The Variable

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What, How, Who: Developing Mathematical Discourse

Spotlight on the Profession: Dr. Marian Small

My Favourite Lesson: The Crazy Calendar

Constructivism in Secondary Math Education: A Teacher's Experience

A Day at the Museum of Mathematics

A (Math) Instructor's Copy for All: There's an App for That

Saskatchewan Mathematics Teachers' Society

AFFILIATE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Recipient of the National Council of Teachers of Mathematics Affiliate Publication Award (2019)

In this issue

Cover Image

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SMTS Executive (2018-2020)

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The Variable welcomes a variety of submissions for consideration from all members of the mathematics education community, including classroom teachers, consultants, teacher educators, researchers, and students of all ages. Submitted material is assessed for interest, presentation, and suitability for the target audience.

Please submit articles by email to thevariable@smts.ca in Microsoft Word format. Authors should aim to limit submissions to 3000 words or less and include a photo and a short biographical statement of 75 words or less with their submission. Accepted contributions are subject to revision. Editors reserve the right to edit manuscripts for clarity, brevity, grammar, and style.

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The Saskatchewan Mathematics Teachers' Society presents ...

#SUM2019

November 1-2, 2019

Who: K-12 teachers, coaches, consultants and coordinators
Where: Circle Drive Alliance Church, Saskatoon
When: November 1-2, 2019
Cost: $175 (early registration) | $220

KEYNOTE PRESENTERS:

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NCTM President

Ruth Beatty and
Lakehead University

Christina Ruddy
Algonquins of Pikwakanagan
First Nation museum and cultural centre

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Editorial

This edition represents the fruition of a series of incremental changes at The Variable. Over the past few years of publication, we have experimented with both format and publication schedule in order to best meet the needs of Saskatchewan teachers. Our efforts were validated through the feedback from our readers as well as recognition from the National Council of Mathematics Teachers in the form of the 2019 Affiliate Publication Award. In many ways, this edition represents a culmination of these efforts.

A bi-annual publication schedule means that this edition goes public at the start the school year, which we feel better aligns with the realities and needs of the classroom teacher. It also means that each edition contains a greater density of high-quality content for classrooms at both the elementary and secondary level written by and for Canadian educators. The other manifestation of our vision to include a more practical focus on the activities of the classroom is the publication of the first installment in the "My Favourite Lesson" column, designed for teachers to share lessons and tell stories directly from their classrooms. In this inaugural installment of the column, Chad Williams shares "The Crazy Calendar," a lesson that builds on students' personal experiences with calendars to further develop their understanding of the passage of time (p. 5).

Also this issue, Jeff Irvine shares his experiences with constructivism in secondary math education (p. 27); Timothy Sibbald recounts his visit to the National Museum of Mathematics, and asks us to consider what we can learn from its success (p. 33); and Kelley Buchheister, Christa Jackson, and Cynthia Taylor share ways to structure tasks to stimulate meaningful mathematical discourse while ensuring equitable participation (p. 21). In the latest "Spotlight on the Profession," Marian Small discusses ways to engage students at different levels in the math classroom. You will also find our regular features, including Shawn Godin's "Alternate Angles," which takes an in-depth look at intriguing problems and their solutions (p. 11); "Intersections," which will bring you up to date on upcoming professional development opportunities (p. 37); and "Tangents," which highlights extracurricular opportunities for K-12 students interested in mathematics (p. 42).

To complete the edition, Egan Chernoff challenges us to reconsider how we view some of the emerging technological tools available to students of mathematics as we teach (and they learn) in an increasingly saturated technological world (p. 45). Of course, the digital climate of education is something we pay a particular attention to being an open access, strictly digital publication.

Ilona & Nat
Editors

The Crazy Calendar

Chad Williams

This is one of my favourite lessons to facilitate with my students because most have personal experiences with seeing calendars at home filled with activities, appointments, and important dates.

The crazy calendar activity connects with the Shape and Space 3.1 outcome and many of the indicators (see Table 1). When used as a pre- or post-assessment, teachers will have a better understanding of what the students understand about the passage of time and where they need to grow as mathematicians.

Shape and Space 3.1: Demonstrate understanding of the passage of time including:

- Relating common activities to standard and non-standard units
- Describing relationships between units
- Solving situational questions

Indicators:
a) Observe and describe activities to self, family, and community that would involve the measurement of time.
f) Create and solve situational questions using relationship between the number of minutes in an hour, days in a particular month, days in a week, weeks in a year, or months in a year.
g) Identify the day of the week, the month, and the year for an indicated date on a calendar.
h) Identify today's date, and then explain how to determine yesterday's and tomorrow's date.
i) Locate a stated or written date on a calendar and explain the strategy used.
k) Create a calendar using the days of the week, the calendar dates, and personally relevant events.
l) Describe ways in which the measurement of time is cyclical.

Table 1: Shape and Space 3.1 outcomes and indicators. Ministry of Education. (2009). *Mathematics 3*. Available at https://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics_3_2009.pdf

Materials

- Handout (see p. 7)
- Blank calendar (see p. 7)
- Pencil
- Glue bottle
- Various stickers
- Various craft items

Task description

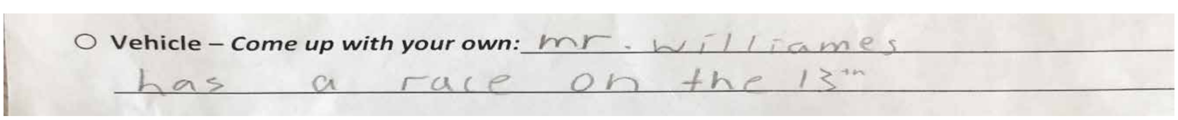
This task was presented to my grade three students on the second day of discussing days, months, and years as measurements of the passage of time. This task was used as a pre-assessment to determine what misconceptions we would need to target over the next several lessons and to determine the next steps as they continued to construct their knowledge about the passage of time.

At the beginning of the lesson, a pair of students were given a calendar for the month of April. The students were to fill in the missing information on the calendar such as days of the week, dates, and year of the calendar. Conversations between students ensued regarding the information that was missing and what they should write to make the calendar correct. This gave wonderful insight into what the students were thinking around the calendar and their knowledge of patterns in the passage of time with days, weeks, and months of the year.

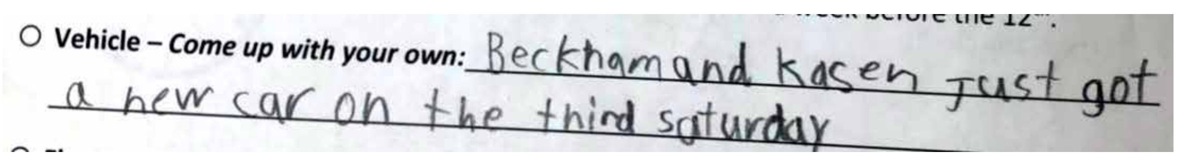
With the calendar completed, students placed stickers on the calendar on dates based on relevant events that the students are involved in inside and outside of school (a trip to the local dollar store allowed me to purchase the necessary stickers for this activity for under ten dollars). Some examples include (names of the students have been changed):

- "Justin and Krista have basketball on the first Saturday of the month and soccer on the last Saturday of the month."
- "Heather has to take care of her neighbour's cat 3 days after Graham's Grandma's birthday."

Students were given two extra stickers (a vehicle and flower) to come up with a couple of their own relevant events to be placed on their calendar using the mathematical language that surrounds the calendar and units of time. Some examples include:



○ Vehicle – *Come up with your own:* mr. williames has a race on the 13th



○ Vehicle – *Come up with your own:* Beckham and kasen just got a new car on the third saturday

The Crazy Calendar

1. Complete the calendar by filling in the missing parts.
2. Place the following items on the appropriate dates on the calendar:

- ❑ **Green Pom Pom** – Jason, Tate, and Danielle have ball hockey on the 12th and two weeks later
- ❑ **Present** – Graham's grandma's birthday is on the 3rd Thursday
- ❑ **Cake** – Austin, Leslei, and Nixon are holding a bake sale two weeks before Heather needs to take care of her neighbour's cat
- ❑ **$ Sticker** – On April 20th, Chris, Cameo, and Ethan put an ad in the newspaper for a garage sale and are having the garage sale 4 days later (place the sticker on the day they are having the garage sale)
- ❑ **Basketball/Soccer** – Justin and Krista have basketball on the first Saturday of the month and soccer on the last Saturday of the month
- ❑ **Football** – Clark and Jennifer have a football game on the 21st of April
- ❑ **Cat** – Heather has to take care of her neighbour's cat 3 days after Graham's Grandma's birthday
- ❑ **Eyes** – Marlon has an eye appointment on the first Wednesday of the month
- ❑ **Rhinestone** – Harriet has gymnastics on the second Saturday of the month
- ❑ **Button** – Park and Bryce are holding a clothing drive on the fifth Thursday of the month
- ❑ **Dog** – Damon and Alexis are volunteering at the animal shelter a week before the 12th
- ❑ **Vehicle** – Come up with your own: ______________________________

  ____
- ❑ **Flower** – Come up with your own: ______________________________

  ____

April 20

	Mon		Wed	Thu		Sat
				1	2	3
4	5	6	7		9	10
11	12		14	15	16	17
18	19	20	21	22	23	24
25	26		28	29	30	

Anticipated student action

Misconceptions that often surface with students throughout this activity include:

- "Snaking" forwards or backwards on the calendar one day at a time, like on a game board, as opposed to jumping down to the next week.
- When determining future or past dates, students include the current date in their counting, which results in them being one day shy of the correct date.
- Counting empty spaces on the calendar when determining dates. (When a month begins on a Thursday and students need to determine the third Wednesday. They will begin with the empty space prior to the month beginning as the first Wednesday of the month).

Student strategies that may emerge from this lesson include:

- To determine a week, students may count 7 days, or move up and down the column. For example, if they need to find a date two weeks after another, they will count 7 days, then another 7 days to find the date, while others may count 14 right away or move two rows down the calendar in the same column.

Figures 1-2 show students working on the task together, while Figure 3 features a calendar completed by a pair of students.

Figure 1: Students coming up with a situation together.

Figure 2: Students determining where the basketball sticker goes on the calendar.

Wrap-up and next steps

When students felt they had placed the stickers on the correct dates they would get together with another group to compare their work. If any discrepancies between the groups' calendars arose, they would discuss them together. As a whole class, we looked at a completed calendar and discussed the strategies that students used when determining where to place the sticker on the calendar. As I circulated throughout the activity it was important to understand what strategies students were using to place their stickers on the calendar to help me determine in what sequence I wanted groups to share their strategies. This sequencing allows students to move to a more efficient strategy and deepens their understanding of the passage of time (Smith & Stein, 2018).

Figure 3: A calendar completed by a pair of grade three students.

We talked about the common misconceptions that many of the groups had while completing this activity and attended to any errors or omissions that needed to be addressed to ensure the calendar was correct. The information I gathered from this activity will help me plan my next activities and tasks as we continue to build on our understanding and use of appropriate mathematical language when dealing with the passage of time with the calendar.

During the lesson one of my students was asked how he enjoyed the lesson and he stated that "it was chaos"… which is exactly what I was hoping for as I engaged students in productive struggle through the creation of the crazy calendar. I encourage you to tweak, change or add to my favourite lesson to create chaos in your math classroom.

References

Smith, M., & Stein, M. (2018). *5 Practices for orchestrating productive mathematical discussions.* Reston, VA: National Council of Teachers of Mathematics.

Chad Williams is a grade three teacher at Clavet Composite in Prairie Spirit School Division. He finds joy and passion in sharing what he is doing in his classroom and in learning from his students and experiences..

Contribute to this column!

The Variable exists to amplify the work of Saskatchewan teachers and to facilitate the exchange of ideas in our community of educators. We invite you to share a favorite lesson that you have created or adapted for your students that other teachers might adapt for their own classroom. In addition to the lesson or task description, we suggest including the following:

- Curriculum connections
- Student action (strategies, misconceptions, examples of student work, etc.)
- Wrap-up, next steps

To submit your favourite lesson, please contact us at thevariable@smts.ca. We look forward to hearing from you!

Alternate Angles

Alternate Angles is a column on problems from multiple perspectives: various methods that could be used to solve them, insights we get from their solution, the new paths that they can lead us to once they have been solved, and how they can be used in the classroom.

Peculiar Products

Shawn Godin

Welcome back, problem solvers! Last issue, I left you with the following problem:

> Three consecutive numbers are multiplied together to form a new number: for example, $7 \times 8 \times 9 = 504$. Discover all you can about numbers of this type.

This problem was inspired by a problem from Brilliant.com, as well as many contest questions and exercises from introductory number theory books. The problem can be investigated at many levels, using different tools.

We will start by looking at the divisibility properties of these numbers. Listed in Figure 1 are the first ten examples of these numbers, factored into primes.

$$\begin{array}{rcccl}
1 \times 2 \times 3 & = & 6 & = & 2 \times 3 \\
2 \times 3 \times 4 & = & 24 & = & 2^3 \times 3 \\
3 \times 4 \times 5 & = & 60 & = & 2^2 \times 3 \times 5 \\
4 \times 5 \times 6 & = & 120 & = & 2^3 \times 3 \times 5 \\
5 \times 6 \times 7 & = & 210 & = & 2 \times 3 \times 5 \times 7 \\
6 \times 7 \times 8 & = & 336 & = & 2^4 \times 3 \times 7 \\
7 \times 8 \times 9 & = & 504 & = & 2^3 \times 3^2 \times 7 \\
8 \times 9 \times 10 & = & 720 & = & 2^4 \times 3^2 \times 5 \\
9 \times 10 \times 11 & = & 990 & = & 2 \times 3^2 \times 5 \times 11 \\
10 \times 11 \times 12 & = & 1320 & = & 2^3 \times 3 \times 5 \times 11
\end{array}$$

Figure 1: The first 10 examples factored into primes

Without even looking at the prime factorization you should have noticed that every single one of these numbers is even. This can be explained quite easily. In terms of parity, (evenness or oddness) we can see that there are only two possibilities: (odd) × (even) × (odd) or (even) × (odd) × (even). In both cases, there is at least one even number involved, hence our answer will be even every time.

We can go even further and say that if the first number is even, then there will be two even numbers multiplied together so the result must be a multiple of 4. When we check the results against Figure 1 we notice that when the first number is even the result is divisible by 8 or a higher power of two. Why does this happen?

Figure 2 looks at the first few even numbers with the even part factored out. Notice that the largest power of 2 that divides every *second* even number is 2. The largest powers of 2 that divide the even numbers go: 2, 4, 2, 8, 2, 4, 2, 16, 2, 4, 2, 8, 2, 4, 2 , 32, 2, … Similarly, if we looked at the multiples of 4, the highest power of two that would divide every second one would be 4. All the others would be divisible by higher powers. The pattern continues for all powers of two.

$$\begin{aligned}
2 &= 2 \times 1 \\
4 &= 4 \times 1 \\
6 &= 2 \times 3 \\
8 &= 8 \times 1 \\
10 &= 2 \times 5 \\
12 &= 4 \times 3 \\
14 &= 2 \times 7 \\
16 &= 16 \times 1 \\
18 &= 2 \times 9 \\
20 &= 4 \times 5
\end{aligned}$$

Figure 2: The first 10 even numbers factored into the product of an even and an odd

What does that mean for the previous problem? If the first number is even, then so is the last number, which means that they are consecutive even numbers. As a result, the highest power of 2 that divides one of them will be 2. The other one will be divisible by 4 or a higher power of 2. Hence, when the first number is even, the result is divisible by 8 or some higher power of 2.

Pretty neat! We went from the obvious: the evenness of the results, to a justification of that property, to a further generalization. Can we go any further? Looking back at Figure 1, we will quickly notice that not only is every number even, it is also a multiple of 3 (or some higher power of 3). Why does this happen?

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**

Figure 3: The multiples of 3

Looking at Figure 3, each multiple of 3 is coloured red. Imagine circling any three consecutive numbers. What do you notice? Since there are only two numbers between each multiple of 3, no matter where you start, you will always need to circle a multiple of 3.

Notice also that for multiples of two, we can have two types of number: even (a multiple of 2) or odd (not a multiple of 2). We could have a similar classification for multiples of 3: something is either a multiple of 3 or it isn't. Unlike when we were examining the multiples of 2, however, there seem to be more non-multiples of 3 than multiples of three. Consider the integers rewritten in a different form, like Figure 4. Now we have three groups: multiples of 3, 1 above a multiple of 3, and 2 above a multiple of 3 (or 1 below a multiple of 3). Note that we can also interpret the numbers at the top of each column (0, 1, 2) as the remainder when we divide numbers in that column by 3.

0	1	2
3	4	5
6	7	8
9	10	11
12	13	14
15	16	17
18	...	...

Figure 4: Integers modulo 3

Carl Friedrich Gauss introduced the idea of *modular arithmetic*. Some interesting things happen when you start playing with this system. Pick any two numbers in the table, add them together, and note where the sum is in the table. Now look at the numbers at the top of the columns of your original numbers. Add these two numbers together and note where the sum ends up. For example, 7 + 11 = 18, which is in the first column. The top of 7's column is 1, and the top of 11's column is 2. Adding, we get 1 + 2 = 3, which is also in the first column. This will work for any two numbers you pick, and it will work for addition, subtraction, and multiplication.

What does that mean for our previous problem? If a, b, and c are three consecutive integers multiplied together, we must have one number from each column. This can be written using Gauss' notation as

$$a \times b \times c \equiv 0 \times 1 \times 2 \equiv 0 \ (mod\ 3)$$

This means that, if we are only interested in the 3-ness of a number (modulo 3, or mod 3 for short), then the product of the three consecutive numbers will be equivalent to (≡) the product of the tops of the columns they occupy, which gives 0. So, since our result is 0, there is no remainder and the number must be a multiple of 3.

We can use this idea for any number as our modulus. If we look at modulo 5, that is at the 5-ness of a number, we see that it is possible to have three consecutive numbers whose product isn't a multiple of 5. For example, we see from Figure 5 is we start with something congruent to 1 (mod 5) or 2 (mod 5), then no multiple of 5 is in our list. On the other hand, if we start with something congruent to 0 (mod 5), 3 (mod 5), or 4 (mod 5) then our result will be a multiple of 5. Checking back to Figure 1, we see that when we start with 3, 4, 5 or 8, 9, 10 the result is indeed a multiple of 5.

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24
25	…	…	…	…

Figure 5: Integers modulo 5

We can also use integers to note that $3 \equiv -2 \ (mod\ 5)$ and $4 \equiv -1 \ (mod\ 5)$ to see that for *any* integer n, then if we take numbers congruent to $-2, -1$, or $0 \ (mod\ n)$, then the result will be a multiple of n. Figure 1 will show this is true; for example starting with 5, 6 or 7 yields a multiple of 7 whereas starting with 7, 8 or 9 yields a multiple of 9.

Now, let's attack the problem from a different point of view. Figure 6 is a table of values where x represents the first number in the trio and y represents the product. Looking at the differences, the third difference is constant, so y is a cubic polynomial in x. We could use Desmos to graph the data and fit the data with a cubic curve of best fit. In this case, it would fit exactly.

x	y	**1st diff.**	**2nd diff.**	**3rd diff.**
1	6			
		18		
2	24		18	
		36		6
3	60		24	
		60		6
4	120		30	
		90		6
5	210		36	
		126		6
6	336		42	
		168		6
7	504		48	
		216		6
8	720		54	
		270		6
9	990		60	
		340		
10	1320			

Figure 6: A table of values for our numbers

However, it isn't necessary to use technology. Since x represents the first number, the other two numbers must be $x + 1$ and $x + 2$. Therefore, we have

$$y = x(x + 1)(x + 2) = x^3 + 3x^2 + 2x.$$

Sometimes, when we model things algebraically, the "natural" choice of variables may not be the only one, or may not be the most useful one. Suppose, instead, we let X represent the *middle* number. Then we would have

$$y = (X - 1)X(X + 1) = X^3 - X.$$

Here we see our result is the difference between a cube of a number and the number itself. This is a bit easier to calculate and gives us a simpler relationship than the first cubic. Yet we can see they are equivalent since $X = x + 1$ can be substituted into the second expression and it will yield the first. Similarly, substituting $x = X - 1$ into the first yields the second.

Different representations of a problem often lead to different insights. The difference in the representation may be the difference between the prime factorization of the numbers and the graph of the relation formed. It can also be the difference between the two different ways we can handle things algebraically. It is often a worthwhile exercise to look at a problem in different lights or search for different solutions to the problem. There is still more we could do with this problem: look at a physical representation of the numbers (for example using linking cubes), extend to 4 or 5 consecutive numbers, or make the connection to permutations. Have fun exploring!

Now, time for your homework:

> The following information is known about ΔOBC:
> - O is at the origin, and points B and C lie in the first quadrant;
> - ΔOBC is an isosceles right triangle with $OB = BC$ and $\angle OBC = 90°$; and
> - the hypotenuse OC is on a line segment with slope 3.
>
> Determine the slope of line segment OB.

Until next time, happy problem solving!

Shawn Godin teaches and is a department head at Cairine Wilson Secondary School in Orleans, Ontario. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: leading workshops, writing articles, working on local projects and helping create mathematics contests.

Spotlight *on the* Profession

In conversation with Dr. Marian Small

In this column, we speak with a notable member of the mathematics education community about their work and their perspectives on the teaching and learning of mathematics. This month, we had the pleasure of speaking with Dr. Marian Small.

Marian Small, former Dean of Education at the University of New Brunswick, writes and speaks about K-12 math across the country. Her focus is on teacher questioning to get at the important math, to include all students, and to focus on critical thinking and creativity.

Some resources she has written include Making Math Meaningful for Canadian Students: K-8, Big Ideas from Dr. Small *(at several levels),* Good Questions: A Great Way to Differentiate Math Instruction, More Good Questions: A Great Way to Differentiate Secondary Math Instruction, Eyes on Math, Gap Closing *(for the Ministry of Education in Ontario),* Leaps and Bounds toward Math Understanding *(at several levels),* Uncomplicating Fractions, Uncomplicating Algebra, Building Proportional Reasoning, Open Questions for the Three-Part Lesson *(at several levels)*, Fun and Fundamental Math for Young Children, Math that Matters: Targetted Assessment and Feedback for Grades 3-8, *and* The School Leader's Guide to Building and Sustaining Math Success. *Marian is currently authoring, with Rubicon Publishing,* MathUp, *a new digital teaching resource.*

First things first, thank you for taking the time for this conversation!

Although you have explored a wide variety of issues in your work as a consultant, researcher, speaker, and author, one enduring focus has been differentiation. Over the years, differentiation has received increased attention from all levels of education, but as a catchall term, it is likely to be interpreted by different educators in many different ways.

What do you mean by differentiation in the math classroom? Does it involve making multiple lesson plans—i.e., more work for teachers?

My intention is that teachers recognize that different students in any class may have different needs or are ready for/comfortable with different tasks or questions. The teacher tries to accommodate those differences in the interest of those students. It is not reasonable, for example, to believe that every single student in a particular Grade 4 class is at exactly the same "academic" spot in math on a given day.

> Differentiation does not involve making multiple lesson plans; it involves using similar tasks at different levels or questions that are open enough that many students can engage at different levels.

I think it does not involve making multiple lesson plans; it involves using either similar tasks at different levels or questions that are open enough that many students can engage at different levels.

One of the differentiation strategies you have advocated is open questions, which you describe as evoking "a broad range of responses at many levels" (Small, 2009b). Could you expand on this definition, and give a few examples of open questions that would be appropriate at different grade levels?

The notion of an open question is one to which there are many correct interpretations or responses.

There are different styles of such open questions. Some are "super open." For example: "The answer is 2/3 . What might the question have been?" (Possible answers include "Name a fraction.", "Name a fraction between 1/2 and 1.", "Tell the sum of 1/3 and 1/3.", "Name a number.", "Name a number less than 1.", etc.)

Some are somewhat less open, but still have many possible responses. For example: "Tell three things about the number 7 that are true.", or "Name a number you think is big and tell why you think it is big.", or "Describe an equation that you think is really easy to solve and tell why you think that."

Some are open because they allow for choice, such as the following questions: "Choose a radius and height for a cone. Tell its volume." "Choose two 2-digit numbers to multiply. Tell the product."

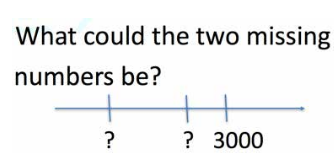
Conversely, what is a "closed" question?

A closed question has one correct answer. For example, "What is 23 × 45?" is a closed question, in contrast to "Choose two 2-digit numbers to multiply and multiply them," which is open. Even though there are many strategies students can use to solve 23 × 45, there is only one possible response that is correct.

In Small (2017a), you explain that "open questions [...] are often deliberately vague and require students to make sense of the question before they can choose the direction in which to go" (p. xiii). Won't struggling students feel overwhelmed by such questions? Why is it that more structure may not be appropriate in meeting the needs of a diverse group of learners?

First of all, it is important to understand what is meant by "deliberately vague." It does not mean careless or not well-constructed; it simply means there is room for interpretation or to make choices.

For example, asking students



is vague in that no scale is given so that there can be many possible answers, but that was purposeful. We want students to learn that without a scale, there are many possible answers. I am sure some students will wonder where 0 is in the previous question and ask about it, to which the teacher can reply, "You get to decide." This can take away the possible concern a student might have had.

As well, I think that there are students who do like structure, but there are also students who enjoy and benefit from choice. Not forcing every student to divide the same two fractions and allowing students to choose the fractions that are comfortable for them helps some students and does not overwhelm them.

Can questions be too *open? (I wonder if you could give an example.)*

I am not sure there is a "too open" question in theory, but I believe that some students prefer less openness than others. For example, rather than asking students to choose a figure and describe its net, some students would wish that you suggest what figures to choose from. I think a teacher can always leave it more open and support students who want more structure by providing only those students with additional structure. By leaving it open, conversation will be richer since some students may choose figures nobody else thought of.

> A teacher can always leave questions more open and support students who want more structure by providing only those students with additional structure.

A question like "How many baby steps make a giant step?" is fine for some students, but others will be nervous until you clarify what you mean by those terms. Again, if we are trying to get students to learn that a measurement can be large for two different reasons—either a small unit or a "large" object—it is best not to tighten it. But if a student finds this too much to deal with, a teacher has the prerogative of giving suggestions.

Besides open questions, another differentiation strategy you advocate is parallel tasks, where students are presented with two or more similar tasks that focus on the same big idea but are at different developmental levels (Small, 2009b). With parallel tasks, students "must make sense of both options to decide with which one to proceed" (Small, 2017a, p. xiii).

Parallel tasks, then, contrast with open questions in that different students may work on different problems simultaneously. How good are students at selecting a task that is appropriate for their developmental level?

Parallel tasks contrast with open questions because they may be convergent rather than divergent, and because different problems are being worked on simultaneously. Sometimes the teacher might choose for the student, but giving students the right to choose is often good; they may actually be able to do more than we thought. At worse, they choose the "wrong" one and switch.

How does assessment change when open and parallel tasks are the norm in a classroom?

I think that it would be natural to include some open questions and some choices in assessment of learning situations if they were a regular feature in classroom instruction. That doesn't mean all questions are open or parallel, but some are.

Another major theme of your work is that of "big ideas" (e.g., Small, 2009a, 2010). As you write in Small (2017b), "by organizing content around big ideas, teachers can teach more efficiently" (p. 15). Could you describe one or two of the big ideas that students will encounter across grade levels? How can focusing instruction around big ideas facilitate the teaching and learning of mathematics, and how can this support differentiation?

> I think that differentiation is not possible unless teachers teach to big ideas. If ideas are too 'tight,' there is not enough room to differentiate.

I think that differentiation is not possible unless teachers teach to big ideas. If ideas are too "tight," there is not enough room to differentiate. Here are three examples:

Big idea: Numbers can be represented in different ways and each representation might reveal something different about the number.

- At a Grade 1 level, students might show some representations for 7 that show that 7 comes after 6 (e.g., on a number line), others to show that 7 is 5 and 2 more (e.g. ,using tally marks), and others to show that 7 is odd (e.g., writing it as 2 + 2 + 2 + 1).
- At a Grade 5 level, students might show representations for 5/3 that show that it is 5 sets of 1/3 (e.g., 5 jumps of 1/3 on a number line), or that show it's slightly less than 2 (e.g., as $1\frac{2}{3}$), or that show that it is 5 ÷ 3 (e.g., as 5 objects grouped into sets of 3).
- At a high school level, students might show one representation for $\sqrt{8}$ that shows it is between 2 and 3 (e.g., on a number line), and a different one that shows that it is twice as much as $\sqrt{2}$ (e.g. showing a 1-1-$\sqrt{2}$ right triangle nested into a 2-2-$\sqrt{8}$ one).

Big idea: There is no way to know how a pattern continues without a pattern rule.

It does not matter which pattern beginning is used—students need to learn that there are always alternate ways to continue the pattern until a rule is given. For example, the pattern below,

3, 5, 7, …

might be continued as

9, 11, 13, …

or as

13, 15, 17, …

Big idea: Measurement formulas allow us to use measurements that are simpler to access to figure out measurements that are harder to access.

For example:

- It is easier to determine the area of a rectangle by measuring the length and width than counting all the squares you could make inside.

- It is easier to determine the circumference of a circle by measuring the diameter than trying to directly figure out the length around.
- It is easier to determine the volume of a cylinder by determining the height and radius and using the formula than to dip it in water and use displacement.

Lastly: As a speaker, you have addressed not only teachers, but also parents, many of whom are unfamiliar with some of the strategies you propose. Some feel that renewed curricula are not rigorous enough, or do not focus enough on the "basics" or standard algorithms. How do you address parent concerns about new, unfamiliar mathematics curricula and teaching practices?

First of all, none of the curricula we are using today are new; they have been around for a while. Secondly, if we believe that education is a profession, why would "lay people" (parents) get to choose educational directions? Patients rarely tell doctors what to do, but take their advice. Thirdly, we are talking about the basics for a society that is changing. Many employers are now saying that what they need in the workplace is people who can figure things out, not people who can memorize rules. So while there is nothing wrong with standard algorithms, working on problem-solving skills may be more rigorous and a better use of learning time.

> While there is nothing wrong with standard algorithms, working on problem-solving skills may be more rigorous and a better use of learning time.

As far as standard algorithms go, some curricula do require standard algorithms; some do not. It helps parents when they see that what is a standard algorithm in one country is actually different than what is the standard in another, so there is no one perfect algorithm. It also helps them see that they often do not use standard algorithms, e.g. if they were subtracting 2 from 200.

Interviewed by Ilona Vashchyshyn

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What, How, Who: Developing Mathematical Discourse [1]

Kelley Buchheister, Christa Jackson, & Cynthia E. Taylor

Kyra and Jamea closely examine the numbers and letters in four rectangles (see Figure 1). They point to different areas, share what they notice, and—without prompting from their seventh-grade teacher, Ms. Boyana—make conjectures about what their observations mean.

Kyra: The Q just has a letter and no numbers. All the others have letters and numbers.
Jamea: Maybe it means there is just one. But, there is a 1 next to the N in the other square.
Kyra: Maybe because it has other letters and numbers in it?
Jamea: Maybe Q means "questions" because we do questions in here and P means "people." Do we have 25 people?
Kyra: [Counting] No, we only have 19.
Jamea: Maybe it's days for D and nights for N. Like one of these is 2 days and 1 night and another one is 6 nights.
Kyra: But that doesn't make sense. Q doesn't fit. What other things start with Q? Queen? Quiz? Question? Quarter? Quail?
Jamea: Wait! Quarter? A quarter is 25 cents! But, that doesn't fit. There's no C.
Kyra: [Takes the paper] Cents are pennies, so maybe the P is pennies. Do you think the other ones are money, too?

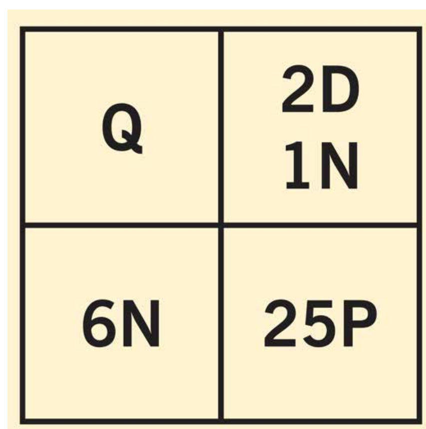

Figure 4: Students were asked to analyze these numbers and letters in the first "What Do You Notice?" task.

With an increased focus on using social discourse to enhance students' mathematical thinking and reasoning (NCTM, 2014; Staples & King, 2017), teachers are looking for discussion strategies that encourage middle-level students to make sense of mathematical concepts. However, structuring these valuable discussions is complex. "Mathematical discourse should build on and honor student thinking, and provide students with opportunities to share ideas, clarify understandings, develop convincing arguments, and advance the mathematical learning of the entire class" (Smith, Steele, & Raith, 2017, p. 123). In other words, teachers must carefully consider what tasks provide meaningful opportunities to explore ideas, generate hypotheses, and promote questions within a collaborative environment. Then, teachers need to consider how to structure the activity to encourage discussions and incorporate responses that contribute to understanding specific mathematical objectives. Additionally, teachers must select who will speak to "advance the mathematical storyline of the lesson" (NCTM, 2014, p. 30). By intentionally focusing on these elements in mathematics instruction, middle-grades teachers can develop a classroom culture that not only emphasizes sense making but also values the intellectual capacity that students bring to the classroom (Gutiérrez, 2013; Lemons-Smith, 2008). In this article, we describe how teachers can promote meaningful discussions using the what-how-who

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structure while giving students opportunities to make sense of mathematical ideas within a social context.

What tasks stimulate meaningful discourse?

One of the greatest contributions to students' opportunity to learn is the selection of tasks (Lappan & Briars, 1995). Mathematics teachers must analyze the standards to determine what content to teach and identify which tasks embody the desired content and skills because different tasks promote different kinds of thinking (Stein, Smith, Henningsen, & Silver, 2000). Thus, to provide a strong foundation for what mathematics students will learn (Hiebert et al., 1997), it is imperative that teachers intentionally identify what tasks (a) provide relevant connections to students' funds of knowledge, (b) stimulate meaningful opportunities to explore mathematical situations, (c) encourage students to generate questions, and (d) promote sense making through collaborative discussions.

An open-ended task, such as "What Do You Notice?" discussed in the opening vignette, is a logical format for removing barriers (Sullivan, 2003). By providing multiple entry points, each and every student can gain access to and engage in discussions of mathematical content. The task, adapted from Danielson's (2016) book Which One Doesn't Belong? prompts students to investigate the similarities and differences among each representation and discuss their observations in a social context. For example, after making several observations and hypotheses to make sense of the letters and numbers, Kyra and Jamea concluded that the initials related to money: Q represented quarters, P symbolized pennies, and the N and D represented nickels and dimes. Finding this connection prompted the students to generate additional observations: "All the coins make 25 cents except this one; it's 30" (i.e., 6N represented 6 nickels, or $0.30), "the quarter is the one that makes 25 cents with the least number of coins," and "6 nickels is the only one with an even number of coins and an even number of cents." Not all students identified that the letters represented coins and their values. Some students speculated that the letters were abbreviations, whereas others argued that they were variables. Some students also noted the lack of numbers in the Q frame or noticed the color of the coins—"pennies are the only [coins] that are not silver."

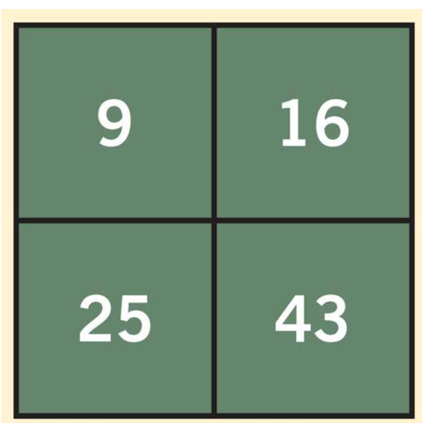

Figure 5: In the second "What Do You Notice?" task, only numbers appeared.

Such open-ended tasks not only provide access for a diverse group of students to engage in conversation but also offer flexibility to teachers to adapt them to a range of mathematical concepts, such as number sense (see Figure 2) and geometry, and use them at different times. Implementing a task during the initial phase of a lesson can also serve as a formative assessment, review, or introduction to a new concept. For example, the opening task could be used to introduce the differences among coefficients, abbreviations, and variables; to show how variables represent quantities; and to solve simple equations (Common Core State Standards Initiative, 2010).

This example can also be extended into problem-solving explorations that can last the entire class period. "What Do You Notice?" can be extended to (a) justify an "imposter" by identifying which representation does not belong (Danielson, 2016; Wyborney, 2015) and (b) create new problems containing multiple "imposters." When Boyana integrated an extension with her seventh graders using the numbers-only task (see Figure 2), new conversations and mathematical discussions emerged. Students discovered that there could be multiple reasons why different numbers did not belong. During small-group discussions, she encouraged continuing investigations, such as identifying how each number could be the "imposter," which then generated additional questions referring to mathematical relationships (e.g., "Does 25 not fit because it's the only number that can be represented with a single coin?" "Is it because 25 is the only number in the set that is a factor of 100?"). Structuring classroom activities using open-ended tasks provides a flexible foundation that can positively contribute to developing discussions that enhance students' mathematical reasoning.

How do we structure tasks to encourage discourse?

The variable nature of open-ended tasks can stimulate mathematical conversations and allow students to negotiate a shared meaning and understanding of the mathematics. However, it is the teacher who takes an active role in purposefully facilitating activities to promote social discourse. Applying the "wonder" component to open-ended problems like "What Do You Notice?" serves as a pedagogical strategy. It can pique students' curiosity and encourage new questions and inquiries as students make sense of representations without risk of failure. For example, when Boyana gave her students Figure 2, she overheard them ask their partners: "Are these numbers supposed to all fit together?" "I wonder why they are all together. Do they follow a pattern?" She then asked her students to take two to three minutes to examine the four numbers closely and write down as many observations as they could. Once time was up, she asked the students to turn and share with a partner. Some students noticed that 9 was the only single-digit number and the only number with digits not totaling 7. They also noticed that 16 was the only even number. As students shared their observations, they clarified their thinking and generated questions to negotiate meaning of the mathematics embedded in the task.

> The variable nature of open-ended tasks can stimulate mathematical conversations. However, it is the teacher who takes an active role in facilitating activities to promote discourse.

Presenting tasks by first asking students to make individual observations allows them time to make sense of the representations. Following the individual reflection, students can discuss their observations with partners or in small groups. Students in Boyana's class willingly shared their observations. By first prompting students to record what they noticed about the numbers in the task, she provided a safe environment in which students were more comfortable sharing their thoughts without extensive pressure on identifying a solution. Approaching tasks by first making observations, then sharing what questions emerge encourages students to construct mathematical knowledge through social interactions with meaningful problems.

Who is speaking?

Such tasks as "What Do You Notice?" allow teachers to engage students in meaningful conversations that "develop language to express ideas, represent evidence, and clarify their reasoning" (Staples & King, 2017, p. 38). Therefore, it is critical that each and every student is given an opportunity to engage in the classroom's social discourse. Without explicitly and purposefully attending to whose voice is represented in classroom conversations or valuing the out-of-school knowledge that students bring, teachers are not giving students the support, confidence, or opportunities necessary to reach their highest levels of mathematical success (NCTM, 2014). Thus, teachers must be cognizant of who answers questions, solves tasks, or shares mathematical strategies while implementing instructional decisions that recognize a variety of students' contributions.

The design of the problem allows students to take risks because they recognize that all contributions are valued and that each and every voice is heard. While listening to her students' conversations during the number- only version of "What Do You Notice?" Boyana heard one student share that 16 was the age of her sister, and another student commented that 25 was his favorite number. At the other end of the room, she overheard another student wondering about the relationships among the numbers as he noticed characteristics such as 43 being the only prime number and 16 being the only even number. She asked each student to share one observation with the class. Sharing simple observations, such as "there are four numbers and they are all different," allowed students to feel more comfortable in the social setting because each response was valued. Boyana not only valued students' voices but also empowered students by exploring student-generated questions. As the class continued to share, one student noticed the equations $3 \times 3 = 9$, $4 \times 4 = 16$, and $5 \times 5 = 25$ on a peer's paper. She exclaimed, "I did that, too! Three squared, four squared, and five squared are all perfect squares! I wonder if 43 is a perfect square, too?" Boyana encouraged students to work in small groups to explore this question. Michalla stated, "Well, 6×6 is 36 and 7×7 is 49, so no. Forty-three can't be a perfect square." Boyana asked Stefan if he agreed with Michalla's conjecture. Stefan said yes. She continued to push Stefan's thinking and asked why he agreed. Stefan replied, "Well, 43 is between 36 and 49, and those numbers are perfect squares. But, you would have to square a number between 6 and 7 to get 43." By first identifying and sharing what students noticed, and then exploring the questions students generated, Boyana provided valuable opportunities for students to negotiate meaning as they analyzed the reasonableness of different mathematical arguments (Staples & King, 2017).

> Without explicitly attending to whose voice is represented in classroom conversations, teachers are not giving students the opportunities necessary to reach their highest levels of success.

Finally, Boyana extended the activity by asking students to create their own "What Do You Notice?" task to share with the class to further encourage and validate each and every student's voice.

J. T.: Remember when all of the digits added to 7? What if we make a grid where the digits add to a different number, like 9?
Christy: OK, what numbers will work? I like 18, 27, 63, and 90. But in the first problem, we did not have any three-digit numbers. What if we add 108?

The group initially decided to use the numbers 5, 18, 45, and 63 but were not satisfied with the task they created. Trevon commented, "This will be way too easy for them because they just did the other problem." The group finally decided to use pi because they thought it would be more challenging for their peers to figure out the pattern.

J. T.: So, pi is 3.14, right?
Christy: Remember, we should use 3.1415 [erases boxes and records].
Trevon: OK, what if for the number in the next box we multiply pi by 3?
Christy: No, let's make it harder. Let's multiply each digit by 3.
J.T.: I don't get it. What do you mean each digit?

Christy: Well, if we take 3 × 3, that is 9. Then 1 × 3 is 3; 4 × 3 is 12; 1 × 3 is 3; and 5 × 3 is 15. So, the next number would be [Christy writes and reads] 9.312315.

Trevon: That's cool. So, if we take 3 times each digit in that number Christy just said, we would get something like 27.936945.

J. T.: [Looks at Trevon's paper] I don't see how you got 45 at the end because 1 × 3 is 3, and 5 × 3 is 15; so, shouldn't it be 27.9369315?

Trevon: No, I like it where we take the last two digits and multiply them by 3 because then people will really have to wonder where we got this number.

Christy: Oooo, that sounds so good. OK, so, the last number would be 81.2791827135 [see Figure 3].

Trevon: Awesome. Ms. Boyana, we are ready!

Although this student-generated example demonstrated creativity using single-digit multiplication, it also caused confusion because students assumed the pattern followed the traditional multiplication algorithm. Because the example stumped the entire class, Boyana chose to discuss this task more in depth and asked students to analyze and make sense of the underlying mathematics. Allowing students an opportunity to create their task can elicit richer conversations. Developing this culture of learning enhances sense making and motivates each and every student to remain engaged in creative brainstorming while discussing similarities, differences, and relationships among the observations.

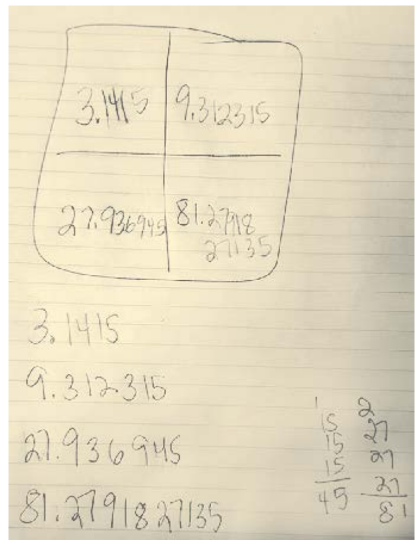

Figure 6: A student generated this "What Do You Notice?" task.

Final thoughts

"Mathematical discourse is a critical practice through which students develop mathematical communication and argumentation skills and the ability to critique the reasoning of others" (Staples & King, 2017, p. 37). Boyana developed a culture of discourse using the what-how-who structure by attending to "what" tasks she selected, "how" she structured the classroom conversations, and "whose" voice was heard during the discussion. Open-ended tasks, similar to "What Do You Notice?" allow teachers to facilitate mathematical discourse using the what-how-who structure and empower students to explore mathematical content within a social context. By integrating this structure into mathematical activities, middlelevel teachers can build a classroom culture that not only emphasizes sense making but also recognizes the intellectual capacity that all students bring to the classroom (Gutiérrez, 2013; Lemons-Smith, 2008).

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Constructivism in Secondary Math Education: A Teacher's Experience

Jeff Irvine

Twenty years ago, the Ontario Ministry of Education mandated massive policy changes in Ontario education, similar to changes that occurred around the same time in Saskatchewan. Designed to be coherent across curriculum, assessment, and pedagogy, these changes were intended to support constructivism, a theory of learning based on the work of Jean Piaget (1954). Piaget, who Ernst von Glasersfeld (1990) called "the father of constructivism," espoused the philosophy that students learn new material by building upon what they already know. For example, students learn that $5 \times 4 = 20$ by building on their knowledge that $5 \times 1 = 5$, $5 \times 2 = 10$, $5 \times 3 = 15$, and so on. Constructivists further believe that students know about $5 \times 4 = 20$ in various ways: for example, by skip counting by 5s (5, 10, 15,…); by recognizing the number fact as 4 groups of 5 objects (or 5 groups of 4 objects); by counting upwards on a number line; by seeing 20 as the area of a rectangle 5 units by 4 units; etc. Boaler (2008) found that students who learn this way not only understand why $5 \times 4 = 20$, but are better able to recall and use their number facts, as compared to facts memorized by rote that are often misremembered. Of course, students still need to know that $5 \times 4 = 20$ when the fact is needed. For an example from secondary education, students develop their understanding of the effects of transforming sinusoidal functions based on their prior understanding of transformations of quadratic functions.

> The theory of constructivism shifted the teacher's role from telling, and the students' role from passively receiving knowledge, to teachers guiding students' personal development of understanding.

Teachers who support this theory of learning offer children multiple opportunities to understand concepts in a variety of ways through investigation and problem solving. This approach to teaching, sometimes erroneously called "discovery learning," was pioneered by John Dewey in his book *The Child and the Curriculum* (1902) and supported by the work of Lev Vygotsky (1934), who highlighted the need for learning through social interactions in groups, as well as the zone of proximal development, which gave guidance on how to scaffold activities to provide challenge to students while keeping the learning goal within reach. The theory of constructivism shifted the teacher's role from telling, and the students' role from passively "receiving" knowledge, to teachers guiding students' personal development of understanding. (The foundation of constructivism can be seen in Ontario curriculum expectations (outcomes, in Saskatchewan curriculum parlance) that encourage "investigation," "problem solving," and "using a variety of methods.") Supporting constructivist learning, therefore, does not mean unrestricted "discovery." Students are unlikely to discover the formula for the volume of a cylinder just by playing with some cylindrical cans. However, with the guidance of their teacher, and with some choice in their activities some of the time, multiple studies (e.g., Fosnot & Dolk, 1995) have shown that students understand concepts better and retain the information longer. They also tend to be more engaged in school and be more motivated to continue their studies (e.g., Fredricks, Blumenfeld , & Paris, 2004). It is therefore vital that students take an active role in their own education, especially if we want students to become lifelong learners.

When the Ontario Ministry of Education enacted the policy changes, many teachers were already using investigations and problem solving in their classrooms. However, some teachers found it difficult to implement this new pedagogy. The dominant learning theory in mathematics teaching was (and often still is) behaviorism, emphasizing rote learning of facts and procedures, with an emphasis on speed of recall on demand. This theory relies on a transmission style of teaching, with the teacher demonstrating procedures and stating facts and students memorizing the procedures and facts for future recitation. This was the predominant teaching style for much of the twentieth century. The bad news is that it didn't work well. Working with adults, Cathy Bruce, Dean of Education at Trent University, found that up to 80% of math students taught in this way are either math uncomfortable, math averse, or math phobic (Bruce, personal communication, April 9, 2009). To get a sense of the scope of the problem, consider that "I was never good at math" is a socially-acceptable statement for many well-educated adults to make (Dweck, 2006).

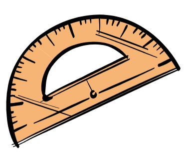
> Many of today's teachers were taught in the transmission and memorization era, when students learned to mimic the teacher's way of solving problems, often without understanding why the method worked.

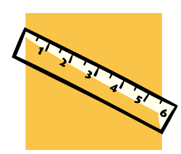
Many of today's teachers were also taught in the transmission and memorization era, when students learned to mimic the teacher's way of solving a math problem, often without understanding why the teacher's method worked. This resulted in a piecemeal implementation of the policy changes in Ontario, with teachers choosing what parts of the policy that they would implement, often based on their own personal teaching philosophies. The result was pockets of teaching excellence in Ontario along with teachers, schools and sometimes school boards that lagged in pedagogy and/or assessment changes. In addition, many math teachers were convinced that the changes did not fit the needs of secondary math classrooms, especially in the senior grades. As such, this article provides some examples of how senior math class concepts can be enacted through the lens of constructivist theories of learning. Using these activities results in deeper understanding while still addressing content concerns.

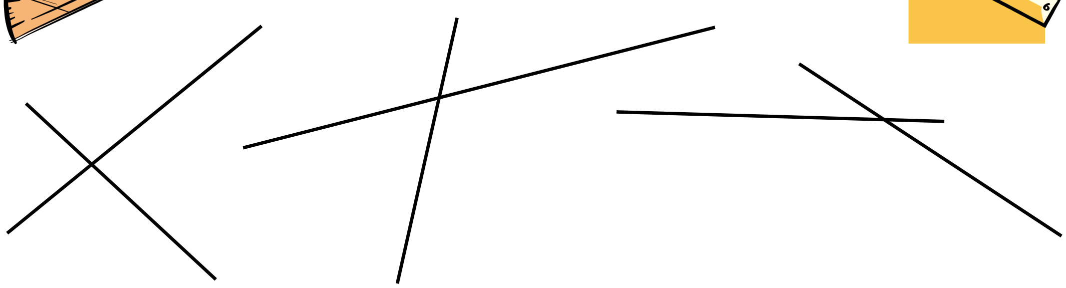
Here is an example of such an activity.

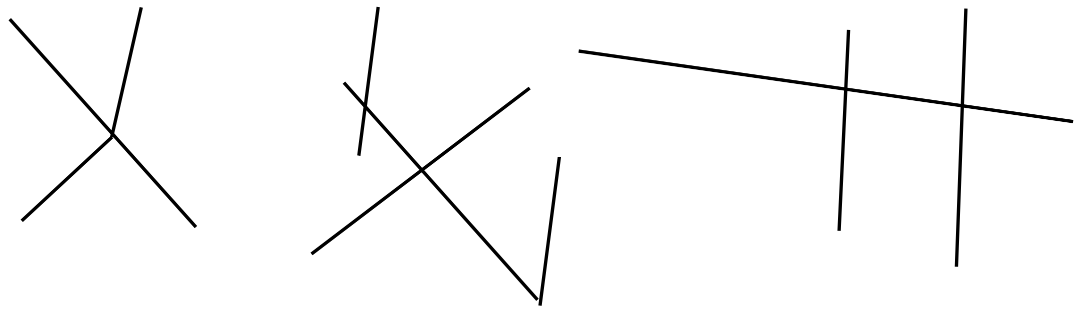
Investigating Angle Relationships and Posing Theorems About Angles

Measure everything and look for patterns.









Use large chart paper to record the patterns you found. Construct a group display using your chart paper for a gallery walk.

During the gallery walk, your group should identify:

- any patterns that your group found;
- any patterns that other groups found that your group did not;
- other groups' patterns with which you don't agree. Be sure to give reasons, or counterexamples.

Students typically cite angle relationships such as opposite angles are equal; when lines are parallel, alternate angles are equal, corresponding angles are equal, interior angles on the same side of the transversal are supplementary; supplementary angles sum to 180°. Students initially may not use correct mathematical language such as supplementary, alternate angles and so on; this provides an opportunity for students to conduct brief research to find the correct terminology. Another motivating strategy is to name the pattern found after the student or group who first discovered it. This activity demonstrates the power of teachers supporting a constructivist theory of learning. Rather than just telling students about vertically-opposite angle relationships, students investigate and find patterns that the teacher can confirm during whole-class consolidation. Similar activities can be used for triangle properties and classification; quadrilateral properties and classification; and properties of angles in circles. With students more involved in their own learning, constructivist-based activities tend to result in more active classrooms.

Accompanying the adoption of the constructivist theory of learning was an increased emphasis learning in groups and asking students to explain and justify their decisions. Research by Hedges (2012) found that the quality of group decisions and group products was superior to any single member of the group, even the highest-achieving group member.

Here is an activity involving the concept of probability for Grade 12 that relies on group processing.

How Rare is Rare?
Individually, students complete the following chart. They then seek consensus from their group members. Once everyone agrees on the likelihood of each event, the groups identify what information they would need to determine whether their answers are accurate, formulate a strategy for solving each problem, then determine the likelihoods in question. Initially students may select a response based only on intuition. However, when challenged by other students, very quickly students resort to constructing tree diagrams or lists, internet research, calculations and logical arguments. Because the situations are real-world

and related to the student's own lives, students have reasons to engage with the mathematics and persist to find solutions that can be justified to their entire group and class. This can then be supported by whole class discussions concerning what data are required to compute more accurate values, and what mathematical procedures are appropriate.

Event	Not at all rare	Fairly rare	Rare	Extremely rare
All 3 children in a family are girls				
A person you know is hit by lightening				
You win a major lottery prize				
A coin is flipped 6 times and comes up heads every time				
It is cloudy for 7 straight days in July				
The same number comes up twice in a row on a roulette wheel				
It snows for four straight days during January				
You are involved in a fender-bender accident				
You win a doughnut in a coffee shop promotion				
A horse wins 5 or more races each year for 3 years				
Your favourite TV show is cancelled				
You get a royal flush in poker				
When rolling 2 dice, you get a total of 7 four times in a row				

One of the arguments that teachers raise against pedagogy that involves more student agency is that it takes too much time. Indeed, there is relentless pressure to cover content, especially in the senior math courses. However, I have found that addressing constructivist learning through activities, investigations, and problem-solving actually *saves* instructional time because it results in improved engagement and understanding. The activity below, for the Mathematics of Data Management course, compresses to three days content that typically takes a week or more.

Arrangements

Card Set #1 consists of sets of three cards with different letters; Card Set #2 has four different letters; Card Set #3 has five different letters; Card set #4 consists of four cards with repeated letters (e.g., the letters THAT).

Student Instructions: Record your group's questions and answers on the large paper. Keep a record for your own notes. Look for patterns.

1. Using Card Set #1, build all of the possible arrangements (permutations) of the letters using every card in every arrangement. Make a list of the arrangements on your paper.
2. How many of the arrangements start with a vowel? List them.
3. How many of the arrangements end with a consonant? List them.
4. How many of the arrangements start with a vowel and end with a consonant? List them.
5. How many arrangements do not start with a vowel? List them.
6. How many of the arrangements neither start with a vowel nor end with a consonant? List them.
7. If your letters have more than one vowel, how many of the arrangements have all of the vowels together? List them. If your letters have only one vowel, skip this question.
8. Explain how you organized your list in Question #1 to ensure that you didn't miss any arrangements or accidentally count the same arrangements more than once.
9. Repeat questions #1 through #8 for Card Set #2 and Card Set #3.
10. Draw conclusions from your data and identify any patterns.
11. Conjecture the number of different arrangements for each of the sets of letters in Card Set #4, assuming every letter is used in every arrangement.
12. In your journal, summarize what you have learned from this activity and any additional questions that you have.
13. Present your group's findings to the rest of the class.

I use a similar activity in the data management course to differentiate permutations from combinations. Students write their names on slips of paper and then three names are withdrawn from a bag. We start with five names in the bag. The first situation has the first student drawn designated class president, the second vice president, and the third class secretary. Whole-class discussion leads to the number of possible class executives that can be formed. Then the question is posed: "What if we just want a committee of three people, and the order of drawing is not important?" After discussion leads to the correct number of possible committees, groups investigate the impact of having more than five names in the bag. Consolidation compares the results of permutations (order matters) to combinations (order doesn't matter) and leads to the generation of computational formulas.

Activities like the ones in this article allow teachers to focus on the big ideas of processes and content in mathematics. So, the answer to "Can secondary math teachers support the theory of learning called constructivism is "absolutely." Not only can we, we must. Supporting student agency in mathematics does not mean that students no longer need to have important facts and procedures at their fingertips. However, the deeper understanding that is a consequence of supporting a constructivist theory of learning provides students with a better fundamental set of tools and strategies to enable them to transfer their learning to new problems and new situations and make them more able mathematics learners, mathematics users, and mathematics producers.

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Jeff Irvine has been a secondary math teacher in two Ontario boards of education, a secondary math department head, and a secondary vice principal. Jeff has taught at 3 faculties of education and at Sheridan College. For several years he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was grades 7 to 12 mathematics for the Province of Ontario. Jeff is a coauthor or contributing author for 11 high school mathematics textbooks. He is currently an instructor of mathematics education at Brock University, where he is pursuing a PhD.

A Day at the Museum of Mathematics

Timothy Sibbald

With more than a million visitors since it opened in December of 2012, in addition to the million people reached through its outreach programs, the National (U.S.) Museum of Mathematics in New York City has clearly been successful in its mission to enhance public interest in mathematics. While it may not be surprising to math teachers that such an interest exists, this article recounts a visit to the museum and asks what we can learn from its success.

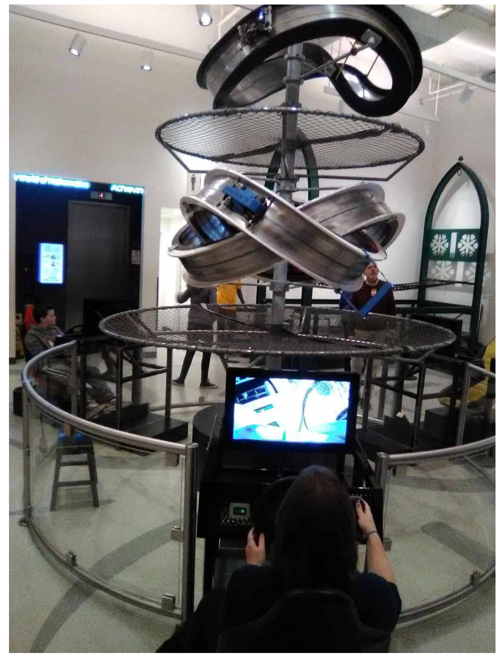


Figure 1: The twisted racetrack with POV driving console

My visit to the Museum of Mathematics (MoMath) took place on a Saturday morning in February. Although I arrived shortly after it opened, it was already buzzing with children and parents! From the moment I grasped the π shaped door handles and hung my coat beside an oscillating wall displaying a slowly-moving vertical sine wave, I knew the experience was going to be rich.

The centerpiece of the main floor, shown in Figure 1, is a racing set-up featuring two twisted tracks, with race cars secured to the metal track using magnets. A driving console allows one to control a car on the track and to see a video screen from a camera mounted on the car. The camera shows the driver, on one of the two tracks, what it's like to drive on a Mobius strip (a surface with one continuous side)!

While much of the museum is oriented to K-12 students, it is not constrained by curriculum considerations and therefore provides a much richer connection to the larger scope of mathematics, which the race cars on the Mobius strip exemplify. Throughout the museum, there are many such hands-on activities, and each is accompanied by a display screen that starts by offering a simple explanation of the given concept that children can understand. A tap reveals another screen with a little more detail, generally aimed at children who are more engaged. Another tap reveals a screen that is nominally for parents, drawing out the mathematical idea a little more. An additional screen that will interest teachers provides a sense of the connection of the concept behind the activity to the larger field of mathematics. A final screen tends to offer a brief sense of where the mathematics fits into the larger world of research, outlining perhaps what is not yet known.

Figure 2: Display of hologram images

Figure 3: Interactive video floor

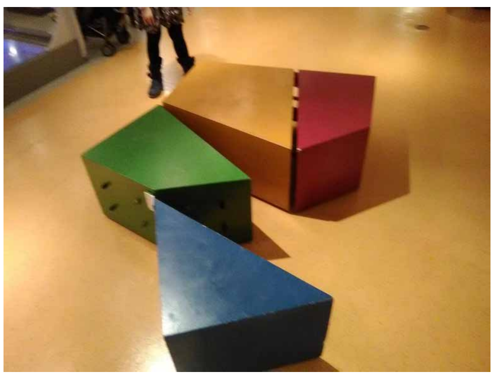


Figure 4: MoMath tangram seat

Figure 5: Galton board with a bias factor (shown numerically at the top right)

The museum consists of two floors that are linked by, of course, a spiral staircase. The layout tends to draw one through the middle of the upper floor and down the middle of the lower floor. Around the edges there are less pronounced, but equally intriguing activities and exhibits. For instance, a display of hologram images (see Figure 2) near the entrance is easily overlooked initially, but is quite impressive as moving left to right changes the perceived images. Similarly, a smaller activity involving a tray rolling on unusual shapes, all the while remaining horizontal, captures attention… after the unusual track designed to work with square-wheeled tricycles has been thoroughly explored.

The basement welcomes visitors with a large interactive video floor (see Figure 3), which displays activities supported by the interactive nature of the video screen. One such activity involved generating contour lines according to where people were standing on the display floor, which were actively changing as the people moved. Another activity displayed a maze and challenged people to enter it and try to escape by only turning left.

Some activities will be familiar to teachers, but are presented with a new twist. For example, tangrams are relatively common in classrooms, but have you seen one as shown in Figure 4, constructed using prisms and creating a hinged seat? Similarly, one wall of the museum features a large Galton board (See Figure 5), a tree of pegs where falling balls pass left or right and thereby demonstrate a binomial probability that can approximate a normal distribution. Unlike most Galton boards, the activity in the Math Museum allows a person to make a small mechanical alteration that introduces a bias at the top of the tree. There is a measurement of the bias and one is able to see what impact it has on the distribution of balls at the bottom of the tree. The activity not only demonstrates that chaotic behaviour can repeatedly lead to the same distribution; it also shows something akin to the butterfly effect, where a small alteration in the form of a bias can have significant consequences for that distribution.

One activity that is well suited to math or science fairs is a doorway that has many directional lights shining across the opening. There was a collection of clear plastic objects that one could place in the doorway to observe the resulting cross section. For example, placing the plastic cone in the doorway at different orientations revealed conic sections. Also available for cross-examination were a plastic tetrahedron and a dodecahedron. After some experimentation with these objects, few people can resist thinking about their own cross-section as they pass their hand or whole body through the doorway!

Figure 6: Conic sections in the light doorway

Two activities used interactive cameras set up with feedback loops, allowing one to participate in a fractal! Standing in front of one of the cameras with my arms up, I generated the fractal tree shown in Figure 7. To create the effect, a smaller copy of the original image was reproduced on both of my arms. Since each of the smaller images also has two arms, the process repeated itself with each of them. The process allows one to engage in the construction of the fractal by moving one's body and seeing what the consequence to the overall geometric shape is.

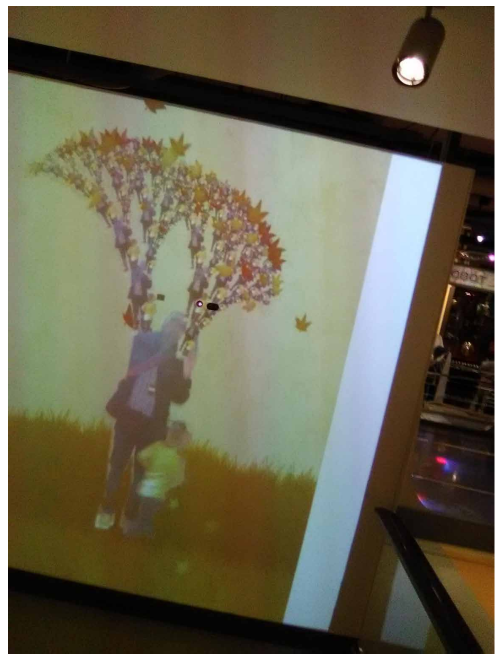


Figure 7: The author becomes a fractal tree!

There were many other activities, including an activity that allowed visitors to build three-dimensional solids, a parabolic calculator, a basketball shooting experiment, and an adjustable racetrack for seeing how fast a vertical car can reach the horizontal end. In the special exhibits section, there was a collection of math cartoons. Few children visited the cartoons, but the display demonstrated how the museum takes into consideration the broader scope and impact of mathematics.

I left the museum feeling invigorated by the energy of the place. And yet, I wonder why we don't see more of these kinds of activities in our own communities. While an entire museum dedicated to mathematics may be beyond the scope of most towns, I think more could be done to highlight mathematics in our local science centers. More could also be done in universities and colleges in the form of hands-on outreach. And in schools—why not have more pentagonal sinks (see Figure 8) to infuse daily life with geometric intrigue?

Figure 8: Geometric intrigue as encouragement to wash one's hands!

While I left the museum thinking about how much more we could do to publicly encourage mathematical thinking and curiosity, an immediate actionable item is to encourage any school trips to New York city to include the Math Museum in its itinerary, because it is both educational and highly entertaining. Also consider making the trip anytime you need a dose of mathematical inspiration. Beyond that, even if you can't visit, then consider its success as encouragement to foster mathematical curiosity through hands-on activities that allow you to interact with ideas in insightful and interesting ways.

Dr. Timothy Sibbald is an associate professor at the Schulich School of Education, Nipissing University. His interests focus on classroom instructional issues, content development and delivery, as well as teacher development. He is the editor of the Ontario Association for Mathematics Education Gazette.

Intersections

In this column, you'll find information about upcoming math education-related workshops, conferences, and other events. Some events fill up fast, so don't delay signing up! For more information about a particular event or to register, follow the link provided below the description. If you know about an upcoming event that should be on our list, please contact us at thevariable@smts.ca.

Within Saskatchewan

Saskatchewan Understands Math (SUM) Conference
November 1-2, Saskatoon, SK
Presented by the SMTS

The Saskatchewan Understands Math (SUM) conference is for mathematics educators teaching in Grades K-12 and all levels of educational leadership interested in mathematics curriculum, instruction, number sense, problem-solving, culturally responsive teaching, and technology integration, and will bring together international and local facilitators to work in meaningful ways with participants in a variety of formats. This year, SUM is proud to welcome keynote speaker Robert Berry, president of the National Council of Teachers of Mathematics (NCTM), and featured speakers Anne Schwartz, educator at Escondido Union High School, and Shauna Hedgepeth, consultant with Hedgepeth Consulting.

More information at www.smts.ca/sum-conference/

Accreditation Seminars
Accreditation seminars are offered to enable qualified teachers to become accredited, the process by which teachers are granted the responsibility of determining the final mark of students in a Grade 12 (level 30) subject. The seminar provides an opportunity for teachers to challenge, extend, enhance and renew their professional experience, with an emphasis on assessment and evaluation. Participation in this seminar results in partial fulfilment of the requirements for accreditation in accordance with the Ministry of Education's publication Accreditation (Initial and Renewal): Policies and Procedures (2017).

Mid-Year Accreditation
September 26-27, 2019
Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

More information at www.stf.sk.ca/professional-resources/events-calendar/saskatoon-mid-year-accreditation

Accreditation Seminar – Initial
October 3-4, 10-11, 2019
Weyburn, SK
Presented by the Saskatchewan Professional Development Unit

More information at www.stf.sk.ca/professional-resources/events-calendar/accreditation-seminar-initial-1

Accreditation Seminar – Renewal/Second
October 3-4, 2019
Weyburn, SK
Presented by the Saskatchewan Professional Development Unit

More information at www.stf.sk.ca/professional-resources/events-calendar/accreditation-seminar-renewalsecond-1

Beyond Saskatchewan

Mathematics: A Natural High
October 25-26, 2019
Jasper, AB
Presented by the Mathematics Council of the Alberta Teachers' Association (MCATA)

Join MCATA in celebrating their annual fall conference in Jasper, Alberta. This year's keynote speakers are Francis Su, Professor of Mathematics at Harvey Mudd College, and James Tanton, Mathematician-at-Large and founder of the Global Math Project. On Friday, sessions will each focus on one essential question: 1) How do you get students excited about where they are going? 2) How do you find out where they are? 3) How do you close the gap? On Saturday, don't miss the mathematical playground: a one-session time block dedicated to the exploration of mathematical concepts in a choose-your-own-adventure-style activity. A variety of stations will be set up around the room providing you with an opportunity to engage in mathematical conversations and activities.

More information at www.mathteachers.ab.ca/information-and-registration.html

BCAMT Fall Conference 2019: Math for All
October 25, 2019
Surrey, BC
Presented by the British Columbia Association of Mathematics Teachers

The BCAMT is proud to present the 2019 edition of their Fall Conference in Surrey, British Columbia! Given the recent fundamental shifts in our curriculum and a world-wide focus on equity in the classroom, we have chosen the theme "Math For All" for this year's conference. Our goal for this conference is to support teachers in implementing the revised curriculum while creating mathematical experiences that value and reflect the diversity of British Columbians. This year also features the Mini-Conference within the conference: Reggio-Inspired Mathematics, facilitated by Janice Novakowski and BC Teachers.

More information at bcamt.ca/nw2018/

Online Workshops

Education Week Math Webinars
Once a month, Education Weekly has a webinar focusing on math. They also host their previous webinars on this site. Previous webinars include Formative Assessment, Dynamic vs. Static Assessment, Productive Struggling, and Differentiation.

More information at www.edweek.org/ew/marketplace/webinars/webinars.html

Global Math Department Webinar Conferences
The Global Math Department is a group of math teachers that organizes weekly webinars and a weekly newsletter to let people know about the great stuff happening in the math-Twitter-blogosphere and in other places. Webinar Conferences are presented every Tuesday evening at 9 pm Eastern. In addition to watching the weekly live stream, you can check the topic of next week's conference and watch any recording from the archive.

More information at www.bigmarker.com/communities/GlobalMathDept/conferences

Problems to Ponder

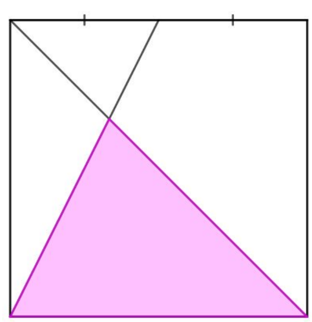
Six Marbles
You have three bags that each contain two marbles.

- Bag 1 has two red marbles.
- Bag 2 has two blue marbles.
- Bag 3 has one red marble and one blue marble.

First, you choose a bag at random. Then, from that bag, you choose a marble at random and end up with a red marble. What is the probability that the other marble in the bag is also red?

Source: brilliant.org/100day/day92/

Simple Enough
What fraction of the square below is shaded?



Source: www.futilitycloset.com/2019/07/05/simple-enough-5/

It All Starts Here: Register your Students for the Canadian Open Mathematics Challenge

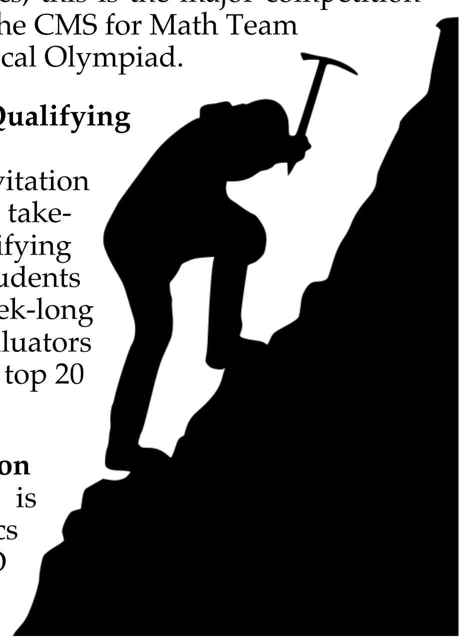
Termeh Kousha
Executive Director, Canadian Mathematical Society

Mathematics competitions are a fun activity for students of all ages. Since 1969, the CMS has been staging national math competitions to encourage students to explore, discover, and learn more about mathematics and problem solving. Along the way, thousands of students have become more comfortable with math and more confident in what they can achieve. The most popular of the CMS national competitions is the Canadian Open Mathematics Challenge (COMC) held in November each year, and is the first step on the path to representing Canada on the international stage.

Step 1: COMC Competition
This flagship national competition is open to any primary and secondary student around the world. It attracts thousands of participants from across Canada and internationally each year. Although the competition is targeted at upper-level high school students, performance awards are available at multiple grade levels. Furthermore, every student in Canada who participates is equally eligible for prizes. Top-performing students receive certificates and their school receives a plaque. In addition, students may be considered for an invitation to a CMS regional, speciality, or national math camp.

The COMC takes place in November, and registration opens in September each year. While the 2.5-hour competition, normally staged at a school in early November, is nationally focused, performance is recognized as 'best in Canada' and 'best in grade in Canada, as well as 'best in the province' and 'best in grade in the province.' In addition to awards, plaques, certificates, and prizes, top-performing students are also automatically invited to participate in a more advanced CMS competition (see Step 3).

For students with an advanced interest in mathematics, this is the major competition that can eventually lead to a student being chosen by the CMS for Math Team Canada and competing in the International Mathematical Olympiad.



Step 2: The Canadian Mathematical Olympiad Qualifying Repêchage
Students who come very close to qualifying for an invitation to the CMO (Step 3) are invited to participate in the take-home Canadian Mathematical Olympiad Qualifying Repêchage (CMOQR) in early February. About 75 students are given eight problems to solve. The CMOQR is a week-long exam completed through email. It is not scored, but evaluators choose the most insightful correct exams and offer the top 20 an invitation to the CMO.

Step 3: Canadian Mathematical Olympiad Competition
The Canadian Mathematical Olympiad (CMO) is Canada's premier national advanced mathematics competition. Candidates are invited to write the CMO based upon excellent performance in COMC or CMOQR. This three-hour advanced competition is

usually written in each student's school in late March and typically consists of five challenging math problems.

Step 4: Competing on the World Stage
Candidates with excellent performance in the CMO, the CMS Math Training Camps, and, in part, in other mathematics competitions, are selected to be part of Math Team Canada and compete on the world stage at the International Mathematical Olympiad (IMO) or the European Girls' Mathematical Olympiad (EGMO).

International Mathematical Olympiad
The IMO is the world championship mathematics competition for high school students. Math Team Canada is chosen from top ranking students. Canadian students have consistently performed very well on the international stage at the IMO competition. IMO is an intense, world-class two-day contest. Each day, students have 4.5 hours to solve three questions. CMS selects six students to be on the Canada team and assembles training and coaching staff to provide an intense preparation program. Training takes place at the Banff International Research Station (BIRS) or at the University of Waterloo and then the team travels to the IMO venue.

European Girls' Mathematical Olympiad
Canada is one of the countries participating in the EGMO. The EGMO is an international competition in mathematics, which focuses on female high school students whose commitment to mathematics goes beyond the usual curriculum. The four competitors who represent Canada are selected based on excellent performance in the COMC and an additional EGMO team-selection examination held in January.

Preparing for the contest
In order to assess the difficulty of a typical COMC exam, you can view the COMC exam archive at http://comc.math.ca/2018/practice. You may also visit https://cms.math.ca/Competitions/problemsolving/ to learn about resources you and your students can use when preparing for our competitions. Moreover, beginning in the first week of September, *Problem of the Week* will be posted on the COMC website each week leading up to the competition as a tool to prepare for the competition.

Registration
The 2019 COMC will be written on November 7th, and registration will open in early September. To register your students, visit www.COMC.math.ca. To stay informed, subscribe to the math competitions email list through the COMC home page.

If you have any questions, please contact us at contests@cms.math.ca.

This column highlights local and national extracurricular opportunities for K-12 students interested in mathematics, including collaborative, individual, online, and in-person challenges, contests, and camps. For dates, registration procedures and applications, and other information about the contests listed, please head to the contest websites, included below the descriptions. If we have missed an event that should be on our list, please contact us at thevariable@smts.ca.

If you are looking for contests available in other provinces, head to the Canadian Mathematical Society website (cms.math.ca/Competitions/othercanadian). The CMS also maintains a list of resources for students who are looking to build their problem-solving skills and succeed in competitions: see cms.math.ca/Competitions/problemsolving.

Canadian Math Kangaroo Contest
Written in March

The purpose of this competition is to introduce youngsters from Grade 1 to Grade 12 to math challenges in a fun and enjoyable way, thus inspiring their further interest and advancement in mathematics. The competition is held yearly in more than 50 Canadian cities. Students may choose to participate in English or in French.

More information at kangaroo.math.ca/index.php?lang=en

Canadian Open Mathematics Challenge (COMC)
Written in early November, registration opens in September
Organized and presented by the Canadian Mathematical Society

The COMC is a high-school-level math competition that encourages creative problem-solving and mathematical discovery that often goes beyond the curriculum. Excellent results at the COMC may lead to an invitation to write the prestigious Canadian Mathematical Olympiad (CMO), the results of which constitute one of the main qualifiers for being selected for Math Team Canada—a six-person team of high-school students who represent Canada at the International Mathematical Olympiad. The contest is available in English and French.

For more information and to register your students, visit www.comc.math.ca.

Canadian Team Mathematics Contest
Written in April
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Canadian Team Mathematics Contest (CTMC) is a fun and challenging competition for teams of 6 secondary school students in any combination of grades. One teacher and groups of six students participate at their own school over 3 consecutive hours. The curriculum and level of difficulty of the questions will vary. Junior students will be able to make significant contributions but teams without any senior students may have difficulty completing all the problems.

More information at www.cemc.uwaterloo.ca/contests/ctmc.html

Caribou Mathematics Competition
Held six times throughout the school year

The Caribou Mathematics Competition is a worldwide online contest that is held six times throughout the school year. Each of these days, five contests are offered, one for each of the grade levels 3/4, 5/6, 7/8, 9/10 and 11/12 and each one in English, French and Persian. The Caribou Cup is the series of all Caribou Contests in one school year. Each student's ranking in the Caribou Cup is determined by their performance in their best 5 of 6 contests through the school year.

More information at cariboutests.com

Euclid Mathematics Contest
Written in April
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Euclid Mathematics Contest is an opportunity for students in their final year of secondary school and motivated students in lower grades to have fun and to develop their mathematical problem solving ability. Most of the problems are based on the mathematical curriculum up to and including the final year of secondary school. Most of the problems are based on curricula up to and including the final year of secondary school. Some content might require students to extend their knowledge and the best way to familiarize oneself with commonly appearing topics is to practice using past contests.

More information at www.cemc.uwaterloo.ca/contests/euclid.html

Fryer, Galois, and Hypatia Mathematics Contests
Written in April
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Fryer, Galois and Hypatia Math Contests are an opportunity for students to write a full-solution contest. They are fun way to develop mathematical problem solving skills through a written mathematical activity. For students in Grades 9 (Fryer), 10 (Galois) and 11 (Hypatia). Questions are based on curriculum common to all Canadian provinces. Rather than testing content, most of the contest problems test logical thinking and mathematical problem solving.

More information at www.cemc.uwaterloo.ca/contests/fgh.html

Gauss Mathematics Contests
Written in May
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Gauss Contests are an opportunity for students in Grades 7 and 8, and interested students from lower grades, to have fun and to develop their mathematical problem solving ability. Questions are based on curriculum common to all Canadian provinces. The Grade 7 contest and Grade 8 contest is written by individuals and may be organized and run by an individual school, by a secondary school for feeder schools, or on a board-wide basis.

More information at www.cemc.uwaterloo.ca/contests/gauss.html

Opti-Math
Written in March
Presented by the Groupe des responsables en mathématique au secondaire

A French-language mathematics challenge for secondary students who would like to exercise and develop their problem-solving skills.

Les Concours Opti-Math et Opti-Math + sont des Concours nationaux de mathématique qui s'adressent à tous les élèves du niveau secondaire (12 à 18 ans) provenant des écoles du Québec et du Canada francophone. Ils visent à encourager la pratique de la résolution de problèmes dans un esprit ludique et à démystifier, auprès des jeunes, les modes de pensée qui caractérisent la mathématique. Le principal objectif des Concours est de favoriser la participation bien avant la performance. La devise n'est pas : « que le meilleur gagne » mais bien « que le plus grand nombre participe et s'améliore en résolution de problèmes ».

More information at www.optimath.ca/index.html

Pascal, Cayley, and Fermat Contests
Written in February
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Pascal, Cayley and Fermat Contests are an opportunity for students in Grades 9 (Fryer), 10 (Galois)m and 11 (Hypatia) to have fun and to develop their mathematical problem solving ability. Early questions require only concepts found in the curriculum common to all provinces. The last few questions are designed to test ingenuity and insight. Rather than testing content, most of the contest problems test logical thinking and mathematical problem solving.

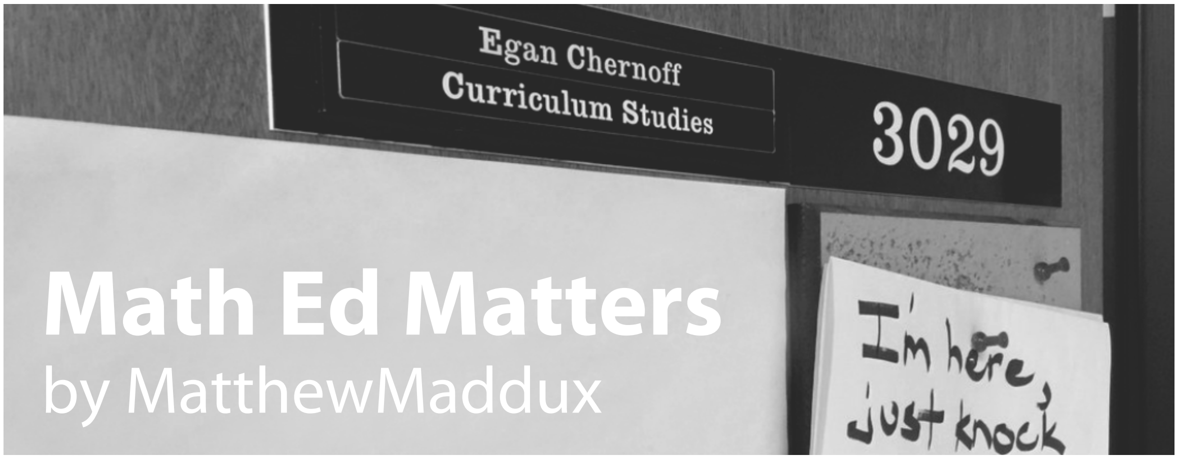
More information at www.cemc.uwaterloo.ca/contests/pcf.html

The Virtual Mathematical Marathon
Supported by the Canadian National Science and Engineering Research Council

The virtual Mathematical Marathon has been developed by an international team of mathematicians, mathematics educators, and computer science specialists with the help of the Canadian National Science and Engineering Research Council.

The main activity is a competition allowing students to enjoy solving challenging mathematical problems all year around. Students can join the game at any time and at no cost, simply creating an individual profile with an individual username and a password. Available in French.

More information at www8.umoncton.ca/umcm-mmv/index.php



Math Ed Matters by MatthewMaddux *is a column telling slightly bent, untold, true stories of mathematics teaching and learning.*

A (Math) Instructor's Copy for All: There's an App for That

Egan J Chernoff
egan.chernoff@usask.ca

In the autumn of 2014, the release of a new app for phones and tablets caused a frenzy on social media and in the news: Photomath. The premise of Photomath was, and is, quite simple: point the camera of your phone or tablet at a math equation, and the app solves the equation for you almost instantly. But that's not all: Not only does the app solve the equation for you, it also shows you all the steps. Wow!

Intrigued by the app, I tried to keep tabs of the coverage surrounding the release of Photomath. Presupposing that others might also be interested in the story, as I do, I posted all of these articles on my Twitter feed (follow me @MatthewMaddux). As time went on, I noticed that the narrative surrounding the app shifted several times.

Shifting Storylines

When Photomath was first released, most stories covering the app were focused on the technological feat: that is, being able to just point a phone or tablet camera at a math equation, have it process the equation, and then solve the equation showing all the steps. Again, wow! Shortly thereafter, though, the tone of the stories began to change. The second wave of stories covering the app shifted the focus from marveling at the technological feat to its potential impact on the teaching and learning of mathematics. Stories in this wave began to question and discuss whether the app would have a positive or a negative impact on students and teachers in the math classroom. Inevitably (in my opinion), in the third wave of stories, discussions about cheating in the math class, and on math homework in particular, were the sole focus of the Photomath coverage. Then, things changed again.

In the fourth, and arguably final wave of coverage, something weird happened. It turned out, as is the case with most early versions of new software or technology, the app did not work perfectly every single time. Soon enough, example after example after example appeared on various social media platforms. These examples showed that, in some instances, the answer supplied to a given problem was incorrect, and, in other instances, even when an equation was solved correctly, the app did not show all of the appropriate steps exactly. And, just like that, a large swath of people associated with the teaching and learning of mathematics seemed to write off the app and (perhaps unsurprisingly, given how fast things move on the Internet) moved on to other stories. Conversations about the teaching and learning of mathematics, as seen in earlier waves of stories, were quickly left behind.

Don't Throw the Shovel Out with the Snow

I continued to share articles about Photomath on my Twitter feed, so much so that at one point someone asked me whether I was involved in its development. In my reply, I indicated that I was flattered, but did not have anywhere near the technical wherewithal to pull something like this app off. To this, the person asking replied, "Phew, it's a little buggy and I didn't want to hurt your feelings." I quickly wrote back to them and explained that, in my opinion, the focus on the initial bugs of the program was a case of burying the lead. To me, it felt as if everyone was inexplicably throwing the baby out with the bathwater.

I think that the release of Photomath was a marker of things to come and shouldn't have been dismissed by many in the math ed community so quickly. After all, I had been using technology like Maple software since I was in university. And, sure, MATLAB and Wolfram|Alpha and other powerhouse math programs were part of my mathematical learning experience. But Photomath just seemed different—there was something about not having to manually enter anything on a computer or mobile device. Just point and solve. But instead of attempting to look ahead even further (note: I'm definitely not a futurist), which we are usually apt to do, in the remainder of this column, I'll make my case for not so quickly dismissing Photomath by looking back, way back, to what I consider the analog version of Photomath and all of the other similar technology that now exists (e.g., Socratic, Mathpix, etc.): flipping to the back of the book.

The Back of the Book

The book, in this instance, is each and every math textbook that I was given during the first few days of school. It would always play out like this: First, we would have to write our names and student numbers on that little, square sticker on the inside of the front cover; then, as I casually flipped through the book, pretending to be interested in the content, I would sheepishly make my way to the back of the book to see what sort of answer key scenario I would be dealing with for that particular year.

> Before the advent of the Internet, there were many occasions where the back of your math textbook was the only—and let me repeat *that*, *the only*—place that the answers existed.

I was not alone in my interest in the back of the book. If, say, your math teacher wasn't around, you weren't in math class, or your friends or family weren't able to help, the only place that had answers for you, the answers that would (arguably) let you know whether you were doing things correctly or not, was the back of the book. Let me repeat that for those readers who may have had access to the Internet for their entire lives: Before the advent of the Internet, there were many, many occasions where the back of your math textbook was the only—and let me repeat that, *the only*—place that the answers existed. Getting back to the parallel that I just drew, you flipped to the back of the book much in the same way that, today, someone could point the camera of their phone or tablet at an equation. Worthy of note, whether flipping to the back of the book was quicker or slower than taking a picture, as is now the case, would depend on a number of factors.

Like most of my friends, I had a dedicated bookmark in the back of my textbook, which I called a back-of-the-book-mark. And, given my years of experience, I contend that flipping to your back-of-the-book-mark could probably produce the answer quicker than the app. With a bit of time on my hands, I recently set up this experiment, back-of-the-book-mark versus camera app (I won't say which one), and can confirm that my years and years of practice dedicated to flipping to the answers in the back of various math textbooks, unlike other skills that I developed but have subsequently eroded over the years, is as sharp as ever. Based on the lack of rigour associated with my experiment, though, I am sure that different scenarios would lead to different results. For example, a person who has not dedicated many, many years to flipping to the answers in the back of math textbooks might, just might, do better in the digital realm than the analog.

The Back of the Book & Me

Before the Internet, as I can attest, a math student's relationship with the back of the book was very personal, but would manifest itself differently in different settings. When doing homework at home, in the privacy of your own room, flipping to the back of the book was unencumbered, naturally. One year, for example, I actually had two textbooks. One was my "regular" textbook, which I took it to school and used in math class. The other one remained at home, in my room, perpetually open to the back of the book, displaying the answers for the particular section I was working on at the time. At school, however, and especially in math class, how you projected your relationship with the back of the book was a delicate dance—because others were watching, always watching.

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My public use of the back of the book was much different than my private use of the back of the book. After all, I had a bit of a reputation to uphold with my peers and my math teacher. As I've mentioned before, I was just an ok math student in junior high and high school, which in British Columbia meant Grade 8-12. Nevertheless, I knew that, while in public, your relationship with the back of the book let everyone around you know how much you relied on the answers. Over the years, I had seen extremes.

On one extreme, there were those individuals whom I never once saw use the back of the book. Wow! Instead, these individuals worked on their homework in class and navigated from one question to the next with a sense of confidence that both made me jealous and, at times, question what was really going on. As I would find out, those who never checked the back of the book were either math geniuses or wanted to be seen as math geniuses. The latter, those who would never flip to the back of the book but still had no clue what they were doing, would eventually get exposed on the unit exams or sometimes on pop-quizzes. It took me a while, but sometime around Grade 9 it finally clicked for me that, just because a person did not check the answers in the back of the book in class did not necessarily mean that they knew all the answers—all you had to do was ask them. There was, of course, the other extreme, but there was also the middle-ground user.

Me, I was a middle-ground user in public. I definitely used my dedicated back-of-the-book-mark (or dedicated second textbook) at home, in the privacy of my own room when doing homework, and did so with impunity, sure. As most kids do, I wanted to get my homework done fast, but I wanted also wanted to make sure that I knew, to the best of my ability, what I was doing. But I would never, ever be caught with my back-of-the-book-mark at school. Working on homework in class was different. Like others, I wanted to get as much done as possible, which reduced the amount of homework to do at home, but I wasn't going to incessantly flip for answers the same way that I would at home. Instead, I would work on a number of questions, anywhere from 3 to 5 in a row, and then, and only then, casually, calmly, flip to the back of the book to see if I was on the right track—never showing surprise or indication as to whether I was on the right track or had no clue as to what I was doing. I would even, in some instances, go through a chunk of questions and, if I even suspected I was on the right track, then I would not flip to the back to check my answers; rather, I would make a note to check at home to see if I had any clue about what I was doing for those particular questions. After all, I had friends, close friends, smart friends, who never flipped to the back of the book. Friends who simply knew without checking whether or not they were right. Wow! As a result of these friends, I was not able to reveal math-homework-at-home-Egan, two-textbook-Egan to the public. Others, though, were not encumbered the same way that I was when working on homework in class.

To this day, I still have a lot of respect for the students at the other end of the spectrum. Those who brought their back-of-the-book-marks inside their math textbooks to math class. I'm not sure why I had a sense of awe for these people. Perhaps it was because I, too, wanted to bring my back-of-the-book-mark to school, but just couldn't bring myself to do it. As I would later learn, my reverence for those who sat there in math class and, when necessary, would quickly flip to see if their answer was right, was based on the type of person who would, who could do such a thing! Ultimately, at a rather young age, these students did not really care what you or others thought about them and what they were doing. Wow! From a developmental standpoint, for most high schoolers, not caring what others think about you is years away. This particular type of unabashed person was also one who liked to figure things out on their own. Sure, there were teachers and classmates around when doing homework in math class, but they preferred to first take a crack at solving the equation on their own, checking afterwards to see whether or not they were correct based on the answers in the back of the book, and delving back into the problem if necessary. Although I'm not sure what these people are doing today (I deleted Facebook years ago), I'm sure that this carefree attitude towards the opinion of others at a young age has served them well.

Reverse-Engineering

As I progressed through my schooling, I quickly came to realize that the back of the book was indispensable to me when it came to learning mathematics. Based on this realization, I concurrently began to expect more and more from the back of the book, especially as the mathematics that I was learning became more and more difficult. With no teacher, family member, or friend to call upon most of the time, and having taken in the lecture and gone through all of the examples in the notes and the textbook, was it sufficient to flip to the back of the book to just see the final answer? The answer, very clearly, is no. However, as Theodore Roosevelt said, "Do what you can, with what you have, where you are."

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And so, as a result, I became very, very adept at using the answers in the back of the book to "reverse-engineer" the solutions to the math problems I was working on. Working backwards—that is, from the answer back to the problem—I would reverse each step that I had done in solving the previous problem, which would result in one of two outcomes. If my efforts resulted in the same problem that was found in the text, my assumption was that I had followed the directions that were given to me correctly and I had applied all the steps where and when I was supposed to. I would then apply these steps in the other direction, the proper direction, to solve all of the other similar problems. However, it was the times when I was wrong—that is, when the answer did not lead to the problem in the textbook—that the learning occurred.

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Knowing full well that the back of the book, with only the final answers, was lacking when it came to my learning school mathematics, the back of the book was, ostensibly, where I started when I got things wrong. I would begin a process of rereading notes, rereading text book examples, going over previous, similar questions to see what I was doing wrong. Sometimes, that worked. More often than not, though, I scoured through my reverse-engineering from answer to question, step by step, in order to find what wasn't working. Perhaps it was a miscalculation, or a step was missing or completed in the wrong order, but I would pore over each of the steps in order to see what each particular step should have resulted in, instead of what I had done. This process would then get repeated for any and all remaining steps required to solve the problem. (At these times, I remember thinking to myself how great it would be if the answer key showed just a bit more than the final answer—just a step or two!) Once I was able to establish the error of my ways, I would start the set of particular problems over again, making sure that I ended up with the correct answers. Once comfortable, I was ready to move on to the next problem set.

To be clear, I was not able to entirely reverse-engineer my way through school mathematics. At school, when possible, when necessary, I would call upon the help of classmates and, in rare instances, the math teacher. At home, I would ask my Mom and Dad and, in a pinch, I would even ask my younger brother. Sometimes, phone calls—via landline!—would be made to friends who, I hoped, were stuck on the same problems. At that point, after all, there was nowhere else to turn. By and large then, my learning of school mathematics—yes, informed by lectures, notes and textbook examples—relied heavily on the answers given in the back of the book. Smoothing out any inconsistencies that occurred while reverse-engineering from answers to questions became my default approach to learning mathematics. Heeding the words of Mr. Roosevelt, correct answers were all I had, so I had to make it work. To this day, I'm pretty amazed that I could figure out how to turn an answer like $x = 1$ or $x = 2$ from the back of the book into the equation $x^3 - 5x^2 + 8x - 4 = 0$ (and not $x^2 - 3x + 2 = 0$, and not $x^4 - 6x^3 + 13x^2 - 12x + 4 = 0$).

Off to University

Looking back at the progression of learning school mathematics, having the answers in the back of the book was somewhat less important in Junior High (Grades 8 to 10 for me). After all, at that time I had a lot of additional support. The teacher would consistently walk around the room, asking plenty of questions that sussed out whether or not we understood what we were doing—not to mention that we spoke with our friends in class a lot and were not as hung up on public perception. In High School (Grades 11 and 12), however, the answers in the back of the book became more important. Classes were mostly dedicated to taking notes based on the teacher's lecture, and any time dedicated to homework was, for whatever reason, silent—at least in my classes. In other words, you became more and more responsible for your own learning. The next step, which for me involved university math classes, would take this important responsibility to a whole new level.

When I went to university, there was only one reliable place to turn (pun not intended) when it came to establishing whether I was on the right track: the back of the book. Consider the following common scenario for a first-year calculus student: You're sitting in a class with a hundred, perhaps a few hundred, other students. If you're lucky, you'll know one or two of them. If you're even luckier, you might form a study group at some point in the semester. There is a lecturer or instructor or professor at the front of the room who has office hours by appointment; and, as if they weren't intimidating enough already, you now have to go up to them in front of hundreds of students and request to ask some questions because you don't understand what just went on. Sure, you took notes during the lecture, which you hope you copied down correctly, and there are a few other examples in that textbook that you paid way, way too much money for at the beginning of the semester. All for naught, though, because the notes and the examples aren't helping you figure out why you're making the mistakes you're making. And, no Internet. No Netscape. No Google. No "Hey, Siri..." All of which leaves you with, you guessed it, the answers in the back of the book.

The back of the book in a university textbook, at least when I went to school, was not that different from the back of the textbook that I had encountered in junior high and in high school. However, the university texts that I encountered only had the answers for the odd-numbered problems. This, to a degree, made sense. University students would be able to see the answers for half of the questions, and instructors or professors would be able to assign the other half of the questions for homework. However, and particularly at this point in my schooling, having just the answers presented, and only half of them now, frustrated me beyond belief.

Seeking Access to Information

To be clear, I wasn't looking for a full solution to each and every question in the back of my university textbooks, which did exist in some of the high school texts that I used. (One could dream, though.) No: What I was looking for was more problems! Let's say that a particular notion that I was supposed to learn for university math class was represented in the textbook by three problems, for example. Since the textbooks only provided the solution to every other question, this meant that I may have only had one answer to reverse-engineer from for the other two homework questions. One was not enough. If, though, there had been 6 (or more) questions on the topic, which would equate to 3 answers to help me reverse-engineer what to do, I would have had a much better shot at understanding what I was doing. After all, in the first year of university you are, by and large, fending for yourself.

Given the sheer number of times that I was getting things wrong when trying to learn particular topics, I was not able to bother my friends or my professor for the hours and hours on end that it would have taken, which left me with nowhere else to turn other than my odd-numbered answers. Looking back on the situation that I was in, I did pretty well, given the circumstances I was working with—that is, the sparse number of answers in my

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university math textbooks. For me, for large parts of my schooling career, the answers in the back of the book were key to my learning of mathematics and, now in university, this approach that I had spent so much time on in junior high and high school was actually starting to pay dividends. I could now attend a lecture, take the notes, and then make my way to the library with my textbook examples and some answers and, eventually, figure out what was going on. All the while, though, I would continue to critique having just the answers to even (or odd) questions, and only a handful. This critique would come to full fruition for me during one particular semester.

The Instructor's Copy

I won't go into details, but, one semester, me and a few of my friends got our hands on an instructor's copy of the textbook for the math class that we were taking. Now, before you make certain assumptions about our integrity, hear me out. First, we went through completely legitimate channels to procure this particular instructor's copy of the textbook. Second, 0% of the mark for this course was based on homework. Third, the professor for the course did not have any questions from the textbook on the tests or the quizzes that we wrote, which comprised 100% of our marks for that semester. So, you might be asking, how did my friends and I benefit from having (1) all the answers to all the questions in the textbook, as opposed to just half of the answers, and (2) more answer-related detail than was found in the students' textbook?

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The key to my success that one particular semester of university mathematics, that one semester we had the instructor's copy of the textbook, I'm proud to say, was me. For the first time in my university schooling career, there was more information housed in the instructor's copy than I needed. The answers to *all* of the questions were available. More detailed explanations in the examples were copied into the notes that I took, especially when the professor decided, for whatever reason, to leave particular details out during their lecture. I also had a group of friends who all worked together, and who were willing to communicate and share information that they had learned. We also had a sense of confidence in knowing, for sure, whether or not we were on the right track. But perhaps the best part was that the textbook never slept; it was always there for us if and when we needed it. That semester, we never ran out of material to aid our learning. It would be through our own understanding of the material, reverse-engineered or otherwise, through our own effort, through our own excitement that we all performed better that we normally did on mathematics tests and quizzes that semester. Sure, we did it, but access to the instructor's copy was a key to our success. Which brings me back to the present day, and to Photomath.

From the Back of the Book to Photomath

The Internet is ripe with thought experiments concerning what would happen if someone from the past got plunked into the present day. What would happen, for example, if a cave person were dropped into the middle of Times Square tomorrow? Questions regarding their reactions, processing ability, and more abound. Getting back to my scenario, I wonder what would happen if I had been shown an app like Photomath at different stages of my schooling. Of course, in this scenario, we are assuming that I could grasp the idea of a cell phone and all of its features (e.g., a phone with no wires, the phone is also a camera, I would only kinda understand the notion of accessing the Internet, etc.). There's no way to know. There is, however, another way to look at this scenario.

Borrowing from a tired twenty-first century trope: Do you hate having just the answers in the back of your mathematics textbook? There's an app for that®. In other words, looking back on it now, I see many parallels between what Photomath provides today and its analog analog, flipping to the back of the book—but especially what I experienced that one semester when I had unfettered access to an instructor's copy of my math textbook. I guess that's why I was so hesitant to so quickly dismiss the app and its' potential, even when it got a few things wrong—just like the back of the book in that old math instructor's copy.

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Call for Contributions

***The Variable* is looking for contributions from all members of the mathematics education community**, including classroom teachers, consultants, and teacher educators. Consider sharing a favorite lesson, an essay, a book review, or any other work of interest to mathematics teachers in Saskatchewan. Articles may be written in English or French. If accepted for publication, your article will be shared with a wide audience of mathematics educators in Saskatchewan and beyond.

We also welcome student contributions in the form of artwork, stories, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom, and for teachers to recognize students' efforts during their journey of learning mathematics.

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Ilona & Nat,
Editors

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