system or at body-scale) so that math can be explored in a meaningful way. To create and facilitate a meaningful context in which children can think deeply and engage in mathematical sense making with their bodies, the following elements should be present:

- The activity explores one or more math concepts at a new scale.
- The math-and-movement lesson provides a structure in which students make choices, converse, collaborate, and reflect verbally on what they did, how they did it, and what they noticed while they were engaged in whole-body activity.
- The body activity is focused on mathematical sense making, not mnemonics (memorization), often through efforts to solve a physical or moving-scale challenge of some kind, and not on illustrating a math idea as it is typically represented on the page.
- The teacher is the facilitator of the activity, pacing, and discussion, supporting learners and their collaborative relationships as they work and, later, connecting the mathematics as experienced in the moving challenge to a new mode or new contexts.
- Students experience the activity at the center of the action and as observers of others' work, providing needed perspective to reflect on the mathematics in question.

- The math-and-movement should be explicitly connected with the same math idea as it is experienced in other scales, contexts and/or modes, often through some kind of written or schematic documentation (like a map), or a table to organize the results of the moving math challenge or investigation.

Many mathematics teachers will have encountered—often, by means of a well-meaning friend—the image of a stick figure posing its arms in the shape of various graphs, with the function (sin(x), x^2, 1/x, and more) written below each figure. In Math on the Move, you suggest that this is an example of "less meaningful" math movement. There is clearly more to Math on the Move, *then, than simply "moving a part of the body or being on one's feet" (Rosenfeld, 2016, p. 10).*

Could you expand on what you mean by "meaningful" and "less meaningful" math movement?

Those stick figures are primarily focused on memorization, which is not necessarily the strongest or most meaningful use of the human body in a math investigation. I've always thought that a more meaningful body-based activity would be to turn those graph shapes into physical pathways and then investigate how different combinations of graphs create beauty or chaos! Even if you don't want to to turn it into a dance activity, my overall point is that both math and dance utilize the ideas of space, time, and motion. Whole-body math learning is not about re-representing math ideas we see on the page. Instead, we can learn new things from exploring those ideas with our whole bodies. The math idea remains the same, but looking at it from a new angle can bring new perspectives to children, teens, and and adults alike. That's where the meaning, and learning, is created.

"Whole-body math learning is not about re-representing math ideas we see on the page. Instead, we can learn new things from exploring those ideas with our whole bodies."

Your work has focused on primary- and middle-grade-level mathematical concepts that can be expressed through whole body learning activities. Have you considered how Math on the Move might be brought into the secondary classroom? If so, what kinds of higher-level mathematical ideas might be explored in this way?

Well, different people see different mathematics in Math in Your Feet depending on their perspective and experience of "what math is." For example, after reading *Math on the Move* David Butler (@DavidKButlerUoA), an Australian mathematician at the University of Adelaide wrote: "The dance moves within the tiny square spaces [used in Math in Your Feet] are abstract mathematical ideas that are explored in a mathematical way. We ask how the steps are the same or different from each other, identifying various properties that distinguish them. We investigate how these new objects can be combined and ordered and transformed. We try out terminology and notation to make our investigations more precise and to communicate both current state and how we got there. These are all the things we pure mathematicians do with all our functions, graphs, groups, spaces, rings and categories. The similarity of [the dance work in Math in Your Feet] to pure mathematical investigation is striking."

So, if you want to investigate pure mathematics, give Math in Your Feet a try!

Barring that, in secondary and middle school classrooms we can also think about transformational geometry, which can be explored in meaningful ways at body-scale. Scott

Steketee and Daniel Scher have written an article published in the NCTM journal Mathematics Teacher titled "Connecting Functions in Geometry and Algebra," and I think their approach would translate very well to a body-based adaptation of that investigation.

Max Ray-Riek (@maxrayriek) of the Math Forum and Michael Pershan (@mpershan) have investigated the complex plane using an open, body-scale number line. Max has also advised me on middle and high school adaptations for the Rope Polygon activity from *Math on the Move*. For middle school classrooms, you might also consider Math in Your Feet pattern creation in a coding context.

Lastly: Where can mathematics teachers learn more about engaging their students in whole-body learning?

My book *Math on the Move: Engaging Students in Whole Body Learning* (Heinemann, 2016) is a thorough resource of information about the #movingmath approach, including 40 videos of the Math in Your Feet classroom action.

The Math on the Move Facebook book group is a resource for conversation with others starting out with a body-based approach to math learning and the potential for a book study and discussion. In addition, the Math on the Move book blog is an ongoing resource for learning more about whole-body math learning.

And, although I'm probably best known for my work with math and dance, the things we think about and do in a moving math context also translate to the art making modality of paper, glue, straws, color, Cuisenaire rods, and tape! To check out examples of my math art activities you can go to my blog Math in Unexpected Spaces (which has some examples of my work with scaled-up polyhedra) or visit the math art projects page on my website where you can read more about the projects I've created. Whatever you end up doing, I hope you have fun making math!

Interviewed by Ilona Vashchyshyn

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Supporting Productive Struggle with Communication Moves [2]

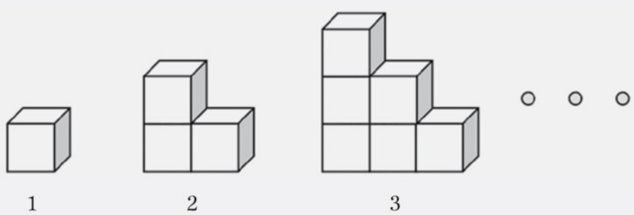
Ben Freeburn & Fran Arbaugh

Think about the following scenario. You have just launched a task called *The Staircase problem* (see Figure 1) during which students are to identify a pattern of growth and justify a rule for the pattern. As you walk around the classroom, you note that most students are making progress on the task. In one group, however, students are no longer talking to one another. Instead, they stare blankly at their desks—they appear to have hit a snag. How do you determine what students are thinking? How do you support the students to make progress with the task without taking over their thinking?

Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014, p. 10) contains eight research-informed teaching practices that have been shown to support students' mathematical thinking and learning. Two teaching practices that a teacher might draw on for the scenario above are to *elicit and use evidence of students' thinking and support students' productive struggle in learning mathematics*. Through enacting these two teaching practices, teachers can support students in ways that do not take over the thinking of the students. Such communication involves teachers determining how students are thinking mathematically and, in the moment, being able to respond in a way that supports students building on their thinking.

The Staircase Problem
Adapted from Annenberg Learner's Teaching Math:
A Video Library, 9–12

Look over the following sequence of figures. (The small cubes used to build each of the figures measure one inch along each edge.)



Complete the table below.

Height of Staircase	1	2	3	4	5	6	7	8	9	10
Number of Blocks										

How many blocks would you need to build a staircase that has a height of 20? 50? 100? n?

Fig. 1 The Staircase problem (Annenberg Learner n.d.) is an example of a high-level task.

In this article, we share a set of communication moves that teachers can use to support efforts to *elicit and use evidence of students' thinking and support students' productive struggle in learning mathematics* and then illustrate how a teacher can implement such moves while working with students.

Assessing Questions, Advancing Questions, and Judicious Telling (AAT)

The three communication moves discussed in this article are (1) assessing questions, (2) advancing questions (Smith, Bill, & Hughes, 2008), and (3) judicious telling (Lobato, Clarke, & Ellis, 2005). Assessing questions enable teachers to find out what students are

[2] Reprinted with permission from "Supporting Productive Struggle with Communication Moves," *Mathematics Teacher 111*(3), copyright 2017 by the National Council of the Teachers of Mathematics (NCTM). All rights reserved.

thinking and understanding during a lesson (Smith, Bill, & Hughes, 2008). Some of these questions might be hip-pocket questions—general questions that a teacher can keep in her or his pocket and use to elicit student thinking. Assessing questions, which are more specific, can aim to discover how students are thinking about a mathematical idea, how to evaluate students' capabilities, and how to encourage students to share their thinking publicly (Freeburn, 2015). Using assessing questions allows teachers to elicit student thinking.

Teachers use advancing questions to build on and extend students' current mathematical thinking toward the mathematical goal of a lesson (Smith, Bill, & Hughes, 2008). More specifically, these questions guide students to new or different ways of thinking, scaffold students' engagement in mathematical practices, and scaffold students' thinking about new mathematical relationships (Freeburn, 2015). Advancing questions are a way that teachers can use students' thinking and support students' productive struggle as they work toward the mathematical goal of the lesson.

Another way that teachers can support students' productive struggle is by using judicious telling, which involves teachers initiating ideas with students in a way that does not take over the students' thinking (Lobato, Clarke, & Ellis, 2005). More specifically, judicious telling can involve revoicing students' contributions to highlight an important mathematical idea, redirecting students to a solution pathway they are unable to find on their own, clarifying directions or contexts in mathematical tasks, and conveying terminology for students' mathematical ideas (Chazan & Ball, 1999; Freeburn, 2015).

A Teacher's Use of AAT in Mathematics Teaching

Baker's lesson preparation around the Staircase problem was foundational for her productive use of AAT. The Staircase problem is representative of the types of high-level tasks (Smith & Stein, 1998) that Baker regularly selects to provide her students with opportunities to think mathematically. Baker planned to support students to reach two specific mathematical goals: (1) conjecture an explicit rule for the number of blocks in a staircase of any given height (where h is the height of the staircase and n is the number of blocks); and (2) justify the explicit rule using connections among the diagram, the table of values, and the symbolic representation. Also during the planning phase, Baker solved the task in a number of ways to help her ask advancing questions that were tightly connected to students' initial mathematical explorations. Figure 2 represents one of four approaches to the task she developed during the planning stage; we chose to include this approach to support readers in understanding the classroom episodes below.

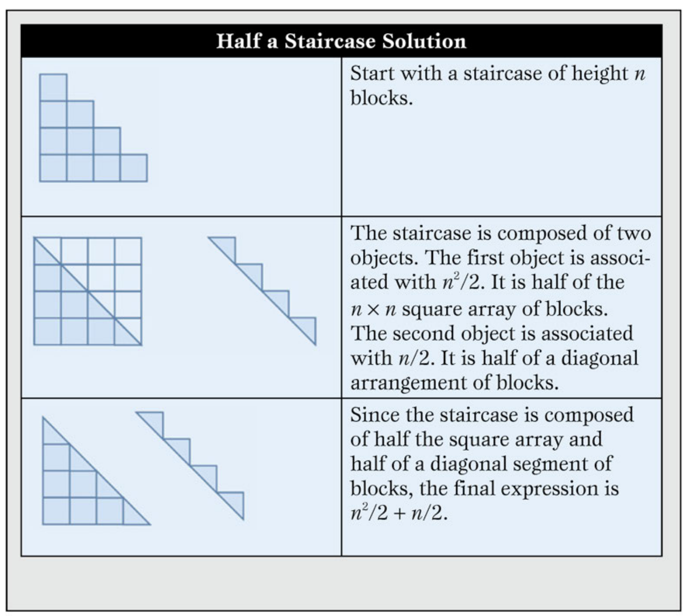


Fig. 2 Baker devised this approach to the Staircase problem during her planning for the lesson.

The lesson began with Baker launching the Staircase problem with an explanation of the mathematical goals for the task and taking time for students to ask any clarifying questions. The illustration of AAT that follows is based on Baker's three conversations with one of the groups of students: Daniela, Nick, and Stefan (all pseudonyms).

The first time Baker approached this group, she noticed all three were sitting silently, staring at their papers and no longer working on the task. She decided to use hip-pocket assessing questions to determine the students' progress on the task, asking such questions as, "How are you doing?" and "Do you feel as if you may have known this [rule] at one point?" Baker's initial assessing questions allowed her to gather *some* information about the students' thinking—the mathematical ideas of the task seemed familiar to them, but they could not remember their prior experiences with the ideas. At this point, however, Baker did not have enough information to be able to support them to move toward the mathematical goal of the lesson. So, Baker asked additional assessing questions: "OK, tell me what you are seeing right now. What have you been thinking about?" In response to Nick sharing his thinking that for each successive staircase, you add a column of blocks to the previous staircase, she asked, "Why does it make sense that you are adding the next height?" This use of assessing questions allowed Baker to see that the students were thinking recursively about the pattern of the number of blocks in the staircases, something that she knew might happen and had planned for.

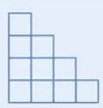
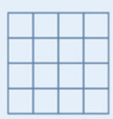
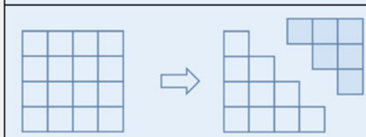
Illustration of Stefan's Explanation	Stefan's Response to Baker's Assessing Question
	Stefan begins his explanation with a staircase height of four blocks.
	"The blocks help, but pictures help me more. If you just complete this staircase, you have sixteen blocks" [he "completes" the staircase to create a four-by-four square].
	"You are taking out six of them to get the one that you need" [He "removes" a set of blocks to arrive at a staircase height of four].

Fig. 3 Stefan responded to an assessing question.

Since Baker wanted to support students' productive struggle with the problem, she decided to use judicious telling in a way that built on and extended the students' thinking. First, in response to students explaining two possible solution paths, Baker said, "That's two ways of thinking about what's going on in the mathematics of the problem." Recognizing that those two ways of thinking were recursive in nature, she provided the correct mathematical terminology by stating, "That kind of thinking is recursive." She then stated that she did not want students to pursue this approach for now because a goal of the lesson was to find an explicit rule. Next, in response to learning that students were working solely with the table of values, Baker stated, "Why not change your viewpoint a little bit and look at the relationships using the staircase figures as opposed to the numbers? See if that helps a little bit." Baker used the judicious telling to direct the students to use a different type of representation—the physical blocks used to build staircases. Baker then walked away to consult with other groups.

Baker used a similar sequence of assessing questions and judicious telling during her next conversation with this group. When she returned to talk to the three about their progress, she used a hip-pocket assessing question and determined that the students were having some difficulty in using the blocks to think about a pattern. She also noticed something that Stefan had sketched on his paper that made her think that he was headed in a productive direction (see Figure 3), and she said, "I think Stefan is thinking about something. Stefan, will you share with us what you are thinking about?" (assessing question).

After Stefan shared his thinking with the group, Baker stated, "So, think about Stefan's idea for a few minutes." (judicious telling). As Baker walked away, Daniella and Nick began talking with Stefan about his approach.

When she made her way back for a third visit to this group, Baker began the conversation with another hip-pocket assessing question to elicit the students' progress with Stefan's complete-thestaircase idea. She heard from them that Stefan's approach was still recursive and did not lead to an explicit rule. They then explained that they went back to focusing on the square, and from responses to a set of assessing questions, Baker recognized that the students were generating ideas that were significant for constructing a generalized rule similar to the Half a Staircase solution (see Figure 2), but they needed support in focusing their thinking about these ideas. Hence, she decided to ask a sequence of advancing questions to extend their thinking toward an explicit rule (see Figure 4).

Note that Baker used her first advancing question to focus students on the square they had drawn and the operation of dividing by two. Nick's response that the physical blocks would have to be cut initiated a subsequent set of advancing questions in which Baker oriented students to a pictorial diagram to extend their thinking about how the operation of dividing a square on the diagonal related to a corresponding, complete staircase. Her last advancing question connected the ideas the group generated and aimed the students at constructing the generalized rule relating the height of a staircase to the number of blocks in the staircase. Ultimately, the students constructed an explicit rule, $h = n^2/2 + n/2$, and an argument for the rule similar to the Half the Staircase solution.

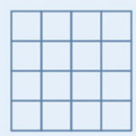
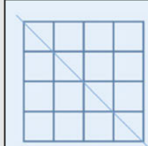
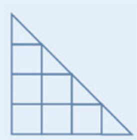
Group's Diagrams	Conversation between Baker and the Group
	*Baker*: What are all the different ways that you could take this figure (a four-by-four square) and divide it by two? What would that look like? (Advancing Question) *Nick*: Wouldn't we have to split a cube in half in order to divide it evenly? Group members agree.
	*Baker*: Is that not possible? What if you drew a picture? (Advancing Question) *Nick*: You could draw a line right down the diagonal and split each of the blocks on the diagonal into two.
	*Baker*: What has that done when you think about the square? (Advancing Question) *Nick*: It's half of *n*-squared. *Baker*: Why is half of *n*-squared not representative of your original four-by-four staircase? (Advancing Question) *Stefan*: That's not the whole thing. You are missing one and a half from *n*. *Nick*: You are missing half of a block from *n* blocks. *Stefan*: *N* over two. *Baker*: So, if you were able to add back in half of a block from n blocks on the diagonal, would you get the original figure back? Think about that. (Advancing Question)

Fig. 4 Baker used advancing questions during the final conversation.

Conclusion

Two considerations are important to discuss about the use of AAT. First, begin thinking about AAT during the planning of a task. A good tool to use in planning for this type of instruction is the Thinking Through a Lesson protocol (Smith, Bill, & Hughes, 2008), which includes several planning activities, including selecting tasks that require a high level of cognitive demand, identifying the mathematical goals of the task, solving the tasks in multiple ways, and crafting questions that can be used during instruction. The second thing to consider is that asking assessing questions comes before asking advancing questions and using judicious telling. Baker's persistence in using hip-pocket assessing questions followed by assessing questions that were more specific enabled her to situate students' thinking in relation to the mathematical goal of the task and to the solution paths she had identified during planning. Once she recognized how students were thinking, she could use advancing questions and judicious telling *on the basis of how students were thinking* to support their productive struggle. Note that Baker supported different solution paths with other groups, also *on the basis of how they were thinking*, which she found out when she asked assessing questions.

AATs are not meant to be a script for conversations with students. Instead, AATs represent a tool that teachers can use to engage in important mathematics teaching practices (NCTM, 2014). It can be a challenge to respond to a student's' in-themoment thinking, but we believe that such a challenge becomes less daunting when teachers talk with students about how they are thinking about a task. Using assessing questions, advancing questions, and judicious telling allows teachers to have access to how student are thinking and thus to support students' learning of mathematics.

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Exploring Measurement[1]

Anamaria Ralph

As an entry point to learning about measuring length, I set up a provocation for my Kindergarten students with the following questions:

Who is the tallest?
Who is the shortest?
How can we find out?

I also placed some non-standard units for length (cubes and Kapla planks) as well as standard units of length (measuring tapes) for added exposure.

As students made their way into the classroom, I sat back and observed. I was excited to see what they would do and use to answer the questions.

A few students read the questions out loud to everyone. They were very excited and quickly started to stand back to back to figure out their height in comparison to their friends. I was surprised that they took a different solution to figuring out the questions than I had anticipated. They didn't use any of the materials I set out. Instead, they worked as a group sorting and lining themselves from tallest to shortest in a line.

[1] A prior version of this article was published on December 7, 2014 on Anamaria's blog, *Wonders in Kindergarten* (http://wondersinkindergarten.blogspot.ca/2014/12/exploring-length-measurement.html). Reprinted with permission.

I was very proud of the way they self-managed and used their critical thinking skills to find the answers to the questions without any guidance.

But now, I wondered how to motivate them to use the materials to measure length. They were no longer interested in measuring each other, since they had answered the questions posed to them.

I decided to add more to the existing provocation and use their interest of building tall structures as a possible motivator for them to use the measuring materials. I posed new questions:

How tall is your structure?
How can you find out?



They enjoyed using the non-standard units of measurement, but were particularly drawn to the rulers and measuring tapes available. I guided their learning by demonstrating the correct way to use a ruler and measuring tape. They wondered why there were numbers on both sides, so I showed them and explained the difference in spacing between the lines of one side versus the other. I didn't go into deep conversation about millimeters, centimeters, and inches, but asked them to choose which side they wanted to use to measure their structures.

After a few days, I noticed that the interest in measuring structures was fading, maybe it wasn't meaningful to them or they didn't see the purpose for it. Instead students started measuring random objects and each other. I took this opportunity to once again change the provocation questions:

What can you measure?
What can you use?

During a group discussion on measurement, H. S. came up with a wonderful theory: "If you're tall you have a bigger shoe, and if you're smaller, you have a smaller shoe."

I asked the students: How we can find out if H. S.' theory is correct? They began measuring their shoes, and then each other to find out. I felt that this was a perfect opportunity to discuss the importance of using the same measuring material (constant non-standard unit) when comparing length. As Marian Small writes,

> Just as it is important for students to have opportunities to measure a length using different units, they need to measure different lengths using the same unit. This will give them measurement data that they can readily compare. (2009, p. 93)

I started thinking about the exposure my students have had to measurement thus far. I couldn't help but feel like something was missing. Something I try my best to do as an educator is make a link between learning in the classroom and its applicability to students' daily lives and the world around them. So, I decided to ask the students the simple question: "Why do we measure?"

The discussion that followed:

> "So we know how tall you are." F. D.
>
> "My cousin is five and I am four and I am taller than him." Z. G.
>
> "We measure how tall we are." S. T.
>
> "So we know how tall we are and how many bones we have because so our doctor knows how much we have grown." S. C.
>
> "If some people are working and are putting in a new door, they have to measure how tall the door is so they know if the door will fit in the doorway." C. C.
>
> "They use a measuring scale that measures height." M. O.
>
> "How do we measure the weight of the class?" W. E.
>
> "When my brother was in my mom's tummy, he came out before me, but I'm taller than him." E. E.
>
> "So people know how big you are cause people want to know how tall you are." K. W.
>
> "You can measure their weight with a measurer. It has a line thing and it's at my doctor's. S. C.
>
> "He wants to know if you're too much weight or not enough weight. It's called a scale. It touches your head and a silver thing goes down." J. K.

"There's different kinds of scales because at my home I do have a scale and you just step on it and it tells you how heavy you are." H. S.

"B (brother) was born first and he's taller than me." M. S.

"I think if you are born first, whoever is born second they are bigger." E. E.

"It depends on who is born first in your family. A (sister) is six and she was born first and she's tallest." M. O.

"I was born first and I'm taller than my brother." C. C.

"But Z. G. is four and I'm four but he's taller?! M. S. is four but he's taller than me still!" E. E.

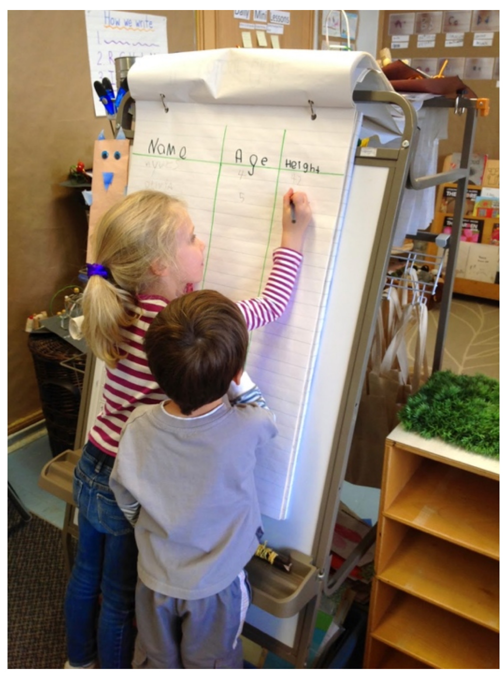
"J. S. is five and I'm four and I'm taller." O. M.

"D. A. is five and Z. G. is four and Z. G. is taller!" W. E.

"Maybe people get born at not the same time, that makes a difference of how old people are?" O. S.

The students' responses gave me a better idea of the knowledge they had about measurement. They seemed to be aware that measuring also entailed mass and time, not just length.

The discussion of age and its relation to height also continued to be debated by the students. This led to the co-creation of a chart displaying their name, age, and height.



After the chart was completed, we studied the data together. Students noticed that being older didn't necessarily mean they were taller. Someone who is older can be shorter than someone who is younger.

> "There is examples for both theories! It tells me maybe both are right!" Z. G.
>
> "Maybe families are different sizes because not everyone is born at the same time." O. S.
>
> "Maybe different families get different things?" M. O.
>
> "I think it depends on how much you eat and how much you grow." H. S.

I am glad I asked the question, "Why do we measure?". It provided insightful responses and generated a purposeful investigation that was meaningful to the students and motivated them to further develop their measuring skills.

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Anamaria Ralph is a Kindergarten teacher for the Toronto District School Board. She teaches at Maurice Cody Public School in Toronto, Ontario. She has taught Kindergarten for nine years and still regards each year as an exciting adventure where many wonders, explorations, and investigations take place. She is passionate about inquiry and play-based learning, and is greatly inspired by the Reggio approach to learning. She shares her students' learning with families and other educators on her classroom blog, www.wondersinkindergarten.blogspot.ca, and can also be reached on Twitter at @anamariaralph.

In this column, you'll find information about upcoming math education-related workshops, conferences, and other events. Some events fill up fast, so don't delay signing up!

For more information about a particular event or to register, follow the link provided below the description. If you know about an event that should be on our list, please contact us at thevariable@smts.ca.

Within Saskatchewan

Workshops

Multi-Graded Mathematics
February 9, 2018, Moose Jaw, SK
Presented by the Saskatchewan Professional Development Unit

How do you address all of the needs within your combined grades mathematics classroom? By looking at themes across curricula, teachers can plan for diverse needs and address outcomes at two grade levels without having separate lesson plans. Curricular through lines and planning templates will be shared that are helpful for identifying how concepts grow over the grades, so that you can build a learning continuum within your instruction.

Head to www.stf.sk.ca/professional-resources/events-calendar/multi-graded-mathematics

Early Learning With Block Play – Numeracy, Science, Literacy and So Much More!
March 2, 2018, North Battleford, SK
Presented by the Saskatchewan Professional Development Unit

This is a one-day workshop for early learning educators from prekindergarten, kindergarten and Grade 1 to discover and deepen their understandings around the many foundational skills that children develop during block play. Through concrete, hands-on activities, participants will experience and examine the many connections between block play and curricular outcomes, and the current research on the topic. Participants will have opportunity to plan for block play and for creating a network of support.

Head to www.stf.sk.ca/professional-resources/events-calendar/early-learning-block-play-–-numeracy-science-literacy-and-0

Technology in Mathematics Foundations and Pre-Calculus
May 7, 2018, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

This workshop is designed to have math foundations and pre-calculus teachers experience a variety of technology tools that allow students to represent and visualize mathematics concepts. Tools highlighted are useful for students to explore, learn, communicate, collaborate and practice in order to enhance their understanding of mathematics in secondary mathematics.

Head to www.stf.sk.ca/professional-resources/events-calendar/technology-mathematics-foundations-and-pre-calculus-0

Number Talks and Beyond: Building Math Communities Through Classroom Conversation
May 11, 2018, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Classroom discussion is a powerful tool for supporting student communication sense making and mathematical understanding. Curating productive math talk communities required teachers to plan for and recognize opportunities in the live action of teaching. Come experience a variety of classroom numeracy routines including number talks, counting circles, quick images and more. Take math conversations to the next level by strengthening your skills as a facilitator of classroom discourse and student thinking.

Head to www.stf.sk.ca/professional-resources/events-calendar/number-talks-and-beyond-building-math-communities-through-0

Saskatoon Summer Initial Accreditation Seminar
March 8-9 & April 12-13, 2018, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Accreditation seminars are offered to enable qualified teachers to become accredited. Accreditation is the process by which qualified teachers are granted the responsibility of determining the final mark or standing of the students in a specified Grade 12 (level 30) subject or subjects. The Accreditation seminar provides an opportunity for teachers to challenge, extend, enhance and renew their professional experience with an emphasis on assessment and evaluation.

Head to www.stf.sk.ca/professional-resources/events-calendar/saskatoon-accreditation-–-initial

Saskatoon Summer Renewal/Second Accreditation Seminar
March 9 & April 13, 2018, Regina, SK
Presented by the Saskatchewan Professional Development Unit

Accreditation seminars are offered to enable qualified teachers to become accredited. Accreditation is the process by which qualified teachers are granted the responsibility of determining the final mark or standing of the students in a specified Grade 12 (level 30) subject or subjects. The seminar provides an opportunity for teachers to challenge, extend, enhance and renew their professional experience with an emphasis on assessment and evaluation.

Head to https://www.stf.sk.ca/professional-resources/events-calendar/regina-accreditation-—-renewalsecond

Beyond Saskatchewan

NCTM Annual Meeting and Exposition
April 25-28, 2018, Washington, DC
Presented by the National Council of Teachers of Mathematics

Join more than 9,000 of your mathematics education peers at the premier math education event of the year! Classroom teachers, administrators, math coaches, supervisors, college professors, and preservice teachers can all benefit from the sessions and learning at this event. Improve your knowledge and skills with high-quality professional development and hands on activities; gain insights by connecting and sharing with like-minded educators; and learn from industry leaders and test the latest educational resources.

Head to http://www.nctm.org/Conferences-and-Professional-Development/Annual-Meeting-and-Exposition/

Online Workshops

Education Week Math Webinars
Presented by Education Week

Once a month, Education Weekly has a webinar focusing on math. They also host their previous webinars on this site. Previous webinars include Formative Assessment, Dynamic vs. Static Assessment, Productive Struggling and Differentiation.

Past webinars: http://www.edweek.org/ew/webinars/math-webinars.html
Upcoming webinars:
http://www.edweek.org/ew/marketplace/webinars/webinars.html

Did you know that the Saskatchewan Mathematics Teachers' Society is a **National Council of Teachers of Mathematics Affiliate**? When registering for an NCTM membership, be sure to support the SMTS by noting your affiliation during registration.

NCTM | AFFILIATE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Tangents

Extracurricular Opportunities for K-12 Students

This column highlights local and national extracurricular opportunities for K-12 students interested in mathematics, including collaborative, individual, online, and in-person challenges, contests, and camps. For dates, registration procedures and applications, and other information about the contests listed, please head to the contest websites, included below the descriptions. If we have missed an event that should be on our list, please contact us at thevariable@smts.ca.

If you are looking for contests available in other provinces, head to the Canadian Mathematical Society website (cms.math.ca/Competitions/othercanadian). The CMS also maintains a list of resources for students who are looking to build their problem-solving skills and succeed in competitions: see cms.math.ca/Competitions/problemsolving.

Local Events and Competitions

Canadian Math Kangaroo Contest
Spring

The purpose of this competition is to introduce youngsters from Grade 1 to Grade 12 to math challenges in a fun and enjoyable way, thus inspiring their further interest and advancement in mathematics. The competition is held yearly in more than 40 Canadian cities. Students may choose to participate in English or in French.

Students in Saskatoon may write the contest at Walter Murray Collegiate; students in Regina may write the contest at the University of Regina. Contact Janet Christ at christj@spsd.sk.ca (Saskatoon) or Patrick Maidorn at patrick.maidorn@uregina.ca (Regina).

Head to https://kangaroo.math.ca/index.php?lang=en

University of Regina Regional Math Camp
Spring

The Math Camp is a full-day event for students in Grades 1 through 12 who are interested in exploring the infinite frontier of mathematics beyond the school curriculum. Participants are guided by professors and students through a fun and enriching day. In a variety of grade appropriate sessions, students will explore mathematical topics in hands-on activities, games, puzzles, and more.

Head to https://www.uregina.ca/science/mathstat/community-outreach/mathcamp/

National Competitions

Canadian Computing Competition
Written in February
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Canadian Computing Competition (CCC) is a fun challenge for secondary school students with an interest in programming. It is an opportunity for students to test their ability in designing, understanding and implementing algorithms. Students may compete in one of two levels: Junior Level – any student with elementary programming skills; Senior Level – any student with intermediate to advanced programming skills.

Head to http://www.cemc.uwaterloo.ca/contests/computing.html

Canadian Team Mathematics Contest
Written in April
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Canadian Team Mathematics Contest (CTMC) is a fun and challenging competition for teams of 6 secondary school students in any combination of grades. One teacher and groups of six students participate at their own school over 3 consecutive hours.

Head to http://www.cemc.uwaterloo.ca/contests/ctmc.html

Caribou Mathematics Competition
Held six times throughout the school year

The Caribou Mathematics Competition is a worldwide online contest that is held six times throughout the school year. Each of these days, five contests are offered, one for each of the grade levels 3/4, 5/6, 7/8, 9/10 and 11/12 and each one in English, French and Persian. Available in English, French, and Persian.

Head to https://cariboutests.com/

Euclid Mathematics Contest
Written in April
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Euclid Mathematics Contest is an opportunity for students in their final year of secondary school and motivated students in lower grades to have fun and to develop their mathematical problem solving ability. Most of the problems are based on the mathematical curriculum up to and including the final year of secondary school. Written in April.

Head to http://www.cemc.uwaterloo.ca/contests/euclid.html

Fryer, Galois, and Hypatia Mathematics Contests
Written in April
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Fryer, Galois and Hypatia Math Contests are an opportunity for students to write a full-solution contest. They are fun way to develop mathematical problem solving skills through a written mathematical activity. For students in Grades 9 (Fryer), 10 (Galois) and 11 (Hypatia).

Head to http://www.cemc.uwaterloo.ca/contests/fgh.html

Gauss Mathematics Contests
Written in May
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Gauss Contests are an opportunity for students to have fun and to develop their mathematical problem solving ability. For all students in Grades 7 and 8 and interested students from lower grades.

Head to http://www.cemc.uwaterloo.ca/contests/gauss.html

Opti-Math
Written in February
Presented by the Groupe des responsables en mathématique au secondaire

A French-language mathematics challenge for secondary students who would like to exercise and develop their problem-solving skills.

Head to http://www.optimath.ca/index.html

Pascal, Cayley, and Fermat Contests
Written in February
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Pascal, Cayley and Fermat Contests are an opportunity for students to have fun and to develop their mathematical problem solving ability. For students in Grades 9 (Fryer), 10 (Galois) and 11 (Hypatia).

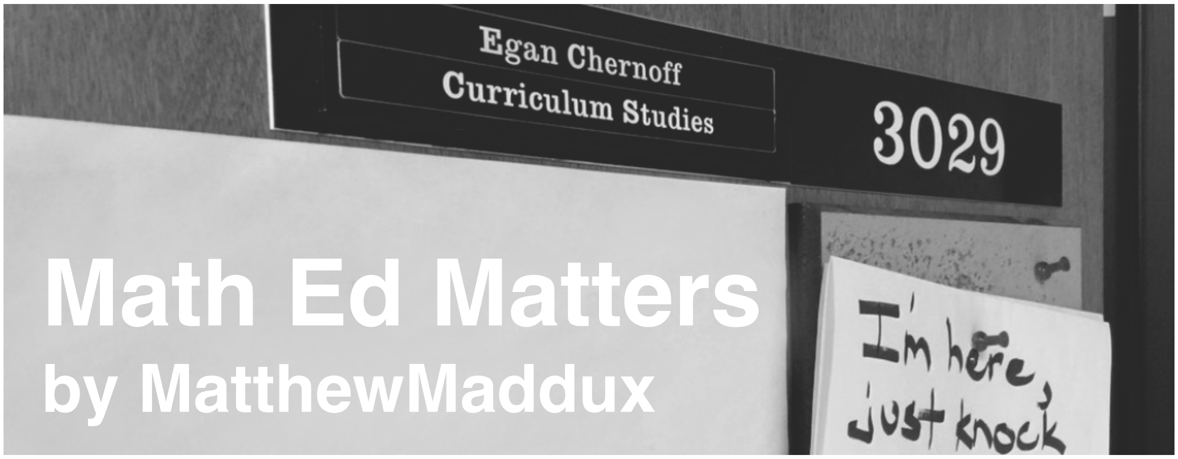
Head to http://www.cemc.uwaterloo.ca/contests/pcf.html

The Virtual Mathematical Marathon
Supported by the Canadian National Science and Engineering Research Council

The virtual Mathematical Marathon has been developed by an international team of mathematicians, mathematics educators and computer science specialists with the help of the Canadian National Science and Engineering Research Council and its PromoScience program.

The main activity is a competition allowing students to enjoy solving challenging mathematical problems all year around. Students can join the game at any time and at no cost, simply creating an individual profile with an individual username and a password. Available in French.

Head to http://www8.umoncton.ca/umcm-mmv/index.php



Math Ed Matters by MatthewMaddux *is a bimonthly column telling slightly bent, untold, true stories of mathematics teaching and learning.*

A Spontaneous Celebration of Learning: Part I

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For over a decade now, I've been teaching math methodology courses (courses that cover content, strategies, and approaches associated with the teaching and learning of mathematics) to prospective elementary and secondary school math teachers. Every semester, the first day of class has been exactly the same. As I find my way to the front of the room, an audible murmur arises, which stems from the students' realization that, yes, I actually am the instructor for the course.

After I assure the prospective teachers that, yes, I do have 'school cred,' I ask the class a question that I learned from a former colleague: "By a show of hands, who here cannot read?" To date, not a single person has raised their hand in response. As a follow-up question, I ask, "By a show of hands, who here cannot do math?" The response—even after a brief discussion about the difference between "can not" and "cannot"—is markedly different. Over the years, lots and lots (and lots) of people have raised their hands to declare to me, and to the others in the room, that they (can not and) cannot do math. The number of hands that are raised varies from year to year, but, on average, represents approximately 30% of the room. But while this number is interesting in and of itself, I find that *how* the hands are raised is even more intriguing.

Specifically, hands that are raised in response to this question are not cautiously or shyly raised—quite the opposite. The arms, hands, and even the fingers of those who raise their hands are unmistakably and purposefully raised straight (and I mean straight!) up in the air. Picture, for a moment, the raised hand of an elementary school student who so desperately wants to grab their teacher's attention because they have the answer (or have to go to the washroom). Now, picture this same purposeful, brazen gesture being made by a group of mostly twenty-something prospective teachers when I ask, "Who here cannot do math?" For many, the raised hand signifies a badge of honour, even of defiance. It is no surprise, then, that what happens next has been described by previous students, especially those who raised their hands to signal their purported inability to do math, as follows: "It's as if my worst nightmare just came true."

"Please put everything away except for a pen or a pencil," I instruct the students. Never have 11 words, I wager, struck more fear into the hearts and minds of prospective elementary and secondary school math teachers—especially on the first day of class.

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Why? Well, having at this point attended school for the greater part of their lives, the students are by now well aware that "Please put everything away except for a pen or a pencil" is synonymous with a Pop Quiz. As the students get settled, the tension in the room becomes palpable. I try to alleviate this tension by letting everyone know that we are not 'really' having a Pop Quiz, but, rather, as the title on the top of the quiz indicates, a "Spontaneous Celebration of Learning" (a phrase I learned from Jim Mennie). I get some laughter. Further, should anyone ask if the quiz is hard, I reply with: "No. See, it's made out of paper." More laughter, which quickly dies down. You see, over the years, I've learned that the students' laughter has little to do with my feeble attempts to alleviate the pressure in the room; rather, the laughter that I hear is a manifestation of the prospective teachers' nervous energy and anxiety that stems from the harsh realization that they are, in fact, going to write a Pop Quiz during their first math methods class of the semester. In other words, their worst nightmare is about to come true!

Although the prospective elementary teachers do not know it at the time, the purpose of the quiz is not, in fact, designed to discern what mathematics they know and do not know. Rather, the purpose of the quiz is quite simple: math-anxiety-inducing pageantry.

Having done this for a number of years now, I have come to fully embrace my role as the math-anxiety-inducer during our quiz. As I walk around and hand out the quiz (face down, of course), I make sure to sternly remind the students that: they must put everything away, except for a pen or a pencil; they are to write their name and student number on the quiz in the top left hand corner, as soon as I indicate that it is OK to turn the quiz over—not a second before!; they will be reminded when there are five, two, and one minute/s remaining to complete the quiz; they are not look at anyone's paper, except their own; there will be absolutely *no* talking during, or after, the quiz; they are to turn their quiz over after they are finished so as to deter other people from looking at their answers; they must put their pen or pencil down as soon as I say that times is up; and finally, that, yes, this will be for marks (spoiler: it never is!). However, the math-anxiety-inducing pageantry does not stop once the "Spontaneous Celebration of Learning" begins.

> "The laughter that I hear is a manifestation of anxiety stemming from the students' harsh realization that, in fact, their worst nightmare is about to come true."

Once the quiz starts is when I really start to shine. As the prospective elementary school teachers work, I make sure to visit the desk of each and every student and, in some instances, loom over them and look disapprovingly at the answers that they put down thus far. (In previous years, I would print the quiz on both sides of the paper, with the result that as students saw me coming their way, they would casually turn the paper to other side so that I would not see their answers. Of course, I made short work of that particular 'out' and now always print the quiz on just one side of the page.) Sometimes, just my movement around the room, highly conspicuous thanks to the squeaky shoes I wear for the occasion, is enough to heighten the anxiety that many prospective teachers are experiencing.

Now, as hard as it may be to believe, I am not a monster. Thanks to years and years of walking around the class during the quiz, I have realized that, although there is not a perfect correlation between the physical manifestations of math anxiety (e.g., grimacing, fidgeting, nail-biting) and the level of math anxiety an individual is experiencing, physical manifestations are, to borrow a poker term, a good tell. Knowing this, I interact differently with different students, depending on the tells I see. As a rule, those who exhibit less anxiety during the quiz are the individuals who I spend more time attempting to induce anxiety from by, for example, looming ominously over their quiz. On the contrary, those who are visibly shaken and disturbed are left alone.

> "Sometimes, just my movement around the room is enough to heighten the anxiety that many prospective teachers are experiencing."

Unbeknownst to the students writing the Spontaneous Celebration of Learning, the amount of time that I declared I would give for the quiz (10 minutes) does not coincide with how much time I actually give for the quiz (8 minutes). Interestingly, other than the handful of individuals who have looked at their watch during the quiz over the past decade, nobody keeps track of time. As a result, when I loudly announce "5 minutes remaining," everybody thinks that five minutes have already gone by, despite the fact that they have only been writing the quiz for three minutes. At the 5-minutes-remaining mark of the quiz, a collective murmur arises, coupled with a collective shift in everyone's seat. By comparison, the "2 minutes remaining" announcement has little impact on the room of prospective teachers. And finally, the "1 minute remaining" announcement divides the room. Certain individuals, with one minute remaining, double down during that final minute and try finish just one more question, re-revisit an answer from earlier, or give their quiz one more look-over to make sure everything is in order. Others take an entirely different approach: they give up. Actually, it is not so much that they give up, but rather that they finally admit defeat, enjoying the sense of relief that comes with knowing that the quiz is over and that there is nothing they can do about it. Most tellingly, those who retire at the one-minute-remaining mark are known for, what I call, the pen drop—an interesting phenomenon, which is also popular among another group of individuals writing the quiz.

Now, as an aside, I'm not that hip. ("Surprise, surprise," I hear you say.) However, in my continuous effort to try and relate to the world my students live in, I do try to keep tabs on what is cool. For example, I make an effort, despite my lack of interest, to watch the Oscars, the Emmys, the Grammys, and other shows of the like. After all, how else does one learn about 'twerking'!?

Among the more interesting phenomena that I have come across has its roots in rap and stand-up comedy. As you may be aware, after a particularly strong performance or set, certain individuals (with blatant disregard for any damage that may be incurred) drop the microphone onto the floor. Essentially, "dropping the mic" is a way for the performer to indicate they have just concluded an excellent performance. The mic drop, which has in recent years morphed into a full-blown meme, even has its analogue in the math classroom.

Given that, typically, nobody has a microphone in this setting (unless it's a lecturer talking to hundreds of calculus students), the microphone in the math classroom has its counterpart in the pen or the pencil that a student is using to write a test or quiz. Unlike the mic, the pen or pencil is usually not dropped onto the floor, but, rather, onto the desk, often as a conspicuous gesture of victory over the quiz in question. After all, how else would a student indicate to the other students in the room, when there is no talking allowed, that they have finished the quiz? Thus, instead of announcing their triumph over the assessment verbally, they drop the mic: that is, once they have finished writing the quiz, they let others know that they are done (read: have just had a good performance) by dropping the pen from a decent height (usually achieved by stretching the arm out at length) so that it makes a sound audible to the others in the room.

> "The 'mic' in the math classroom has its counterpart in the pen or the pencil that a student is using to write a test or quiz."

As I mentioned earlier, there are two instances in which prospective math teachers drop the mic during the Pop Quiz on day one of class. As you might now expect, prospective math teachers who finish my quiz before the others almost always drop the mic. This is especially true of prospective secondary school math teachers. In fact, the earlier an individual finishes the quiz, the higher the chances of them dropping the mic. In my experience, if a student finishes the quiz before I announce the five-minute mark, there is an almost 100% chance that they will fully extend their arm over their desk and drop their pen or pencil so that it makes a sound for others to hear. After the first few mic drops—the gesture is not restricted to the first person to finish—they begin to subside. (My favourite pen drop incident of all time was by an individual who had finished the quiz early and, of course, dropped the mic. At this point, I walked over, looked disapprovingly at some of their answers, then walked away. The student then picked the pen back up, worked on their answers for pretty much the remainder of the time remaining and, just for good measure, dropped the pen again.)

The pen drops pick back up again after my 1-minute-remaining announcement; in fact, there are more pen drops at and just after the one-minute-remaining mark than at any other point in the quiz. However, unlike those who finish the quiz early and use the pen drop to proudly announce to others that they are done, those who drop the pen or pencil at the 1-minute mark—after seven minutes (which they think is nine) of frustration, anxiety, anger, and other emotions—are indicating to others that they are, despite the fact that there is one more minute to go, giving up on the exercise. Whether an individual drops the pen(cil) early on in the quiz as a point of pride or at the one-minute-mark as an expression of "that's enough," the look on their faces once the pen has dropped is almost identical: resolve, mixed with relief, in the knowledge that the quiz, for them, is over.

> "Whether an individual drops the pen(cil) early on in the quiz as a point of pride or at the one-minute-mark as an expression of "that's enough," the look on their faces is almost identical."

"For those of you still writing the quiz [one last jab from me], time is up. Pencils down! I said, pencils down. When I say pencils down, I mean no more writing. *Pencils down*." You see, just because time has run out for the quiz doesn't mean that my anxiety-inducing pageantry is over, which even extends into the collection of the quiz. I offer the following set of instructions, which I imagine to be common in math classrooms around the world: "Please pass all of the quizzes from back to front, and then, people at the front, please pass the quizzes from left to right. Let's make sure that they're all facing up and the same direction." I then collect all the quizzes from the person sitting in the desk at the front (stage) right of the room. By now, the room is abuzz with students discussing (read: telling each other) their answers. However, the buzz in the room always dies down once people start to realize what I am doing at the front of the room.

Standing at the front, I make sure to flip through all the quizzes one last time to make sure that they are all facing the same direction and the same page is facing up. The purpose of this activity is two-fold. First, even if all the papers are facing up and in the same direction, I will take one of the quizzes and purposely turn it upside down, taking the chance to offer a disapproving look to the room. Second, my long, disapproving look gives me an opportunity to scan the room and get a sense of the atmosphere that I have created. Having conducted this "exercise" for nearly a decade, I have never experienced the same atmosphere from two different classrooms, but, with that said, I have noticed some patterns.

"At first, I was impressed that the activity had not unsettled them. But, looking back, I became less impressed and more concerned that young citizens leaving our schooling system believed in the same testing values that I had projected."

First, prospective secondary school mathematics teachers are more hardened to the quiz than prospective elementary school mathematics teachers. I suspect that this hardening stems from being more accustomed to the relentless, monotonous routine of homework-quiz-test-homework-quiz-test-[repeat] which continues, and even ramps up, as they move on from high school to university. In fact, one year, I gave the quiz to a class of students who seemed to be completely unfazed by—let me reiterate—a pop quiz on the very first day of class, from an instructor whom they had never met, in a course they knew little about, on material they had likely never encountered before, which was, to boot, for marks. At first, I was very impressed that the activity had not unsettled them. But, looking back, I became less impressed and more concerned that young citizens leaving our schooling system believed in (or perhaps, more accurately, had resigned to) the same testing values that I, the instructor, had projected and, as such, did not perceive me as a threat.

The atmosphere in a room full of prospective elementary school math teachers after a quiz on the first day of class is markedly different. While standing at the front the of room, (needlessly) turning over a few quizzes that are facing the right direction, just as I'm about to look up to a room of future elementary school math teachers who have just written a quiz on the first day of class, I have a good idea of what I'm about to see. For the most part, I expect a large number of people in the room—especially those who proudly raised their hands when I asked, "Who here cannot do math?"—to look spent, as though they have just finished running a marathon or have been in a car accident, or even some combination of the two.

Physically, these individuals are, at this point, still in the room, but really, they are somewhere else—you can see it in their eyes. When it comes to these particular individuals, I have no doubt: they have just lived through a traumatic experience. (In fact, there have a been a few instances where I was extremely concerned that I may have induced irreparable damage.) The quiz, that is, the psychological trauma that they have just experienced, appears, in some instances, to have damaged their psyche. At this point, you may be thinking, "Hold on a second: You said you weren't a monster." I'm not. Keep in mind that these future math teachers did *not* just live through a car crash. They are young, healthy, smart university students who have just written a short pop quiz (only 10 questions) under good lighting while sipping on expensive lattes in an air-conditioned building—far from an accident scene. Further, other students in the class—the majority of the students, in fact—are doing just fine.

Having flipped through all of the quizzes, now standing silently at the front of the room, I can sense that we have reached the breaking point. Me, I am exactly where I want to be. I even know what the students are thinking at this very moment. "Well, I'm off to the registrar's office as soon as I get out of here to drop this [expletive] class," decide some, or many, of the students in the room. (Fortunately, for me, my course is a requirement for their degree and, as a result, they are unable to drop the class.) Even those who are not thinking about dropping the class are likely thinking along a similar vein: "What have I gotten myself into here!?" It is at this point that I reveal to the class, just as I repeatedly assured you earlier, that I am not a monster. But it's a slow reveal.

> "Even those who are not thinking about dropping the class are likely thinking along a similar vein: 'What have I gotten myself into here!?'"

With the quizzes in one hand, my other hand reaches into my leather satchel (a little on the nose, yes) and pull out a letter-size manila envelope. After stuffing all of the quizzes into the envelope, I seal it. After sealing the envelope, I hand the sealed envelope full of quizzes to a student sitting in the front row of the classroom and ask that she or he sign their signature over the part of the envelope that has just been sealed. "This way, we will know if it has been tampered with or opened," I explain. Then, holding the sealed, signed envelope in my hand I let the class know that, currently, I am not able to mark their quizzes. After the collective sigh of relief, someone asks, "How come?" I can't give them the real reason right now—so I spin another yarn…

To be continued

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Saskatchewan Mathematics
Teachers' Society