The
Variable

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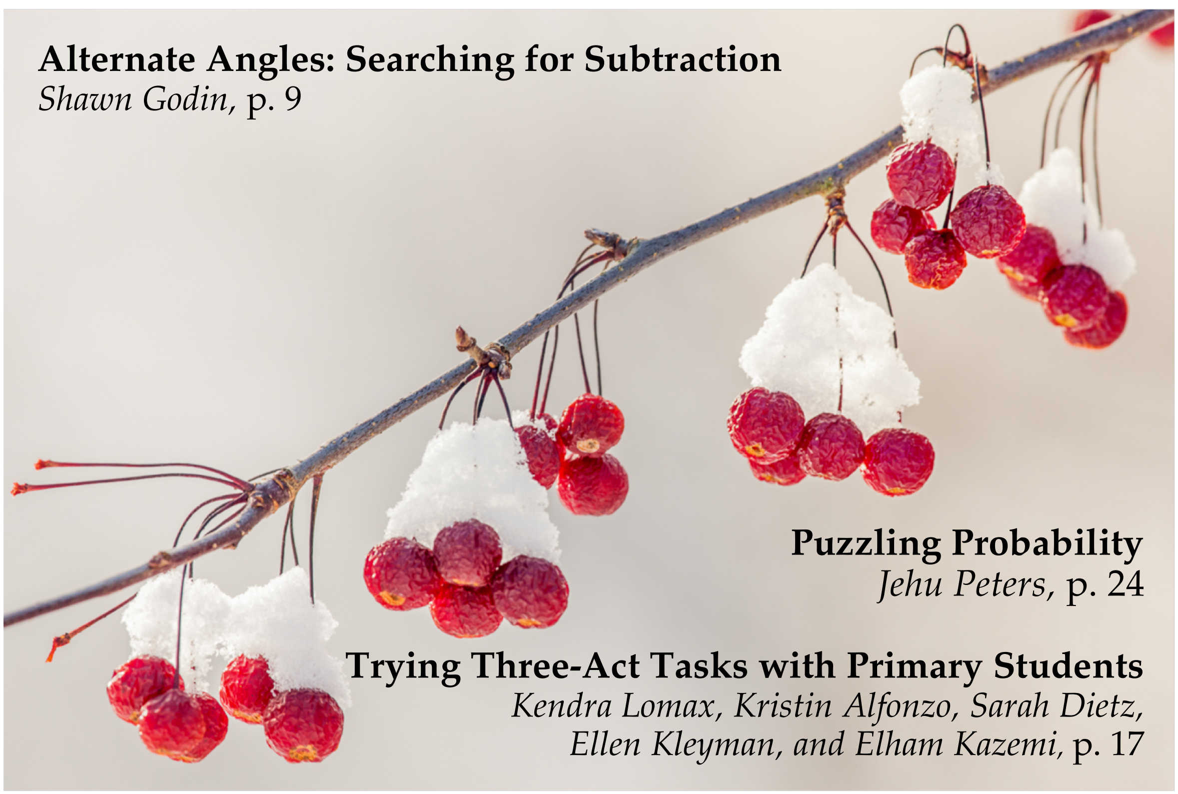

Saskatchewan Mathematics Teachers' Society

AFFILIATE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Cover Image

This month's cover photo, "Winter Berries" (2013), was taken by Barbara Friedman at the taken at the Cornell Laboratory of Ornithology in Ithaca, NY. Retrieved from www.flickr.com/photos/btf5/11119913084. See more of Barbara's work at www.flickr.com/photos/btf5/.

SMTS Executive (2016-2018)

The Variable

Notice to Readers

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Notice to Contributors

The Variable welcomes a variety of submissions for consideration from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants, teacher educators, researchers, and students of all ages, although we encourage Saskatchewan teachers of mathematics as our main contributors. Submitted material is assessed for interest, presentation, and suitability for the target audience.

Please submit articles by email to thevariable@smts.ca in Microsoft Word format. Articles should be limited to 3000 words or less; authors should also include a photo and a short biographical statement of 75 words or less. Accepted contributions are subject to revision. Editors reserve the right to edit manuscripts for clarity, brevity, grammar, and style.

Call for Contributions

The Variable **is looking for contributions from all members of the mathematics education community**, including classroom teachers, consultants, and teacher educators. Consider sharing a favorite lesson, an essay, a book review, or any other work of interest to mathematics teachers in Saskatchewan. When accepted for publication, your article will be shared with a wide audience of mathematics educators in Saskatchewan and beyond.

We also welcome student contributions in the form of artwork, stories, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom, and for teachers to recognize students' efforts during their journey of learning mathematics.

All work is published under a Creative Commons license. **To submit or propose an article, please contact us at thevariable@smts.ca**. We look forward to hearing from you!

Ilona & Nat,
Editors

Message from the President

Hello, and happy March! I hope everyone had a restful and rejuvenating February break, and if nothing else, that your toes have finally thawed a bit in these last few days. I certainly am appreciating all of the extra sunlight!

The extended sunlight and warmer temperatures have me feeling more playful. Finally, we can get outside and actually enjoy the winter and the snow! Coincidentally, my son's school is doing a great job of reminding me to stay playful in my approach to mathematics, too. While you wouldn't think I'd need reminding, home routines get busy; thankfully, my son's school has started including in their weekly update a suggestion for a fun and easy math game families might try at home. While some have been old favorites—SET, for example—there have been some new ones, too. This week's suggestion, BUMP, is a fun game that builds fluency with addition and multiplication.[1]

This got me thinking: What can we do sustain playfulness not only in our homes, but in our math classrooms, too? Maybe it's time to shake up your usual and try out or refine a 3-Act task. Or maybe your students might like to participate in one of the many contest and other extracurricular mathematics opportunities highlighted in the Tangents column (see p. 34). Or perhaps the extra sunlight has you ready for the next-level challenge of organizing something larger for your school: A regular math blurb in the newsletter? A math challenge corner in the school? A club? A Pi Day celebration? If you'd like more inspiration, this edition of *The Variable* has absolutely delightful Problems to Ponder to try out for all grade levels (see p. 5).

Whatever you're thinking, I hope you're finding ways to engage in mathematics playfully and joyfully with your students. And if you tried out a great activity, be sure to share it with our network of math educators either on Twitter (@SMTSca) or here in *The Variable*. 'Till next time, stay mathy and have fun!

Michelle Naidu

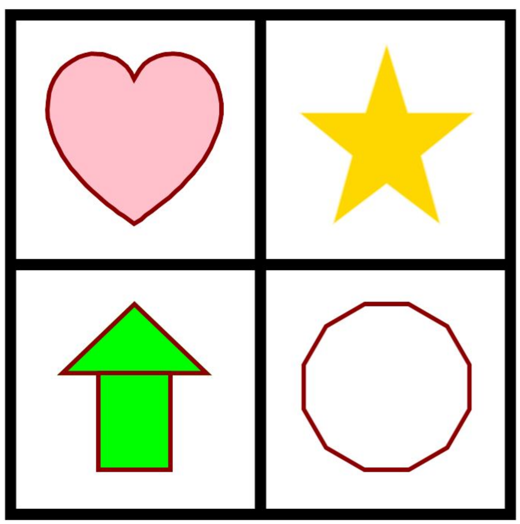
[1] For instructions and a printable game board, head to http://ccssmathactivities.com/wp-content/uploads/2017/01/BUMP-Instructions.pdf

Problems to Ponder

Welcome to this month's edition of Problems to Ponder! Have an interesting solution? Send it to thevariable@smts.ca for publication in a future issue of The Variable!

Primary Tasks (Kindergarten-Intermediate)

Which One Doesn't Belong? [2]
Display the image below and ask the class: "Which one doesn't belong?"



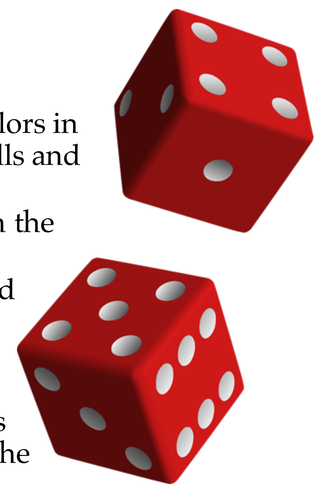
This task encourages students to use descriptive language in their reasoning and to consider multiple possible answers. See wodb.ca for more details and more images.

[2] Bourassa, I. (n.d.). Shape 5 [Digital image]. Retrieved from www.wodb.ca/shapes.html

Prime Time [3]
In this game, students color patterns in a hundred chart, showing multiples of a number they have rolled.

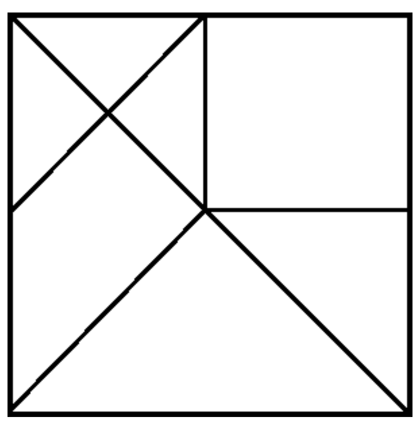
Materials
- Two dice
- One hundreds chart
- Two different colored highlighters

Task Instructions



- Player 1 and Player 2 each pick a different colored highlighter.
- Player 1 rolls the dice and adds the two numbers. Player 1 then colors in every multiple of that number on the hundred chart. If a player rolls and gets the sum of 2, they color in all of the prime numbers.
- Player 2 rolls the die and colors every multiple of that number on the hundred chart.
- If a number is already colored, the player skips that number and continues coloring any available multiple of their number to 100. If a player rolls and there are no multiples available for their number they lose their turn.
- When the hundred chart is completely colored, each player counts the number of squares they have highlighted. The player with the greatest number of colored squares wins the game.

Fraction Talks [4]
a) Find the fraction represented by each section of the square below (*teachers: head to the Fraction Talks website for a larger version of the image*).

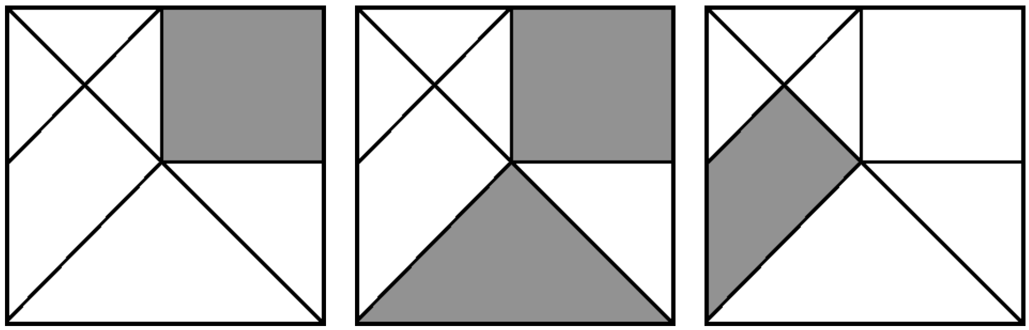


b) Find all of the ways of representing ¼ by shading in parts of the square above.

[3] Prime time. (n.d.). Retrieved from the YouCubed website at www.youcubed.org/tasks/prime-time/

[4] *Image from and questions inspired by* www.fractiontalks.com

c) If the area of the large square is 12 cm^2, what is the area of the shaded section in each image below?



d) If the area of the shaded section is 4 cm^2 for each image above, what is the area of the large square in each case?

e) Which fractions between $\frac{0}{16}, \frac{1}{16}, \frac{2}{16}, \ldots, \frac{15}{16}, \frac{16}{16}$ can you represent by shading in one or more of the sections in the first (unshaded) image above?

Intermediate and Secondary Tasks (Intermediate-Grade 12)

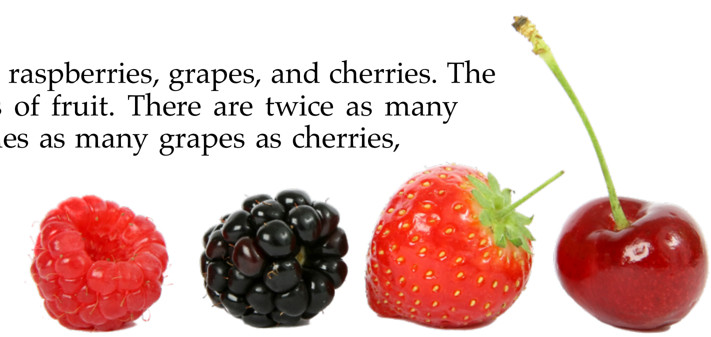
Egyptian Fractions [5]

Ancient Egyptians used unit fractions, such as $\frac{1}{2}$ and $\frac{1}{3}$, to represent all fractions. For example, they might write the number $\frac{2}{3}$ as $\frac{1}{2} + \frac{1}{6}$. While we often think of $\frac{2}{3}$ as $\frac{1}{3} + \frac{1}{3}$, the ancient Egyptians would not write it this way because they didn't use the same unit fraction twice.

How many ways can you express $\frac{1}{6}$ as the sum of two unique unit fractions? How about $\frac{1}{8}$? Can all unit fractions be made in more than one way? Choose different unit fractions of your own to test out your theories.

Fruit Salad [6]

A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad?



[5] *Adapted from* Egyptian fractions. (n.d.). Retrieved from the Illustrative Mathematics website at www.illustrativemathematics.org/content-standards/5/NF/A/1/tasks/839 *and* Keep it simple. (n.d.). Retrieved from the NRICH website at nrich.maths.org/6540

[6] Fruit salad. (n.d.). Retrieved from the Illustrative Mathematics website at www.illustrativemathematics.org/content-standards/tasks/1032

The Prince and the Trolls [7]

A prince picked a basketful of golden apples in the Enchanted Orchard. On his way home, the prince came to a troll who guarded the orchard. The troll stopped him and demanded payment of one-half of the apples plus 2 more, so the prince gave him the apples and set off again. A little further on, he encountered a second troll. The second troll demanded payment of one-half of the apples the prince now had plus 2 more. The prince paid him, and set off once more. Just before leaving the enchanted orchard, a third troll stopped him and demanded one-half of his remaining apples plus 2 more. The prince paid him and sadly went home. He had only 2 golden apples left. How many apples had he picked?

Extensions: What if the prince had 1 apple left? 3 apples? Design an algorithm to determine the number of apples the prince picked for any number, n, of apples remaining.

Have a great problem to share?

Contribute to this column!
Contact us at thevariable@smts.ca.
Published problems will be credited.

[7] *Adapted from* Kelemanik, G., Lucenta, A., & Janssen Creighton, S. (2016). *Routines for reasoning: Fostering the mathematical practices in all students.* Portsmouth, NH: Heinemann.

Alternate Angles

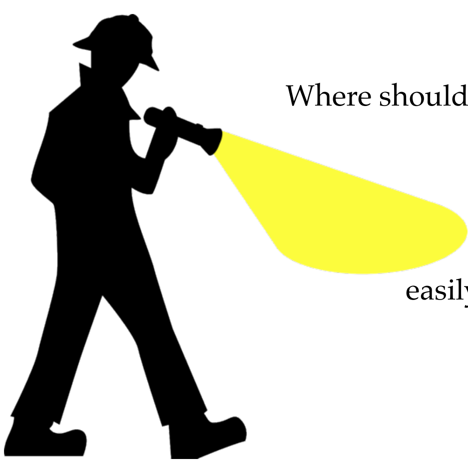
Alternate Angles is a bimonthly column on problems from multiple perspectives: various methods that could be used to solve them, insights we get from their solution, the new paths that they can lead us to once they have been solved, and how they can be used in the classroom.

Searching for Subtraction

Shawn Godin

Welcome back, problem solvers. In the last issue, I left you with the following problem[1]:

In the expression below, six out of the seven "$\square$"s contain addition signs, and the remaining "$\square$" contains a subtraction sign.



$$1 \square 2 \square 3 \square 4 \square 5 \square 6 \square 7 \square 8 = 30$$

Where should the subtraction sign go to make the equation true?

At first glance, it may seem that all we need to do is to try things until we get a correct solution. In fact, there are seven possibilities, which students can easily test. This gives the results in Figure 1 below.

$$1+2+3+4+5+6+7-8=20$$
$$1+2+3+4+5+6-7+8=22$$
$$1+2+3+4+5-6+7+8=24$$
$$1+2+3+4-5+6+7+8=26$$

[1] This problem came from the website Brilliant.org, which offers a wide variety of science, mathematics, and computer science problems and courses. A free membership to Brilliant.org gives you access to some of the material, including scores of problems submitted by Brilliant.org users. You can check out a TEDx presentation on developing students in STEM subjects by Brilliant's CEO Sue Khim at https://youtu.be/EnQCYZ8Oz8Q.

$$1 + 2 + 3 - 4 + 5 + 6 + 7 + 8 = 28$$
$$1 + 2 - 3 + 4 + 5 + 6 + 7 + 8 = 30$$
$$1 - 2 + 3 + 4 + 5 + 6 + 7 + 8 = 32$$

Figure 1: All possibilities

We thus have our answer: the subtraction sign should be placed in the box before the number 3. However, looking more closely at the results, you may also notice an interesting pattern related to the number being subtracted and the result. If we let x represent the number being subtracted and let y represent the final result, we get the table in Figure 2.

x	y
2	32
3	30
4	28
5	26
6	24
7	22
8	20

Figure 2: Table of results

From the data in Figure 2, we can see that we have a linear relation with equation $y = 36 - 2x$. If we knew this ahead of time, we could have easily solved the problem by setting $y = 30$ and solving for x. The question then becomes, "Why is the equation of this form?"

We may either know, or discover with a little experimentation, that $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$. Is this result related to the 36 in the linear relation above, or is it just a coincidence? Let's compare this expression with the answer to the original problem, $1 + 2 - 3 + 4 + 5 + 6 + 7 + 8$. The only difference between them is that the first adds a 3, while the other subtracts it. When we subtract the two expressions, the reason for the form of the linear relation becomes clearer.

$$\begin{array}{r} 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \\ -\ \underline{1 + 2 - 3 + 4 + 5 + 6 + 7 + 8} \\ 0 + 0 + 6 + 0 + 0 + 0 + 0 + 0 \end{array}$$

Figure 3: Subtracting expressions

Since $6 = 2 \times 3$, it is clear where our formula comes from. We could even go further and let $y = 1 + a_2 \times 2 + a_3 \times 3 + a_4 \times 4 + a_5 \times 5 + a_6 \times 6 + a_7 \times 7 + a_8 \times 8$, where all of the a_i's are $+1$, except one that is $a_x = -1$. Then, if we subtract the equation for y from the equation $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$, we get

$$\begin{aligned}36 - y &= 1 + 2 + \cdots + 8 - (1 + a_2 \times 2 + \cdots \times 8)\\ &= (1 - a_2) \times 2 + (1 - a_3) \times 3 \ldots + (1 - a_8) \times 8\\ &= 2x,\end{aligned}$$

which can easily be rearranged into the equation we found for the data in Figure 2.

Where do we go from here? One possible extension is to push students towards discovering that if we have a problem of this type, where all terms are being added except one, which is subtracted, the relationship $y = S - 2x$ holds, where S is the sum of all of the numbers involved in the problem. Thus, if we were tasked with the similar problem

$$1 \square 2 \square 3 \square 4 \square 5 \square 6 \square 7 \square 8 \square 9 \square 10 = 39,$$

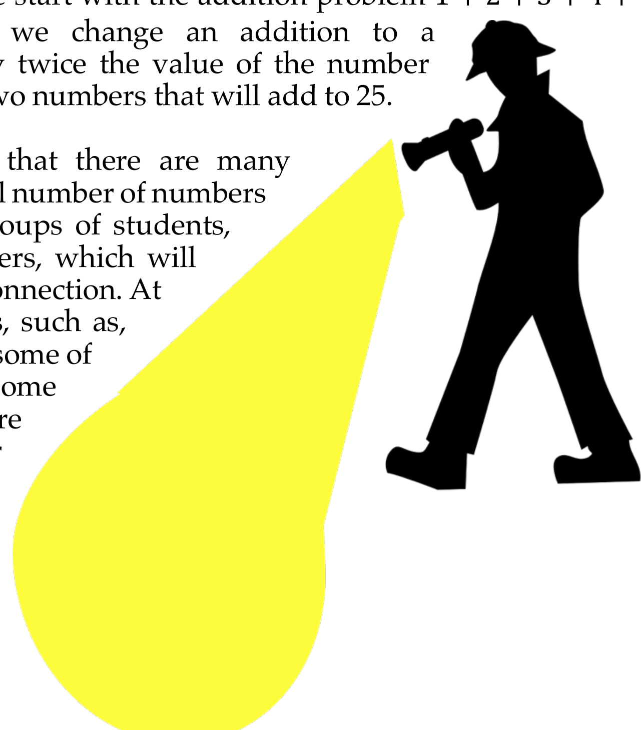
where one of the operations is subtraction and the rest are addition, then we know that $55 - 2x = 39$, since the sum of the first 10 positive integers is 55. We can then quickly find that $x = 8$, which tells us that the subtraction sign should come before the number 8.

It may seem like we have trivialized the problem. We certainly don't want to give students a bunch of homework "testing" the idea that they discovered. However, we can still get to some deeper thinking. Imagine that we start with the following problem:

$$1 \square 2 \square 3 \square 4 \square \cdots \square 98 \square 99 \square 100 = 5000$$

If asked which number must be subtracted for the equation to be true, we can quickly work out that it must be 25 (as long as we know the sum of the first 100 positive integers is 5050). However, what if the problem states instead that *two* of the operations are subtractions? Now, we need to realize that, again, if we start with the addition problem $1 + 2 + 3 + 4 + \cdots + 98 + 99 + 100 = 5050$, every time we change an addition to a subtraction, the final result decreases by twice the value of the number being subtracted. Thus, we need to find two numbers that will add to 25.

The nice thing about this problem is that there are many solutions. Thus, even if we reduce the total number of numbers involved to 20, different students, or groups of students, will likely come up with different answers, which will hopefully lead them to make the deeper connection. At this point, you can ask further questions, such as, "How many different ways can we make some of the operations addition and some subtraction?", or "Five of the signs are subtraction. What is the largest number that could be subtracted?" There are many extensions that will get students thinking more deeply about operations and relationships. I hope you have fun exploring this problem!



And now for some homework:

> One day, Alice encountered the number $4\frac{4}{3}$. She went to simplify it, but mistakenly thought that the expression meant $4 \times \frac{4}{3}$. When she showed it to her friend Bob, he explained her error, but when she simplified the problem correctly, she was amazed to find that she got the same result!
>
> Find some other numbers that have the same property.

Until next time, happy problem solving!

Shawn Godin teaches and is a department head at Cairine Wilson Secondary School in Orleans, Ontario. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: leading workshops, writing articles, working on local projects and helping create mathematics contests.

Spotlight on the Profession

In conversation with Patrick Maidorn

In this monthly column, we speak with a notable member of the mathematics education community about their work and their perspectives on the teaching and learning of mathematics. This month, we had the pleasure of speaking with Patrick Maidorn.

Patrick Maidorn has been a mathematics and statistics instructor at the University of Regina for the past twenty-one years. Apart from teaching undergraduate classes, Patrick has also been involved in the development of several mathematical outreach programs for students in Grades 1-12, including the University of Regina Math Camp and Math Circles, as well as the Canadian Math Kangaroo Contest.

Patrick grew up in Luxembourg, where he attended the European School. He holds a Bachelor of Science from the University of Guelph, a Masters of Mathematics from the University of Waterloo, and a Bachelor of Education from the University of Western Ontario. Realizing that each move took him further west on the map, he made one more westward leap to settle in Regina. He hopes to eventually get used to the cold winters of Saskatchewan. After two decades, he is still waiting.

First things first, thank you for taking the time to have this conversation!

You have been recognized for your hard work in mathematics education and outreach, including by the Pacific Institute for the Mathematical Sciences (PIMS), who awarded you the PIMS Education Prize in 2016. What kinds of mathematics camps, competitions, or other outreach activities are you involved in today?

In the past year, I've focused on expanding the size and scope of the outreach activities offered through the University of Regina. For example, the new Math Circle program is an extension of our evening problem solving sessions. The original sessions were mostly directed toward Grades 7-10, whereas the new format includes four concurrent classes that are available to all students from Grade 1 to high school. Their focus has also changed a little. While problem solving is still a large component, there is now also a bigger emphasis on open exploration of mathematical topics, partially in hopes of developing students' sense of academic curiosity.

The decision, in particular, to include very young children in the Math Circle has led to a lot of interesting experiences. My favourite, by far, is witnessing a young student have a mathematical insight or make a mathematical connection far beyond their current curriculum level. The students' energy and enthusiasm is also wonderfully infectious (if a little exhausting).

The University of Regina is are also continuing to run an annual one day Math Camp, as well as hosting the Canadian Math Kangaroo Contest on campus. Both of these events take place in March of each year.

What do these activities have to offer elementary and high-school students beyond their in-school experiences with mathematics?

Having such events offers those students with an affinity for mathematics an opportunity to explore the subject further. It also brings students into contact with other peers who are enthusiastic about mathematics. If a student has an interest in sports or creative arts, there are many options available to pursue these things outside of school. However, the options for students with academic interests can be more limited, which is why I feel that having these and similar events is so important.

> "There are many options for pursuing sports and creative arts outside of school. The options for students with academic interests are more limited, which is why these events are so important."

Throughout the years, you have been instrumental in organizing several math competitions in the province, and were a lead writer for the Saskatchewan Math Challenge, a provincial math challenge co-sponsored by the Saskatchewan Math Teachers' Society and the University of Saskatchewan for students from Grades 7 to 10. From where do you draw your inspiration for new puzzles and problems?

First and foremost, I love the playful aspect of mathematics. When I see, read, or experience things throughout the day, my first instinct is often to play around with the ideas, look for patterns, and make mathematical connections. For example, when I first moved to Saskatchewan, I didn't know much about the rules of football. When I was watching a game, a friend explained the strategy behind trying for a touchdown or a field goal (i.e., balancing the probability of success and the possible pay-off). My first thought was "This would make a great 'expected value' problem for my stats class!" Now, I still don't know much about football, but I'll always remember the touchdown and field goal point values.

Of course, many new problems are variations of what has come before, and I have to give credit to the mathematical community for being so generous with its resources. There are many authors who will invite others to try out, share, and modify their own problems and puzzles. So, when I come across an interesting problem, I often play around with it and see if it can be expanded or adapted for other purposes.

In your view, what makes a "good" problem?

There are so many ways to answer this question. It really depends on what the purpose of the problem is. Am I posing the problem in a contest to test or challenge the students? Am I using the problem in a classroom to introduce a new concept or to have the problem's solution open up a new avenue to further ideas?

Ideally, a problem should be both accessible and interesting. This means that were we to encounter the problem, we would want to explore it further—even if it wasn't a question on an exam or a problem posed in a class. Something about the problem triggers us to think "Hmm, I wonder how this works?" If there is an internal motivation to start manipulating the pieces of the puzzle before us, we're much more open to discovering pathways to a solution. Also, a good problem has mathematical value. Its solution should expand our existing knowledge, create connections between previously unrelated ideas, or lead to an entirely new problem.

> "A good problem has mathematical value. Its solution should expand our existing knowledge, create connections between previously unrelated ideas, or lead to an entirely new problem."

Take, for example, the classic problem "How many squares can you find on a chessboard?" This question is immediately approachable at any level. It invites us to ask questions to refine the problem, such as "What exactly counts as a square?" It also leads us to start looking for patterns, as soon as we realize that the final answer will be larger than we care to count to using our fingers and toes. Further, working through the solution teaches us the value of being systematic. Finally, having solved the problem, a natural follow-up question might be "Is there a quicker way to do this for larger grids?" This could connect to the idea behind summation formulas, and then you're on your way exploring Gauss' addition algorithm or developing the sum of squares formula.

What advice do you have to offer to students who would like to enhance their problem-solving skills, beyond exposure to a variety of problems and practice? What are some of the skills, habits of mind, or strategies that effective problem solvers rely on?

Even though you have asked me to look beyond "practice," I think this is such an important aspect of learning mathematics that I want to highlight it again. When we see the same kind of problem a number of times, we learn strategies that can become part of our permanent mathematical toolbox. Then, when we encounter a truly new problem, we are much more likely to recognize a familiar element in the problem that might at least suggest a possible solution path.

> "To really benefit from practice, a good guide is important—be it a teacher, a textbook, or another resource."

To really benefit from this type of practice though, a good guide is important—be it a teacher, a textbook, or another resource. A guide can help us select problems that are appropriate to the stage that we're at. Just jumping between random problems can lead to a lot of frustration, as we have to start at "square one" each time.

What drew you to study—and then to teach—mathematics in your younger years? What fuels your teaching and outreach work today?

I enjoyed many subjects in school—science, computing, economics—but I quickly realized that it was the underlying mathematical aspects in each that I enjoyed the most. This led me to the decision to study mathematics in university. It also is a big part of my motivation to teach. I want to help inspire and foster in others the same appreciation I feel for mathematics—whether it is math's many connections to the world around us or its intrinsic beauty. My favourite reaction is when a student tells me that they started a class with trepidation and ended up surprised that they actually enjoyed it. It might not happen often, but, in that moment, I feel that I made a difference.

Interviewed by Ilona Vashchyshyn

Trying Three-Act Tasks with Primary Students [2]

Kendra Lomax, Kristin Alfonzo, Sarah Dietz, Ellen Kleyman, and Elham Kazemi

At mid-morning in Kristen Alfonzo's kindergarten classroom, students are sitting on the carpet with whiteboards and markers, listening to her tell a story. "Jamal is making sandwiches. He makes six sandwiches." Students obediently draw six sandwiches on their boards. "Then Jamal eats two of the sandwiches." Students begin crossing out two of the sandwiches they have drawn. "How many sandwiches are left?" Some students quickly shout out, "Four!" causing others to look around at their neighbors' whiteboards and quickly write a 4 also. In our attempt to engage our kindergartners in solving real-world problems, it seemed they were learning a different lesson: Math is about following directions and getting the right answer quickly.

A new take on problem solving

As primary teachers, coaches, and teacher educators from various school settings in the Seattle area, we discovered we shared common challenges. Experiences like the one in Alfonzo's class prompted us to work together to seek out new approaches to engaging young children in problem solving. The goals of problem-solving activities in the elementary grades often include making sense of story problems, developing a range of strategies, and reaching accurate solutions. These are important mathematical aims, but they do not fully address the demands of *modeling with mathematics* as described in the fourth of the Common Core's eight Standards for Mathematical Practice (SMP 4) (CCSSI, 2010, p. 6–8). Modeling with mathematics also involves identifying mathematical problems in our world, gathering information and determining which details will help us solve a problem, and developing and revising mathematical models of situations. For young children, such models might include diagrams, equations, and using manipulatives to represent the quantities and mathematical relationships in a given situation.

After investigating various approaches and activities, we discovered an exciting problemsolving activity structure specifically designed to engage students in mathematical modeling: three-act tasks (Meyer, 2011) designed by Dan Meyer for use in secondary classrooms and adapted by Graham Fletcher and other educators for elementary school classrooms (Fletcher, 2016). The activity structure comprises three parts or "acts" (see Figure 1).

First, students view an image or video that depicts an interesting real-world situation—such as a child throwing rocks from the beach into the ocean or a "cookie monster" stealing some cookies—and discuss what they notice and wonder. Then students generate mathematical questions about the situation, identify important information they need to answer the questions, and construct mathematical models of the situation. Finally, they discuss their strategies and interpret the results of their modeling in the context of the situation. Students verify how well their modeling helped them answer their mathematical question.

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[2] Reprinted with permission from "Trying Three-Act Tasks with Primary Students," *Teaching Children Mathematics*, *24*(2), copyright 2017 by the National Council of the Teachers of Mathematics (NCTM). All rights reserved.

Since learning about three-act tasks, we have engaged our K–grade 2 students in many of them. The purpose of this article is to share our learning with you. To begin, let's step inside Sarah Dietz's second-grade classroom for a closer look.

FIGURE 1

The authors discovered a problem-solving activity structure specifically designed to engage students in mathematical modeling: three-act tasks designed by Dan Meyer (2011) for use in secondary classrooms and adapted by Graham Fletcher and other educators for elementary school classrooms.

Summary of a three-act task lesson

Act 1

- The teacher shares a compelling multimedia depiction of a situation through a video or photographs.
- Students discuss what they notice and wonder about the video, including mathematical features of the situations.
- Students decide on a mathematical question to answer about the situation.

Act 2

- The teacher provides information or resources that students think they need to work on the focal question.
- Students work to answer the question.

Act 3

- Students discuss their strategies and solutions.
- The teacher may compare and connect students' ideas or "reveal the answer."
- If relevant to the focal problem, students consider why their modeling was different from the real-world resolution.

Act 1: Learning to Ask Mathematical Questions About the Real World

It's Friday afternoon following recess, and Dietz's second graders are squirmy, but she knows that the three-act task she has planned will pique their interest and get them engaged in rich mathematical work. "Mathematicians," Dietz addresses the class, "as you know, my daughter Carrie often comes to help me after school. Well, yesterday she was here, and I made a video of what she did. Watch the video and think to yourself, What am I noticing and wondering?"

Students watch intently as Carrie approaches a stack of crayon boxes on the counter. She takes one of the boxes. She also opens another box and removes a few of the crayons. She then puts the partially empty box back and walks away.

Students are engrossed in the video and ask to watch it again. During the second viewing, they start to excitedly comment to one another:

"Oh! I think there are four boxes."

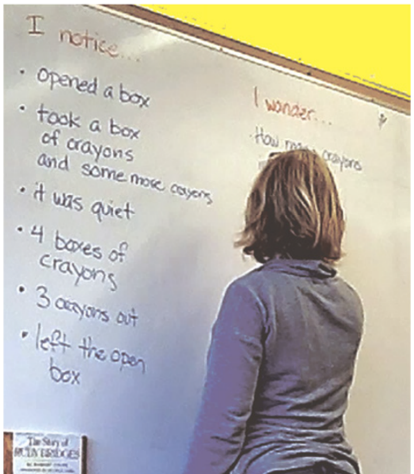
"She took out some crayons before she put it back!"

"She took three! No four!"

These seven- and eight-year-olds have a puzzle to solve.

To launch a three-act task, the teacher shows a short video or an image depicting an interesting situation in the real world and invites students to share what they notice and then what they wonder about the situation. Note that the videos and images do not include explicit mathematical structures like graphs or diagrams. Rather, students are tasked with making sense of the situation and bringing structure to the real-world situations (SMP 1).

Once the video is over, hands shoot into the air, and it is not just the usual hands. All the second graders want to share what they noticed. Kaya, who receives special education services in math and who often avoids eye contact during the subject, is up on her knees with her hand in the air, smiling. Edgar, whose English is still developing and who is usually hesitant to say anything, also has his hand up. In fact, so many hands are in the air that Dietz instructs the class to do a quick turn-and-talk to give everyone an opportunity to share before recording their noticings on the board. “Now I want you to take a moment to think about what you are wondering.” Once again, as Dietz gives students time to turn and share, the classroom erupts into conversations:



KENDRA LOMAX

In Act 1, every idea is recorded, sometimes with words and pictures added to make the information accessible to young learners. After listing students’ wonderings, the teacher guides students toward a single mathematical question that they can all investigate.

“I wonder how many crayons are in each box?”

“I wonder how many crayons were left in the open box?”

“Why did she take the crayons?”

“How many crayons were left after she took some?”

The first act invites every student in the class to participate in a way that supports the classroom community as intellectually and socially inclusive. Some of the children’s observations and questions will be mathematical in nature, and some will not—and that is OK! Learning which types of questions can be answered with mathematics and which cannot is an important aspect of modeling, so every idea is recorded. With young learners, teachers can record students’ noticings, using words and pictures to make the information more accessible. For example, if these were kindergartners, Dietz could sketch the four boxes or the three crayons next to the written words.

After carefully recording their wonderings, Dietz guides the class toward one mathematical question that all the students can investigate. “We have a lot of great questions here! Let’s think about this one: How many crayons were left after Carrie took them?” Together, they have selected a question on which to focus.

In planning for a three-act task, the teacher will anticipate questions that students might ask and, on the basis of the mathematical goal, will identify one to work on (NCTM 2014). However, teachers sometimes revise their plan in the moment if students generate interesting questions that are worth pursuing, even if the teacher did not anticipate them; or the teacher may allow students to choose which question most interests them.

Act 2: Learning to Identify Information Needed to Answer a Mathematical Question

In the second act, students consider what information they need to answer the selected mathematical question. The teacher has typically anticipated these needs and will provide some of the requested information before students begin working independently or in pairs. Students should be offered tools, such as cubes and hundred charts, to use as they see fit (SMP 5).

“OK, if we want to figure out how many crayons are left after Carrie took some away, what will we need to know?” A couple of hands pop up, but most students look unsure. Dietz encourages them to turn and talk to their partner, and then she starts collecting questions.

"How many crayons are in a box?"

"How many crayons are left in the open box?"

"How many did she take out of the open box?"

Dietz reveals some of the information they asked for: Eight crayons are in each of the boxes, and Carrie opened one box and took three crayons. Some students express confusion about the number of boxes at the beginning of the video. "Were there four or five crayon boxes?" The class watches the video once more to confirm that there were four boxes to begin with. Conversation erupts; students clearly want to get started.

Students may not immediately identify all the information they will need to model the situation. Or, after getting started, they may want to confirm details of the situation by reviewing the video. These requests for more information are indications that students are working to make sense of the context (SMP 1 and SMP 4). As needed, the teacher can call the class back together to offer additional details.

As the second graders continue working, Dietz notices that some students are figuring out the total number of crayons and then subtracting the number that Carrie took; other students are summing the crayons in the three remaining boxes. Dietz sees a variety of strategies. Some students, such as Alejandro (see Figure 2a), are directly modeling the situation using cubes or drawings (Carpenter et al. 2014), but many are using number lines (see Figure 2b) or equations (see Figure 2c) to add the groups of crayons. Some students are starting their number line at zero, whereas others, such as Semaiah, are starting at eight. "Is it four hops on the number line?" Semaiah wonders. "There are four boxes . . ." Dietz makes a mental note to discuss this representation as a whole class in the next act.

A teacher's primary role in Act 2 is observing how students approach the problem and which strategies they use. These observations will inform the Act 3 discussion. During this time, teachers might also support students to productively engage with the problem by helping them work through confusion or get started (NCTM 2014).

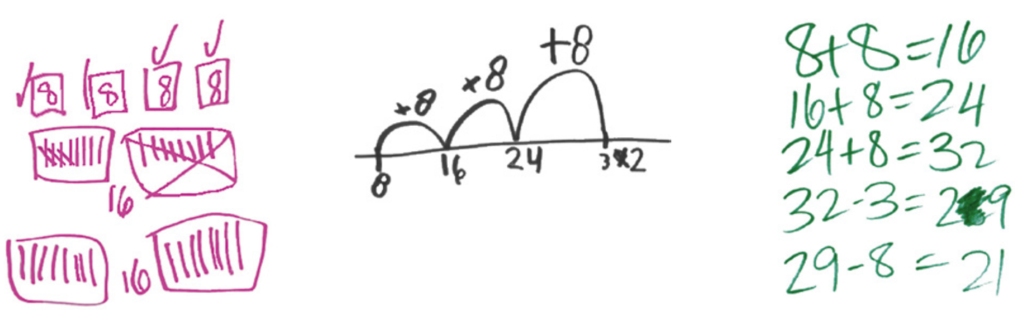
FIGURE 2

Alejandro's clear, visual, direct-modeling approach prepared other students to share their strategies.

(a) Alejandro counted the "crayons" in each box.

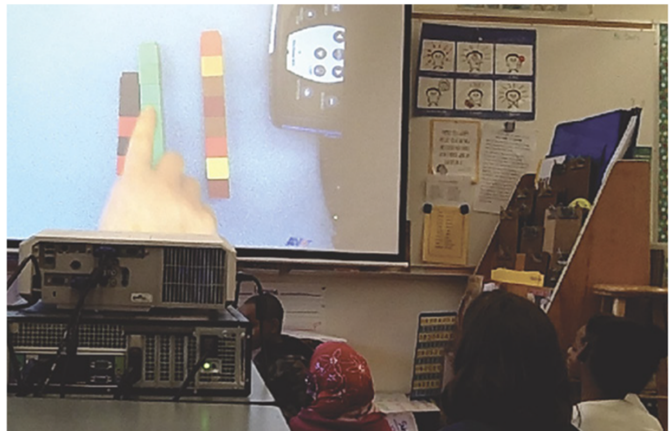
(b) Like many students, Semaiah used a number line to find the total number of crayons, but she started hers at the number 8.

(c) After Ramla explained her equations with respect to the crayon video, the class compared her equations to Semaiah's number line.



Act 3: Learning to Examine, Connect, and Revise Mathematical Models
In the third act, the teacher gathers students to discuss the different models the class has generated to answer their focal question. On the basis of their goals for the lesson, teachers determine which strategies will be most valuable for the class to examine and discuss—and in which order (Smith et al. 2009). As part of this whole-group discourse, teachers might ask students to discuss how their different strategies or representations match the situation, or students could explore the differences and similarities among their models (SMP 3).

Students gather at the carpet, and Dietz asks Alejandro to show his thinking using the document camera. Alejandro used blocks to create four stacks of eight, representing the eight crayons in each box (see Figure 2a). He shows the class how he took away a "whole box" and then removed three blocks, or "crayons," from another stack. Using Alejandro's strategy, the class counts together to determine how many crayons were left. This strategy affords a clear visual model, setting the stage for other students to share their strategies.



KENDRA LOMAX

Alejandro created four stacks of eight blocks to represent eight crayons in each box. He took away a "whole box" and then removed three blocks, or "crayons," from another stack.

Next, Semaiah puts her number line up for the class to see (see Figure 2b). Knowing that some students were working on making sense of this representation, Dietz wants to make sure the class has an opportunity to reason through this representation together: What was the meaning of each hop on the number line? If the number line starts at the number 8, what does that mean in the situation context?

Last, Ramla shares her strategy (see Figure 2c). After she explains how her equations connect to the crayon video, the class compares her equations to Semaiah's number line. The teacher challenges students to find the four boxes of crayons in each strategy and to think about how to show—on Semaiah's number line—the crayons that were taken away.

Another approach to the discussion of Act 3 is to show an image or video clip that reveals the answer to the focal question or reveals the conclusion to the task. The purpose of revealing the answer is not to ask children to "check their work," but to create an opportunity for students to examine how well their models fit the situation and how they could revise their models to improve their accuracy or efficiency (SMP 4).

What We Learned
In our experimentation with three-act tasks, we noticed three important distinctions between this activity structure and other problemsolving activities. These features suggest why three-act tasks are a useful addition to the repertoire of elementary school problem-solving activities:

1. Three-act tasks leverage the wealth of knowledge that young students bring about the world around them. From asking mathematical questions and modeling the story situation, to discussing strategies for answering the question, children's ideas are central to each act.

2. Three-act tasks provide entry points into mathematics for all learners. Three-act tasks use images, action, and sound to convey situations, providing a different set of entry-points for learners than typical story problem solving offers. We have found video and images to be exciting for all students and particularly valuable for our English language learners. All students have access to the story situation and, through structured discourse opportunities, they negotiate meaning and develop increasingly precise language around their shared understanding of the situation. Because students take charge of posing the mathematical questions and modeling the situation, they may choose to work on a "just right" portion of the task. In Dietz's class, some students, such as Semaiah, worked on only one piece of the crayon problem: How many crayons were in the boxes?

3. Three-act tasks engage young children in the work of doing mathematics as described by the SMP. The three-act task lesson structure is designed to engage children in modeling with mathematics (SMP 4), but other important mathematical practices make an appearance as well. Act 1 is all about making sense of contextual problems (SMP 1) and bringing mathematical structure to bear on the situation (SMP 7). In Act 2, children select the information and tools they need to model the situation (SMP 5). In Act 3, they present arguments and analyze the reasoning of others (SMP 3).

Try it Out

When we first introduced three-act tasks, our students were confused about what we were asking them to do. What does it mean to ask a mathematical question about the real world?

Over time, they learned what to expect and how to analyze their world through a mathematical lens. It may take a few tries, but with repeated opportunities, your students will get into the routine. The routine structure of three-act tasks can be repeated with different tasks, streamlining the teacher's planning efforts. Instead of worrying about how the lesson will unfold, teachers can focus on the mathematical ideas that they want students to consider and how to guide conversations for students to work on these ideas.

Tips for getting started with primary-grade three-act tasks

Find tasks or make your own. Such websites as http://www.gfletchy.com and http://www.101Qs.com have great tasks for elementary school students, or you can make your own videos on the basis of your students' interests and the mathematical ideas you are currently working on. Videos and images that show familiar situations with a little mystery work well.

Avoid the temptation to rush students toward *your* noticings and wonderings. Remember, these are opportunities for children to apply their own mathematical thinking to make sense of and solve contextual problems. If we always pose the problem to be solved, offer all the relevant information, and overstructure the problem-solving process, we rob students of the chance to fully engage in problem solving.

Move students gently toward mathematical thinking. If students' noticings and wonderings are not mathematical in nature (e.g., "I have those kinds of crayons!" or "Why did she take them?"), encourage them to focus on the math in the situation. Try this prompt: "One thing mathematicians do is look around the world and ask questions. What kinds of questions might a mathematician ask about this situation?"

Break the activity into smaller parts. Engaging in all three acts is likely to take a full math block. You might find that, depending on their stamina, young students fare better taking a break between acts. For example, they might ask mathematical questions and begin to solve the problem—and then discuss the problem after recess or PE or even on the following day. This has the added benefit of allowing you to carefully look over their work and purposefully select and sequence students' ideas to discuss.

More resources. Check out videos of three-act tasks in action at http://www.TeachingChannel.org, and find more planning resources at http://www.TEDD.org.

For Even the Youngest Learners

Collaborating with one another to better understand our students' mathematical capabilities has been a joyful and productive experience for students and teachers alike. We have found three-act tasks, originally designed for secondary mathematics, to be mathematically engaging and productive for even our youngest learners. If, as we are, you are eager to tap into elementary students' intuitive ability to make sense of problems in a new way that presents entry points for all learners, we invite you to try three-act tasks.

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Puzzling Probability[1]

Jehu Peters

In this article, I am going to explore some interesting probability puzzles. While not taken directly from a secondary mathematics textbook, these problems are closely aligned with the Saskatchewan math curriculum, in particular the probability outcomes of Foundations of Mathematics 30 (FM30.5) and Workplace and Apprenticeship 30 (WA30.11). In deviating from typical textbook problems, however, these examples present a unique and interesting challenge to students studying the basics of probability, especially because they lead to some surprising results!

Balls in an Urn[2]

Curriculum Connection
FM30.5(g): *Determine the probability of an event, given the occurrence of a previous event.*

First, I want to consider a question about selecting balls from an urn, a common context for probability problems in the classroom:

> An urn contains four colored balls: two orange and two blue. Two balls are selected at random without replacement, and you are told that at least one of them is orange. What is the probability that the other ball is also orange?

What an interesting problem! Before reading on, take a guess at what the answer might be.

At first, I was partial to the idea that the probability would be 1/3. After all, after you have been told that one of the balls drawn is orange, there is 1 orange ball left out of the total of 3 remaining balls.

Then, I thought that the answer must be 1/6. The probability of drawing an orange on the first selection is 2/4, and the probability of drawing an orange on the second selection is 1/3. Using the multiplication rule, we get a probability of 1/6.

Unfortunately, neither of these approaches is correct. In fact, the answer is 1/5! Why 1/5, you might ask? Let's make a diagram of the situation. First, I labelled the balls as follows: **O1**, **O2**, **B1**, **B2** (O stands for Orange, B stands for Blue). Although the orange balls are identical, we need to consider *which* balls have been drawn; hence, the additional labels 1 and 2. We are told that of the two balls selected, at least one of the balls is orange. Thus, we must have one of the following situations:

O1, **O2**
O1, **B1**
O1, **B2**
O2, **B1**
O2, **B2**

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[1] This article is adapted from a series of posts on Jehu's blog, *The Math Behind the Magic* (thema thbehindthemagic.wordpress.com): "Puzzling Probability Part 1" (February 5, 2017); "Puzzling Probability Part 2" (February 12, 2017); and "Puzzling Probability Part 3" (February 19, 2017). Reprinted with permission.

[2] Hogg, R., Tanis, E., & Zimmerman, D. (2013). *Probability and statistical inference* (9th ed.). Pearson.

As you can see, there are 5 possible selections where at least 1 of the balls selected is orange. Further, only 1 of these possibilities has 2 orange balls. Thus, the probability that the other ball is also orange is 1/5.

This was very shocking to me. I did not expect such a strange answer from a seemingly simple problem!

Returning to our original guesses, what situation would be required for the guess of 1/3 to be correct? One potential situation could be:

> The first ball removed from the urn was shown to be orange, and a second ball was then secretly removed. What is the probability that both balls are orange?

We can see that for the probability of having two orange balls to increase from 1/5 to 1/3, we must have a situation where we are given additional information. The fact that, in this new problem, we are explicitly shown that the *first* ball is orange ensures that the second ball has a 1 in 3 chance of being orange.

What about the guess of 1/6? In this case, a situation resulting in this probability might resemble the following:

> Two balls are secretly removed from the urn. What is the probability that both balls are orange?

Here, the probability decreases from 1/5 to 1/6, because we are not told any information about the balls. In this situation, using the multiplication rule will ensure that we arrive at the correct answer.

Strange Dice

Curriculum Connection
WA30.11(f): *Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, results of experimental probability, and subjective judgments.*

For the next problem, imagine a typical dice game. You roll two dice and compute the sum of the numbers rolled. For example, if the first die is a 4 and the second die is a 3, the sum is 4 + 3 = 7. From our everyday experience with rolling dice, we know that the number 7 comes up most often, and that numbers such as 3 and 11 are less common.

Now, imagine a pair of strange dice, with the following numbers on their faces:

> Die 1: 1, 3, 4, 5, 6, 8
> Die 2: 1, 2, 2, 3, 3, 4

Is rolling these two dice is the same as rolling two standard dice? Or, better yet, how could we determine whether rolling the pair of strange dice is equivalent to rolling a pair of standard dice?

The first idea I had was to add up all of the numbers on both dice:

$$(1 + 3 + 4 + 5 + 6 + 8) + (1 + 2 + 2 + 3 + 3 + 4) = 42$$

If you add up of all the numbers on two standard dice, you also get a sum of 42. I thought this might be enough to confirm that the strange dice were equivalent. However, consider the following two dice:

Die 3: 6, 6, 6, 6, 6, 6
Die 4: 1, 1, 1, 1, 1, 1

The sum of the all of the numbers on the above two dice is clearly also 42. But the only number that comes up when rolling both is 7. For an alternate pair of dice to be equivalent to rolling a pair of standard dice, there should be a way of rolling a 2, or a 3, or any of the other numbers typically rolled from standard dice.

Creating a table is very helpful for eliciting the answer in this particular situation. Here are all of the possible sums for two standard dice:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Sum	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

The numbers along the outside are the standard dice numbers and the numbers on the inside of the grid are the sums. If we count up the occurrences of each sum, we end up with the table above.

Now, we can repeat the same process with the strange dice:

	1	3	4	5	6	8
1	2	4	5	6	7	9
2	3	5	6	7	8	10
2	3	5	6	7	8	10
3	4	6	7	8	9	11
3	4	6	7	8	9	11
4	5	7	8	9	10	12

Sum	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

How interesting! It turns out that the strange dice have the same sums, occurring with the same frequency as the standard dice. This clearly demonstrates that these dice, when rolled together, are equivalent to a pair of standard dice! These strange dice are called Sicherman dice, discovered by George Sicherman of New York and originally reported by Martin Gardner in *Scientific American* in 1978. However, even if they are mathematically equivalent to standard dice, I doubt I could get away with using these at a casino…

As an extension, can you find a pair of *four*-sided dice equivalent to the standard four-sided dice? (Hint: One of these dice needs to must have the numbers 1, 3, __, 5.)

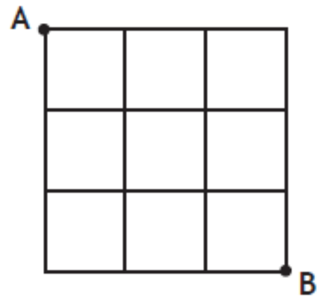
Random Walks [3]

Curriculum Connection
FM30.5(h)*: Determine the probability of two dependent or two independent events.*

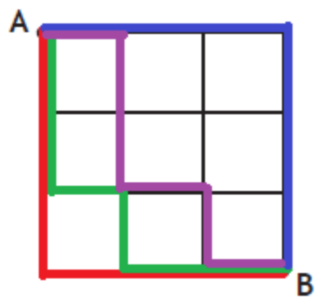
As someone who loves to walk, probability puzzles that involve considering different routes to the same destination are interesting to me. However, this last puzzle requires a warm up. Consider the following problem:

How many ways are there to walk from A to B (assuming no backtracking)?

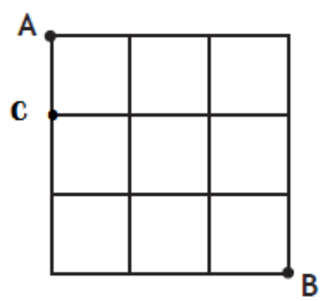
[3] Adapted from Mabillard, B. (n.d.) *Permutations and combinations: The binomial theorem.* Retrieved fromwww.math30.ca/lessons/permutationsAndCombinations/binomialTheorem/binomialTheorem.php



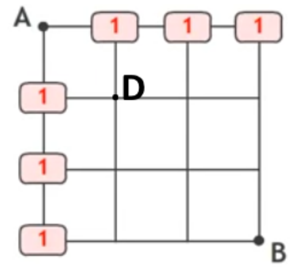
At first, I tried to draw all the possible paths:



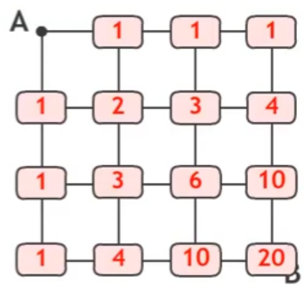
Then, I realized that this was going to take far too long, and I couldn't be sure that I didn't miss a path. I needed another approach. And so, instead of trying to figure out how many ways there were to get from A to B, I asked a simpler question: How many ways are there to get from A to C?



Clearly, we can only get to C in only one way (without backtracking). We place a number 1 at the intersection. Similarly, we only have one way to get to each intersection point along the top and along the left side. Next, we need to determine how many ways to get to point D.

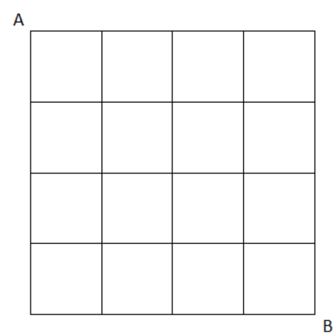


Since there is one way to get to the point above D and one way to get to the point left of D, we add these and obtain two ways to get to D. Continuing like this with each intersection point on the grid, we have:

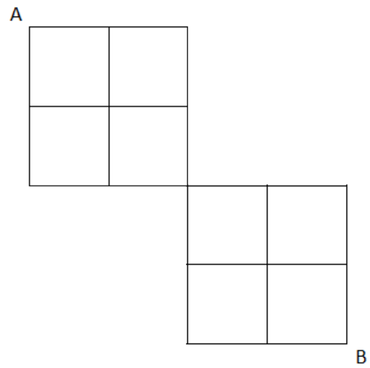


Thus, there are 20 different ways to walk from A to B.

Now to the probability question. What is the probability that a random walk from A to B (assuming no backtracking) passes through the middle of the grid below?



Using the above method, we find that there are 70 ways of walking from A to B. To count the number of ways through the middle, we simply redraw the grid to force us through the middle as follows:



Again, using the above method, we find that there are 36 ways of walking through middle.

Hence, the probability is 36/70. If you try this exercise for a 6 by 6 grid, you will find that the probability is 4900/12870.

This was counterintuitive to me. In a very large gird, if you were to randomly walk from one end to the other, it seemed to me very unlikely that you would pass exactly through the center. However, the above solution shows that even for a 6 by 6 grid, the probability of walking through the exact center is 38%! As an extension, what do you think happens to this probably as the grid grows larger? Does it approach a specific value? Or does the probability eventually tend to zero?

Jehu Peters is a recent graduate from the Faculty of Education at the University of Winnipeg. He teaches Mathematics at Maples and Garden City Collegiate. Jehu is interested in problem solving, algebra, calculus, and enjoys the connections between mathematics and science. He is currently studying for his Masters in Mathematics for Teachers from the University of Waterloo.

Intersections

In this column, you'll find information about upcoming math education-related workshops, conferences, and other events. Some events fill up fast, so don't delay signing up!

For more information about a particular event or to register, follow the link provided below the description. If you know about an event that should be on our list, please contact us at thevariable@smts.ca.

Within Saskatchewan

Workshops

Saskatoon Summer Initial Accreditation Seminar
March 8-9 & April 12-13, 2018
Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Accreditation seminars are offered to enable qualified teachers to become accredited. Accreditation is the process by which qualified teachers are granted the responsibility of determining the final mark or standing of the students in a specified Grade 12 (level 30) subject or subjects. The Accreditation seminar provides an opportunity for teachers to challenge, extend, enhance and renew their professional experience with an emphasis on assessment and evaluation.

Head to www.stf.sk.ca/professional-resources/events-calendar/saskatoon-accreditation-–-initial

Renewal/Second Accreditation Seminar
March 9 & April 13, 2018
Regina, SK
Presented by the Saskatchewan Professional Development Unit

Accreditation seminars are offered to enable qualified teachers to become accredited. Accreditation is the process by which qualified teachers are granted the responsibility of determining the final mark or standing of the students in a specified Grade 12 (level 30) subject or subjects. The seminar provides an opportunity for teachers to challenge, extend, enhance and renew their professional experience with an emphasis on assessment and evaluation.

Head to www.stf.sk.ca/professional-resources/events-calendar/regina-accreditation-—-renewalsecond

Technology in Mathematics Foundations and Pre-Calculus
May 7, 2018
Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

This workshop is designed to have math foundations and pre-calculus teachers experience a variety of technology tools that allow students to represent and visualize mathematics concepts. Tools highlighted are useful for students to explore, learn, communicate, collaborate and practice in order to enhance their understanding of mathematics in secondary mathematics.

Head to www.stf.sk.ca/professional-resources/events-calendar/technology-mathematics-foundations-and-pre-calculus-0

Number Talks and Beyond: Building Math Communities Through Classroom Conversation
May 11, 2018
Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Classroom discussion is a powerful tool for supporting student communication sense making and mathematical understanding. Curating productive math talk communities required teachers to plan for and recognize opportunities in the live action of teaching. Come experience a variety of classroom numeracy routines including number talks, counting circles, quick images and more. Take math conversations to the next level by strengthening your skills as a facilitator of classroom discourse and student thinking.

Head to www.stf.sk.ca/professional-resources/events-calendar/number-talks-and-beyond-building-math-communities-through-0

Beyond Saskatchewan

NCTM Annual Meeting and Exposition
April 25-28, 2018
Washington, DC
Presented by the National Council of Teachers of Mathematics

Join more than 9,000 of your mathematics education peers at the premier math education event of the year! Classroom teachers, administrators, math coaches, supervisors, college professors, and preservice teachers can all benefit from the sessions and learning at this event. Improve your knowledge and skills with high-quality professional development and hands on activities; gain insights by connecting and sharing with like-minded educators; and learn from industry leaders and test the latest educational resources.

Head to www.nctm.org/Conferences-and-Professional-Development/Annual-Meeting-and-Exposition/

45th Annual OAME Conference: Infinite Possibilities
May 3-4, 2018
Humber College, Toronto, Ontario
Presented by the Ontario Association for Mathematics Education Conference

This year's keynote speakers are Peter Liljedahl, Associate Professor of Mathematics Education in the Faculty of Education at Simon Fraser University; Jo Boaler, Professor of Mathematics Education at Stanford University and co-founder of youcubed (youcubed.org); and James Tanton, Mathematician in Residence at the Mathematical Association of America in Washington D.C.. Featured speakers are Fawn Nguyen, Marian Small, Mary Bourassa, and Cathy Bruce.

For more information, head to oame2018.weebly.com/keynote-speakers.html

Online Workshops

Education Week Math Webinars
Presented by Education Week

Once a month, Education Weekly has a webinar focusing on math. They also host their previous webinars on this site. Previous webinars include Formative Assessment, Dynamic vs. Static Assessment, Productive Struggling and Differentiation.

Past webinars: http://www.edweek.org/ew/webinars/math-webinars.html
Upcoming webinars:
http://www.edweek.org/ew/marketplace/webinars/webinars.html

Did you know that the Saskatchewan Mathematics Teachers' Society is a **National Council of Teachers of Mathematics Affiliate**? When registering for an NCTM membership, be sure to support the SMTS by noting your affiliation during registration.

NCTM | AFFILIATE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

This column highlights local and national extracurricular opportunities for K-12 students interested in mathematics, including collaborative, individual, online, and in-person challenges, contests, and camps. For dates, registration procedures and applications, and other information about the contests listed, please head to the contest websites, included below the descriptions. If we have missed an event that should be on our list, please contact us at thevariable@smts.ca.

If you are looking for contests available in other provinces, head to the Canadian Mathematical Society website (cms.math.ca/Competitions/othercanadian). The CMS also maintains a list of resources for students who are looking to build their problem-solving skills and succeed in competitions: see cms.math.ca/Competitions/problemsolving.

Local Events and Competitions

Canadian Math Kangaroo Contest
Spring

The purpose of this competition is to introduce youngsters from Grade 1 to Grade 12 to math challenges in a fun and enjoyable way, thus inspiring their further interest and advancement in mathematics. The competition is held yearly in more than 40 Canadian cities. Students may choose to participate in English or in French.

Students in Saskatoon may write the contest at Walter Murray Collegiate; students in Regina may write the contest at the University of Regina. Contact Janet Christ at christj@spsd.sk.ca (Saskatoon) or Patrick Maidorn at patrick.maidorn@uregina.ca (Regina).

Head to kangaroo.math.ca/index.php?lang=en

University of Regina Regional Math Camp
Spring

The Math Camp is a full-day event for students in Grades 1 through 12 who are interested in exploring the infinite frontier of mathematics beyond the school curriculum. Participants are guided by professors and students through a fun and enriching day. In a variety of grade appropriate sessions, students will explore mathematical topics in hands-on activities, games, puzzles, and more.

Head to www.uregina.ca/science/mathstat/community-outreach/mathcamp/

National Competitions

Canadian Team Mathematics Contest
Written in April
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Canadian Team Mathematics Contest (CTMC) is a fun and challenging competition for teams of 6 secondary school students in any combination of grades. One teacher and groups of six students participate at their own school over 3 consecutive hours.

Head to www.cemc.uwaterloo.ca/contests/ctmc.html

Caribou Mathematics Competition
Held six times throughout the school year

The Caribou Mathematics Competition is a worldwide online contest that is held six times throughout the school year. Each of these days, five contests are offered, one for each of the grade levels 3/4, 5/6, 7/8, 9/10 and 11/12 and each one in English, French and Persian. Available in English, French, and Persian.

Head to https://cariboutests.com/

Euclid Mathematics Contest
Written in April
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Euclid Mathematics Contest is an opportunity for students in their final year of secondary school and motivated students in lower grades to have fun and to develop their mathematical problem solving ability. Most of the problems are based on the mathematical curriculum up to and including the final year of secondary school. Written in April.

Head to www.cemc.uwaterloo.ca/contests/euclid.html

Fryer, Galois, and Hypatia Mathematics Contests
Written in April
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Fryer, Galois and Hypatia Math Contests are an opportunity for students to write a full-solution contest. They are fun way to develop mathematical problem solving skills through a written mathematical activity. For students in Grades 9 (Fryer), 10 (Galois) and 11 (Hypatia).

Head to www.cemc.uwaterloo.ca/contests/fgh.html

Gauss Mathematics Contests
Written in May
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Gauss Contests are an opportunity for students to have fun and to develop their mathematical problem solving ability. For all students in Grades 7 and 8 and interested students from lower grades.

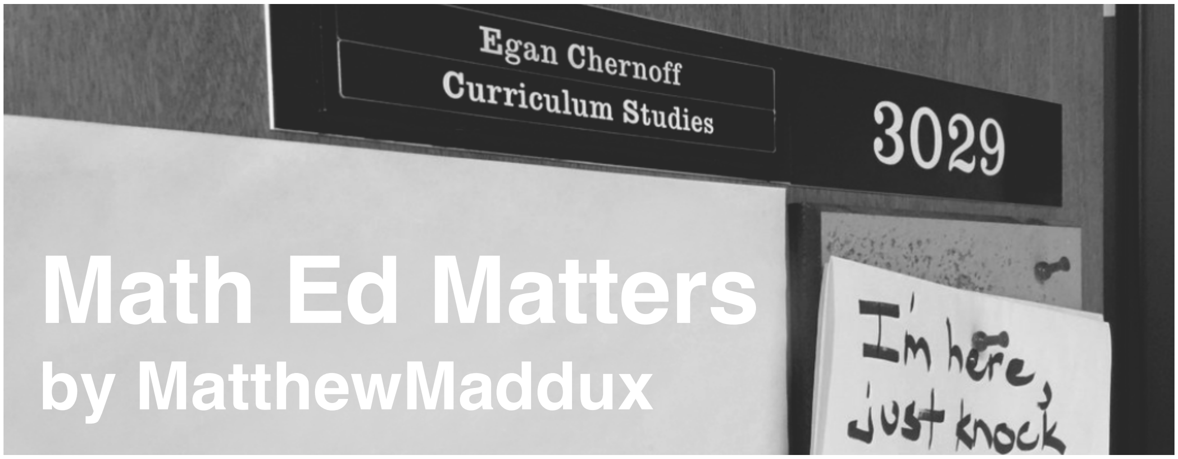
Head to www.cemc.uwaterloo.ca/contests/gauss.html

The Virtual Mathematical Marathon
Supported by the Canadian National Science and Engineering Research Council

The virtual Mathematical Marathon has been developed by an international team of mathematicians, mathematics educators and computer science specialists with the help of the Canadian National Science and Engineering Research Council and its PromoScience program.

The main activity is a competition allowing students to enjoy solving challenging mathematical problems all year around. Students can join the game at any time and at no cost, simply creating an individual profile with an individual username and a password. Available in French.

Head to www8.umoncton.ca/umcm-mmv/index.php



Math Ed Matters by MatthewMaddux *is a bimonthly column telling slightly bent, untold, true stories of mathematics teaching and learning.*

Antiquated Arguments from Math Class: Calculating the Tip

Egan J Chernoff
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As Ferris Bueller famously said, "Life moves pretty fast. If you don't stop and look around once in a while, you could miss it." True. It's also true that if you don't stop and look around once in a while, there are other things you might miss: for example, using antiquated arguments in math class. The "antiquated arguments" that I will be referring to here are responses a math teacher might give to the notorious "When are we ever going to use this stuff?" question. Here's the first, in what will become a series of favourite delusions.

I'll admit, the details are fuzzy. I don't remember exactly when I first heard someone retort, "What are you doing to do, pull a calculator out in the middle of a crowded restaurant to calculate the tip!?" Maybe it was a colleague, or maybe I overheard it in the staffroom; heck, maybe it was even me. Although I've long forgotten the context, I do know that, at the time, the response sounded right. It felt right. Not so anymore.

"What are you doing to do, pull a calculator out in the middle of a crowded restaurant to calculate the tip!?"

Putting aside the debate over whether or not to tip and when, there are numerous angles that reveal that the case against calculators at the restaurant dinner table is now antiquated. First, let's address the implied shame. At the very core of the retort was the implication that you should be embarrassed if you could not calculate a percentage tip mentally. The calculator, in this instance, was seen as a crutch. Certainly, it was assumed, one didn't want to use this crutch in the middle of a crowded room, which would alert to the rest of the world that you weren't smart enough to mentally calculate the tip yourself. And if the tip were 10%? Using the calculator as a crutch in this scenario would bring even more shame—after all, to calculate 10% of something, you just "move the decimal to the left" (mathematical abhorithm alert!!).

Besides shame, this response communicates the seemingly obvious absurdity of pulling out a calculator at a restaurant. Not only would it be ludicrous to bring out a calculator during your night out, who in their right mind would even have a calculator on hand?! The short answer: Nobody. The third angle marries the previous two together: Calculating on the calculator. Even if you had brought a calculator to the restaurant, and even if you somehow brought it out when the bill came, you certainly wouldn't want to be seen, caught dead, if you will, actually using the instrument, actually pressing the buttons to calculate the tip for your bill. For. Shame.

Oh my, how the world has changed.

Ironically, these three reasons that the tip calculator response worked for so long, seemingly failsafe responses to the question of "When are we ever going to use this stuff?," are the very same reasons that the argument is antiquated today. First, let's address the shame. The shame game is not the same. We now live in a world where pajama-wearing airline passengers remove their shoes and socks and stretch their bare feet out far enough to touch the person in the seat ahead. We now live in a world where people clip their fingernails and floss their teeth in public spaces. I could go on with more examples, but I think you get the point. Given the world that we now live in, I contend that not being able to do mental arithmetic, even easy mental arithmetic, simply does not hold the same gravitas as it once did.

> "Not too long ago, it was considered absurd to carry around a calculator. Today, it would be even more absurd if you didn't have a calculator on your person."

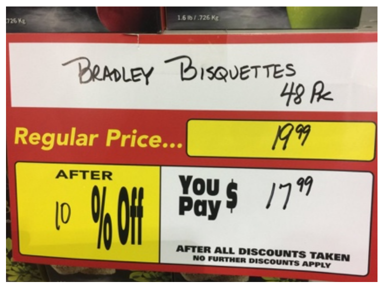
Next, let's address the idea that a calculator does not belong at the restaurant dinner table. I agree that, not too long ago, it was considered absurd to carry around a calculator (unless you were a "nerd"), even a pocket calculator that, yes, would conveniently fit into your pocket. Today, it would be even more absurd if you didn't have a calculator on your person. Not an actual calculator—I'm referring, of course, to the calculator app on your phone. All phones have them; the calculator is a stock app on virtually every cell phone you can buy. And so, the absurdity of having a calculator on you: gone. The absurdity of taking a calculator with you to a nice dinner: gone. Actually, the calculator now often sits on the table throughout the entire meal. Sure, you could look at your phone as a little innumeracy accouterment, especially if the person happens to pick it up once the bill arrives. But, and here's the third reason the tip calculator argument is over: If you were to pick up your phone in front of me, I have no idea whether you are scrolling to find your calculator app, entering in the amount of the bill, and calculating the tip, or if you're responding to a text or hailing a cab or updating your Twitter or any of the other bazillion things you could be doing on your phone. Any shame of actually using a calculator to calculate a tip, thanks to the calculator now being integrated with the phone itself, is yours and yours alone. Sure, there are various doomsday scenarios that we could paint: Maybe your phone doesn't have any power, or you forgot to bring it with you. Maybe the person next to you is looking over your shoulder. Or maybe you're seated beside someone (say, a math teacher) who would judge you for your lack of mental arithmetic skills…

Actually, let's embrace the doomsday scenario for a moment. Say you don't have your phone. Wait, say you're on your phone, which is about to lose power, and you're sitting next to someone who is not-so-surreptitiously sneaking a glance at what you're doing on said phone. Suppose even that you know that they will judge you for your lack of mental arithmetic skills. Somewhere, a math teacher's voice reverberates: "What are you doing to do, pull a calculator out in the middle of a crowded restaurant to calculate the tip!?" It is at this point that you may begin to feel the shame.

> "Our society is becoming less dependent on elementary arithmetic. Signs, literal signs, are everywhere."

Fear not.

After all, we're steadily moving towards a cashless society, which means that the server will undoubtedly ask: "Do you need the machine?" (A tad dystopian, no?!) As I mentioned in previous column, our society is becoming less and less dependent on elementary arithmetic. Signs, literal signs, are everywhere. Take, for example, these conversion charts that many stores now "helpfully" provide:



To reiterate: The world has changed.

The machines are everywhere, too. These days, the server hands you the machine, which tells you exactly how much money is on your bill. You press OK. And, suddenly, the tip calculator argument becomes even more antiquated, as tip options pop up on the screen for you to choose. Not just the percentage: the percentage and the actual amount, right next to each other on the screen. All that's left for you to do is to choose the amount of money you deem appropriate. And if that annoying friend, the one looking over your shoulder, still wants to see what you're doing, politely tell them that your finances are a personal matter.

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Call for Contributions

***The Variable* is looking for contributions from all members of the mathematics education community**, including classroom teachers, consultants, and teacher educators. Consider sharing a favorite lesson, an essay, a book review, or any other work of interest to mathematics teachers in Saskatchewan. When accepted for publication, your article will be shared with a wide audience of mathematics educators in Saskatchewan and beyond.

We also welcome student contributions in the form of artwork, stories, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom, and for teachers to recognize students' efforts during their journey of learning mathematics.

All work is published under a Creative Commons license. **To submit or propose an article, please contact us at thevariable@smts.ca**. We look forward to hearing from you!

Ilona & Nat,
Editors

Saskatchewan Mathematics Teachers' Society