Integer Numbers
and Temperature Problems

Family Math: Winning
Them Over

My Favourite Lesson:
Introduction to Logarithms

Reducing Inflammation

Saskatchewan Mathematics
Teachers' Society

AFFILIATE
NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

NCTM

In this issue

Cover Image

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SMTS Executive (2020-2022)

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Editorial

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R ather than offer an editorial on this issue of *The Variable*, we felt that it would be more appropriate to provide what we likely all need right now—some space. The contents of the issue, we believe, speak for themselves, and it feels disingenuous to offer any further commentary in the aftermath of the upheaval that classroom teaching has just endured with the sudden shift to remote delivery[1] caused by COVID-19, and, in the midst of this new elephant in the room, the challenges facing every teacher in the province as we return to our classrooms. However, one thing that the school shutdown did provide is a very clear image of the importance of schools. Perhaps, then, we should get into the habit of allowing some space for all this to sink in.

Nat & Ilona
Co-Editors

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[1] Note that we intentionally used the word "delivery" instead of the word "teaching".

My Favourite Lesson

Introduction to Logarithms

Jeff Irvine

> "Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content." (Saskatchewan Ministry of Education, 2012, p. 16)

This lesson uses a modified concept attainment strategy (Bruner, Goodnow, & Austin, 1986) to introduce the notion of logarithms. Students make conjectures about the meaning of logarithms and then successively refine their definitions based on additional information. The key is relating the concepts of base, exponent, and power to log notation and translating between the exponential and logarithmic form.

Outcomes

PC 30.9 Demonstrate an understanding of logarithms, including:

- Evaluating logarithms
- Relating logarithms to exponents

Indicators

a. Explain the relationship between powers, exponents, and logarithms
b. Express a logarithmic expression as an exponential expression and vice versa
c. Determine, without technology, the exact value of a logarithm such as $\log_2(8)$
d. Explain how to estimate the value of a logarithm using benchmarks (e.g., since $\log_2(8)=3$ and $\log_2(16)=4$, $\log_2(9)$ is approximately equal to 3.1)

Materials

- Handout
- Scientific calculators
- Student journals
- Placemats (see p. 5; one per group, enlarged on 11x17 paper)

Mathematical Processes

a. Communication
b. Connections
c. Mental mathematics and estimation

    d. Problem solving
    e. Reasoning
    f. Visualization
    g. Technology

Task Description

By making and revising conjectures based on additional evidence, students will develop their understanding and definition of logarithms.

1. Working in groups, students use scientific calculators to complete Part 1 of the handout (see p. 5), making an initial conjecture about logarithms, then revising once they complete the second table in Part 1.
2. Then, without calculators, students complete Part 2, still with common logarithms.
3. Part 3 has students complete logarithms with bases other than 10, again without using technology.
4. After a whole-class consolidation, in Part 4, students write the definition of a logarithm in their journals based on their work in Parts 1-3 and provide some examples.
5. In Part 5, students work with a placemat activity to estimate logarithms that give rise to non-integer values.
6. The activity concludes with an out-of-class research activity in which students investigate the history and development of logarithms.

Anticipated Student Action

I have found that students quickly develop an understanding of the logarithmic notation. Initially, some students may mistakenly associate logarithms only with powers of ten. However, when students begin Part 3, they realize that logarithms can have other bases. The placemat activity supports student development of this concept by having students share and discuss their ideas with others. The most difficult part of the lesson is Part 5, since it involves estimating non-integer exponents. The teacher may choose to have a whole-class discussion about the resulting estimates. I have found that this sharing reinforces the concept of estimating logarithms based on benchmark values (see Indicator d).

Student Strategies

Most students quickly complete the first three parts of the handout, with math talk within groups supporting concept development. Part 3, which asks students to evaluate logarithms with bases other than 10 may prove a stumbling block, but this usually clarified through consultation within the groups. Various strategies usually emerge in Part 5. Reference to benchmarks helps here. Once again, discussion within groups supports student acquisition of concepts.

Wrap-up and Next Steps

A brief whole-class consolidation at the end of class helps correct any misconceptions that may have arisen. Next steps, in subsequent classes, include development of the laws for logarithms and graphing logarithmic functions.

Conjecture and Refine
What is a Logarithm?

Part 1.

This investigation will use the **log** button on your calculator. "**Log"** is an abbreviation for "logarithm." To use the **log** button, on most calculators, first press the **log** key, enter the number, and then press Enter. Some calculators require you to enter the number first and then press **log**. Experiment with your own calculator to see which syntax works.

Complete the table using your calculator:

Number	Result when **log** button is pressed: log(number) =
100	
1000	
1000000	
10	

Make your first conjecture: What is a **logarithm (log)**?

Complete the table below using your calculator:

Number	Result when **log** button is pressed: log(number) =
0.01	
0.0001	
1	
0.1	
0.00000001	

Refine your conjecture: What is a **logarithm (log)**?

Part 2.

The **log** button on your calculator is in base 10. This can be written as $\mathbf{log_{10}}$. However, logarithms with base 10 were used so frequently for calculations in the pre-calculator days that $\mathbf{log_{10}}$ was often just written **log** and called the *common logarithm*.[1]

Reminder: What is a logarithm?

Complete the table below <u>without</u> using your calculator:

Number	log(number)
1000000000	
1000000000000000	
0.000000001	
0.00001	
1	
10	
0.01	
0.0000000000000001	

Part 3.

Now, use your definition of a logarithm to complete the table below. These cannot be determined using the log key on your calculator. (Why not?)

Number	$\log_{base}(number) =$
8	$\log_2(8) =$
32	$\log_2(32) =$
$\dfrac{1}{4}$	$\log_2(1/4) =$
9	$\log_3(9) =$
81	$\log_3(81) =$
81	$\log_9(81) =$

---

[1] Most calculators have another logarithm button, with a different base. This one is called a natural logarithm, and its abbreviation is **ln**. Natural logarithms use base e, a transcendental number (like π) with a value of approximately 2.718. (It was discovered by Jacob Bernoulli in 1683. The letter e is said to be used as an homage to Leonhard Euler, one of the great mathematicians of the 18th century, who made various discoveries about this number.) So, ln is really $\log_e$ The number e is important in calculus, but we will not be using e in this investigation.

$\frac{1}{27}$	$\log_3(1/27)=$
343	$\log_7(343)=$
256	$\log_{16}(256)=$
256	$\log_4(256)=$
256	$\log_2(256)=$

Part 4.
Label the diagram below using the terms *exponent, base,* and *power*:

In your journal, write your definition of *logarithm* using the terms **exponent, base,** and **power**. Give some examples.

Then, compare your definition with others in your group by completing the placemat activity (*see handout on p. 9*). Each member of the group will write their definition in one of the spaces around the oval. After sharing each of your definitions, discuss and come to a consensus about the definition of a logarithm and write it in the middle.

Part 5.
Estimate each of the following without using your calculator. Explain why each estimate makes sense. Check your first two answers using your calculator.

$\log_{10}(20) =$

$\log_{10}(2) =$

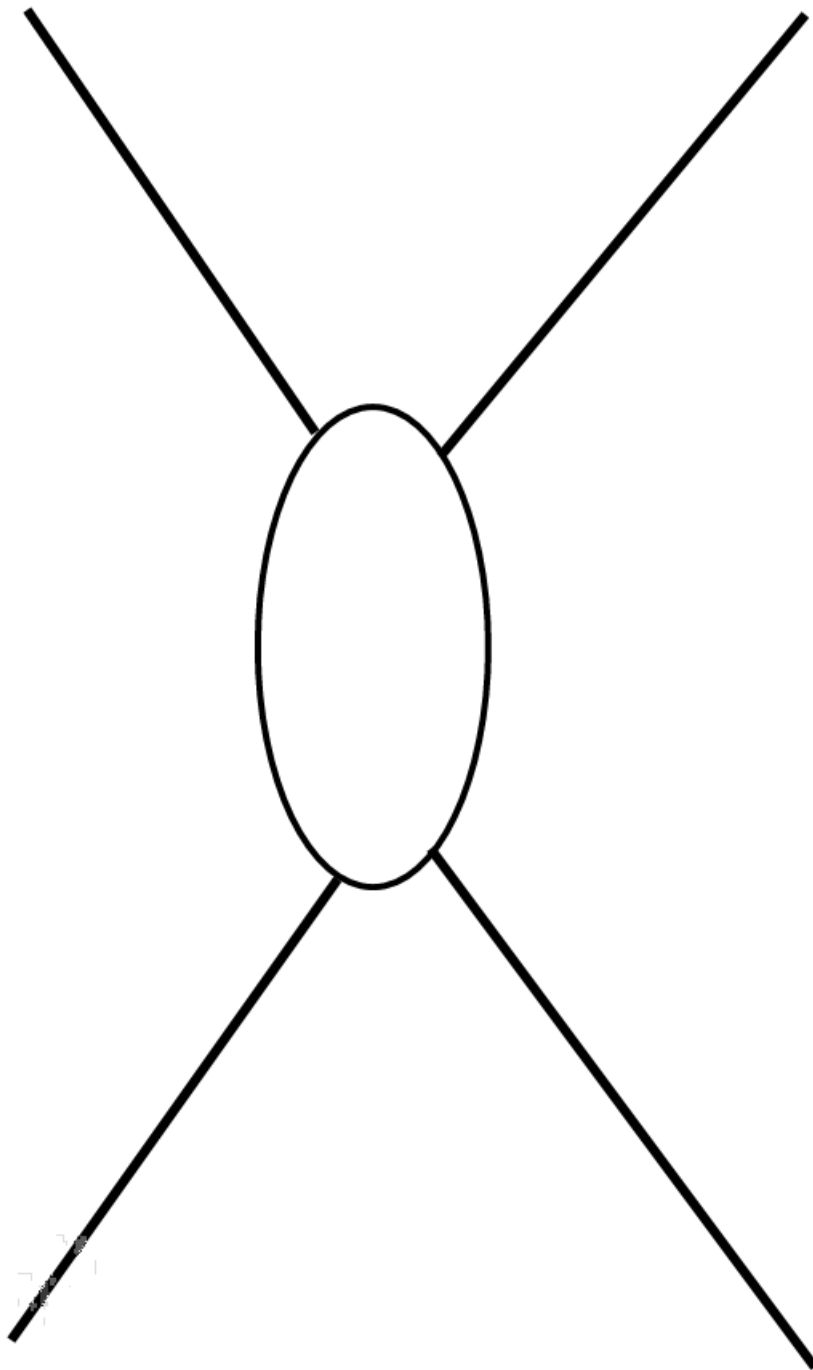$\log_2(55) =$

$\log_3(40) =$

$\log_2(325) =$

Part 6 (out of class).
Conduct an internet search on the history of logarithms, with particular attention to the work of Henry Briggs and John Napier. Write a brief report in your journal.

References

Bruner, J., Goodnow, J., & Austin, G. (1986). *A study of thinking*. New York: Routledge.

Saskatchewan Ministry of Education. (2012). *Saskatchewan Curriculum: Pre-calculus 30*. Retrieved from https://www.edonline.sk.ca/bbcswebdav/library/curricula/English/ Mathematics/Mathematics_Pre_Calculus_30_2012.pdf

Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was grades 7 to 12 mathematics for the Province of Ontario. Jeff is coauthor or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the role of student motivation in mathematics achievement.

Contribute to this column!

The Variable exists to amplify the work of Saskatchewan teachers and to facilitate the exchange of ideas in our community of educators. We invite you to share a favorite lesson that you have created or adapted for your students that other teachers might adapt for their own classroom. In addition to the lesson or task description, we suggest including the following:

- Curriculum connections
- Student action (strategies, misconceptions, examples of student work, etc.)
- Wrap-up, next steps

To submit your favourite lesson, please contact us at thevariable@smts.ca. We look forward to hearing from you!

Problems to Ponder

Walk on By

Two women started at sunrise and each walked at a constant velocity. One went from A to B and the other from B to A. They met at noon and, continuing with no stop, arrived respectively at B at 4 p.m. and at A at 9 p.m. At what time was the sunrise on that day?

Source: Mathematics Education Innovation. (May 2020). *Maths item of the month*. Available at https://mei.org.uk/?section=resources&page=month_item

Alternate Angles

Alternate Angles is a column on problems from multiple perspectives: various methods that could be used to solve them, insights we get from their solution, the new paths that they can lead us to once they have been solved, and how they can be used in the classroom.

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Alphametics

Shawn Godin

Welcome back, problem solvers! I hope everyone is doing well during these trying times. To lift our spirits, let's do some math! In the last issue, I left you with the following problem:

In the product below, each of the variables *P, Q, R, S,* and *T* is a digit in the two six-digit numbers appearing. Determine the values of the variables.

$$
\begin{array}{cccccc}
 & P & Q & R & S & T & 4 \\
\times & & & & & & 4 \\
\hline
4 & P & Q & R & S & T \\
\end{array}
$$

This type of problem is called an *alphametic*. Problems of this type allow students to play with number sense and algebraic reasoning. We will look at an example before we attack our problem.

$$
\begin{array}{ccccc}
 & S & E & N & D \\
+ & M & O & R & E \\
\hline
M & O & N & E & Y \\
\end{array}
$$

This may be the best-known example of an alphametic. Within the problem, each letter represents a unique digit. This problem was created in 1924 by the mathematical puzzle master Henry Dudeney. At first glance you may think, "Where do I start?" It turns out that all we need is our knowledge of addition, along with a little bit of logic and systematic analysis.

We know that when we add two digits together, the largest sum that we can get is 18. If we are carrying a number from a previous addition, then the largest sum we can get is 19, and this would only happen in the case where we were are adding 9 and 9 and had to carry. From this, we can conclude that the only thing we could carry would be a 1, and hence $M = 1$.

This means that in the previous addition, we have either $S + 1 = 10 + O$, or $S + 1 + 1 = 10 + O$, if there was a carry from the previous addition. The largest possible sum we could get would be $9 + 1 + 1 = 11$, but then O and M would be the same number, which is not allowed. This tells us that $O = 0$ and S is either 9, in which case there was no carry from the previous addition, or S is 8, in which case there was a carry.

The previous addition was either $E + O = N$, $E + O = N + 10$ (if it results in a carry for the next step), $E + O + 1 = N$, or $E + O + 1 = N + 10$. Since $O = 0$, the first two equations are impossible. The last equation is also impossible, since it would force N to be zero, but N cannot equal O. This tells us that $E + 1 = N$, and there was no carry from this addition. Hence, $S = 9$.

At the next step, we have $N + R = E$, $N + R = E + 10$, $N + R + 1 = E$, or $N + R + 1 = E + 10$. From the last step, we know that $N > E$, so the first and the third equations cannot be true. Replacing N by $E + 1$ in the other two equations, we find that R is either 8 or 9, but 9 has been used, so $R = 8$ and we must have had a carry from the previous result.

Therefore, $D + E = 10 + Y$. Since 0, 1, 8 and 9 have been used, the sum of $D + E$ can only be 12, 13, 14, 15, 16, or 17. On the other hand, since the largest D and E can be are 6 and 7, $D + E$ could only be 12 or 13. To get 13, we must use 6 and 7 for D and E, which is impossible as it would force $N = E + 1$ to be 7 or 8, which are unavailable. Thus, $Y = 2$, and the only way this is possible is if $D = 7$ and $E = 5$, making $N = 6$. Therefore, the sum must be 9567 + 1085 = 10652.

We can use similar logic and properties of multiplication with the featured problem. We know that $4 \times 4 = 16$, which means that $T = 6$, and we carry 1 to the next operation. Continuing in this manner, we get

- $4 \times 6 + 1 = 25$, so $S = 5$ and we carry 2,
- $4 \times 5 + 2 = 22$, so $R = 2$ and we carry 2,
- $4 \times 2 + 2 = 10$, so $Q = 0$ and we carry 1,
- $4 \times 0 + 1 = 1$, so $P = 1$.

Checking, $102564 \times 4 = 410256$, so it works out. It might be nice, though, to know *why* it works out. To do so, we will look at the problem another way. But first: Another problem.

You may have seen the following "trick" before: Take any two-digit number where the digits are different, switch the digits to make a new number, and find the difference between the original and the new number. Finally, add the digits of the resulting number together. No matter which numbers you choose, you will get 9 every time! For example, if we picked 73, our next number would be 37. Subtracting yields $73 - 37 = 36$, and $3 + 6 = 9$. Why does that work?

We will represent our number as AB, where $A > B$. Then the new number is BA, which means that $AB > BA$. Next, we want to find $AB - BA$, but how do we do this? Recall that, since AB represents a two-digit number, not a product, we can represent AB as $10A + B$ and BA as $10B + A$. The difference between these two numbers is

$$AB - BA = (10A + B) - (10B + A) = 9A - 9B = 9(A - B)$$

Therefore, no matter what we do, our result will be a multiple of 9. And, if you check all two-digit multiples of 9, you will find that their digits add up to 9. (It turns out that the digits of *all* multiples of 9 add up to a *multiple* of 9. This is because our number system is base-10, which is one more than 9.)

The *digital sum* of a number is the sum of its digits. An interesting relationship between a number and its digital sum is that they share the same remainder when you divide by 9. For example, if we start with the number $238\,742\,115 = 26\,526\,901 \times 9 + 6$, its digital sum is $33 = 3 \times 9 + 6$. We define the *digital root* of a number as the digital sum of the digital sum of the digital sum... and so on, until the result is a single digit. For our example, we would get $238\,742\,115 \rightarrow 33 \rightarrow 6$, which is the remainder when we divide by 9. This occurs every time, because the base of our number system is one more than 9. To understand this, let's look at an easier example: $853 = 94 \times 9 + 7$, which has a digital sum of 16. Rewriting this in a different form, we get

$$853 = 8 \times 100 + 5 \times 10 + 3 = 8 \times (99 + 1) + 5 \times (9 + 1) + 3,$$

which can be rewritten as

$$[8 \times 99 + 5 \times 9] + (8 + 5 + 3) = 9 \times [8 \times 11 + 5] + 16.$$

Since the first part of the equation is a multiple of 9, 853 and 16 have the same remainder when divided by 9. If we keep the process going, the digital sum of the digital sum will be 7, which has the same remainder as 853 and 16 when divided by 9.

If a number is a multiple of 9, its remainder will be 0 when we divide by 9, but the digital root will never be 0 (unless we start with 0). When a number is a multiple of 9, its digital root must be 9.

Returning to the previous problem, recall the digital sum of a two-digit number that is a multiple of 9 will be a multiple of 9. Since the number in this case has two-digits, the largest possible value for the digital sum would be 18, but that could only happen if the difference is 99, which is impossible.

Now, let's return to our original problem. Using similar logic, we can rewrite the problem as

$$(10^5 P + 10^4 Q + 10^3 R + 10^2 S + 10T + 4) \times 4 = 400\,000 + 10^4 P + 10^3 Q + 10^2 R + 10S + T$$

Rearranging the equation yields

$$390\,000P + 39\,000Q + 3900R + 390S + 39T = 399\,984.$$

Dividing everything by 39 gives us

$$10\,000P + 1000Q + 100R + 10S + T = 10\,256$$

Hence, $P = 1$, $Q = 0$, $R = 2$, $S = 5$, and $T = 6$.

Now, if we put our thinking cap back on, we may realize that $PQRST$ from the original problem is just a 5-digit number that we can represent by x. Now, our original problem can be written as

$$(10x + 4) \times 4 = 400\,000 + x,$$

which solves to $x = 10\,256$, and we again have our desired result.

If we look at what we have found, we have found a number that, when you add a 4 to the end and multiply by 4, it seems that the 4 "moves" to the start of the number. Was there something special about 4? Did we have to have a five-digit number to start?

Let's simplify the problem and see whether, for example, $PQ4 \times 4 = 4PQ$. Using the method from our last solution, we would write this as

$$(10x + 4) \times 4 = 400 + x,$$

which leads to $39x = 384$, which does not have an integer solution. Perhaps, then, there are no more solutions of this type? A little more thinking leads us to consider an n-digit number of this type. Such a number would satisfy

$$(10x + 4) \times 4 = 4 \times 10^n + x,$$

which leads to

$$39x = 4 \times 10^n - 16,$$

or

$$x = \frac{4}{39}(10^n - 4).$$

Using Wolfram Alpha, I determined that an integer solution exists when n is 5, 11, 17, ... These yield the following results

$$102564 \times 4 = 410256$$
$$102564102564 \times 4 = 410256\,410256$$
$$102564102564102564 \times 4 = 410256\,410256\,410256$$
$$\vdots$$

At this point, we should be able to convince ourselves that this pattern continues.

A little computation reveals that

$$\frac{4}{39} = 0.\overline{102564}.$$

There is a strong connection between this fraction and our problem, but it is beyond the scope of this article. I have written a column that will appear in the October 2020 issue of *Crux Mathematicorum* (cms.math.ca/publications/crux) discussing a problem whose solution uses a similar technique. There are similar families of solutions to related problems, but the smallest member of the family might be quite large. For example,

$$1012658227784\mathbf{8} \times 8 = \mathbf{8}101265822784$$
$$10126582277848101265822784\mathbf{8} \times 8 = \mathbf{8}10126582277848101265822784$$

Are the smallest two solutions in the family of the form $(10x + 8) \times 8 = 8 \times 10^n + x$? Good luck hunting to anyone interested in finding some more relations of this type.

And now, it is time for your homework.

> On an island, there are two types of inhabitants: Heroes, who always tell the truth, and Villains, who always lie. Ten inhabitants are seated around a table. When they are asked "Are you a Hero or a Villain?", all ten reply "Hero." When asked "Is the person on your right a Hero or a Villain?", all ten reply "Villain". How many Heroes are present?

Until next time, stay safe, and happy problem solving.

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Shawn Godin teaches at Cairine Wilson Secondary School in Orleans, Ontario. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: leading workshops, writing articles, working on local projects and helping create mathematics contests.

Problems to Ponder

19 Not Out
Some positive numbers add up to 19. What is the maximum product?

Source: Mathematics Education Innovation. (December 2006). *Maths item of the month.* Available at https://mei.org.uk/month_item_06

Integer Numbers and Temperature Problems [1]

Nicole M. Wessman-Enzinger

Consider the number sentences –2 – –5 = __ and –2 + 5 = __. In what ways are these two equivalent? And in what ways are they different? Yes, –2 – –5 = __ and –2 + 5 = __ are number sentences that have solutions of 3. However, if you were to pose a story for these two number sentences (see Figure 1), –2 – –5 = __ and –2 + 5 = __, would you pose the same story? Let's say we focus on a specific context, such as temperature: What kind of temperature story would you pose for these two number sentences? Figure 1 highlights two temperature stories. What equivalence refers to in the realm of negative integers and contexts, like temperature, is complicated (Wessman-Enzinger & Tobias, 2015; Wessman-Enzinger & Salem, 2017). Integer number sentences that may have equivalent solutions are not necessarily contextually equivalent. The first story (see Figure 1), for –2 – –5 = __, represents a context in which two relative numbers (or temperatures) are given. We must then find the directed distance (state-state-translation) between the relative numbers. The second story, for –2 + 5 = __, illustrates a relative number (or temperature) that is translated, and we must then find the new relative numbers (state-translation-state). Because both –2 – –5 = __ and –2 + 5 = __ have equivalent solutions, we need to consider pedagogical moves to help students think about the precision of number sentences in terms of the contexts, not just obtaining a correct solution (Wessman-Enzinger & Salem, 2017).

The three vignettes below illustrate a small group of grade 5 students who worked on different types of temperature problems (Wessman-Enzinger & Tobias, 2015). Vignettes 1 and 2 refer to state-translation state; vignette 3 involves state-state translation. The Common Core State Standards for Mathematics (CCSSM) recommends including contexts, such as temperature, paired with the introduction to integers in grade 6 and integer operations in grade 7 (CCSSI, 2010). The grade 5 students described here (Alice, Jace, Kim; names are pseudonyms) are ideal examples for highlighting the initial integer conversations about consistency that may occur organically. These discussions are presented in the context of temperature; I facilitated as the teacher-researcher in students' small-group work. Specifically, these vignettes highlight the complexities of learning how to be contextually consistent in the context of temperature. That is, although –2 – –5 = 3 and –2 + 5 = 3, these number sentences are not contextually equivalent when the number sentence is consistent with the story (see Figure 1). The vignettes offer a perspective on how students think about consistency (i.e., the precision of the operations used in number

Fig. 1 Two different stories indicate the differences between the number sentences –2 – –5 = __ and –2 + 5 = __.	
–2 – –5 = __	–2 + 5 = __
The temperature is –2 in Boston and –5 in Chicago. Which place is warmer? And how much warmer is it?	It was –2 degrees this morning and increased 5 degrees in the afternoon. What is the temperature in the afternoon?

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sentences and how it matches the contexts) as they write integer number sentences for temperature problems. The vignettes also offer insight into how the teacher and students worked through the problems.

Vignette 1: Are 8 – 2 and –2 + 8 the Same?
Alice, Jace, and Kim solved the following temperature problem and wrote number sentences (see Figure 2).

> The temperature was –2 degrees Fahrenheit in the morning. The temperature rose 8 degrees in the afternoon. What was the temperature after it rose?

As the teacher/researcher, I supported students as they worked through solutions in their own way. Alice, Jace, and Kim first solved this temperature problem individually and then shared their solutions.

> *Kim:* 8 – 2 = 6 is easier, but all three work.

> *Alice:* It works [referring to 8 – 2 = 6], but it doesn't match. It works because it equals 6 and you are using the same numbers, but it doesn't match. Because the temperature rose, you are adding more onto –2. This [pointing at 8 – 2 = 6] is where the temperature is 8 and went down 2.

> *Jace:* It works and doesn't work. When you solve –2 + 8, you do 8 – 2 and you get 6, but it doesn't show the temperature increasing.

This exchange led the students into a discussion about the commutative property, after which Jace explained, "You can reverse the numbers, 8 + 2 and –2 + 8, but you cannot reverse the symbols."

Reflections on Student Thinking in Vignette 1
At first, when the students constructed number sentences, all that mattered was the solution and not the number sentence. Because these were students' beginning experiences with negative integers, they had not learned the procedures for integer operations. Consequently, rather than being consistent with the context, their number sentences reflected how they thought about solving the problem. Through their connection of similar solutions, but different number sentences, Alice and Jace initiated the idea that consistency of a number sentence involves more than just a solution. This powerful statement helps extend mathematical thinking and learning beyond the answer.

Reflections on Teaching in Vignette 1
Letting students who derived the same solution, but through different number sentences, discuss how this is possible is an important step. The different number sentences that

Fig. 2 The fifth-grade students wrote number sentences for the Rising Temperature problem.

The temperature was –2 degrees Fahrenheit in the morning. The temperature rose 8 degrees in the afternoon. What was the temperature after the temperature rose?

$-2+8$ $6°$

(a) Alice

$8°+2°=6°$

(b) Jace

$8-2=6$

(c) Kim

students produced for the same answer prompted a discussion about consistency of a number sentence with its context. Alice and Jace explored this concept when they reflected on the differences between $8 - 2$ and $-2 + 8$. Through their own observations and discussion, they began attending to precision in their mathematics.

Vignette 1 Highlights

Students can create solutions and number sentences without prior instruction. Eliciting student thinking provides an opportunity for students to reflect on consistency.

Vignette 2: How to Include +7 (Not −7) in the Sentence

Alice, Jace, and Kim then solved the following temperature problem and wrote number sentences (see Figure 3).

> The temperature was –4 degrees Fahrenheit in the morning. The temperature dropped 7 degrees in the afternoon. What was the temperature after it dropped?

In this vignette, students struggled with deciding how to include a positive 7 in the number sentence while also including a dropped temperature. This vignette illustrates the complexity of coordinating being correct with being consistent. When considering the correct answer, students shared their thoughts about which number sentence is best for the context. Although all three students eventually obtained the correct solution, they again wrote different number sentences. However, Kim and Jace focused on the correctness of their solutions rather than the consistency. After sharing the number sentences that they wrote, the students engaged in a discussion about the differences in their number sentences.

Fig. 3 Students struggled a little more with this "how to +7 degrees?" problem.

The temperature was –4 degrees Fahrenheit in the morning. The temperature dropped 7 degrees in the afternoon. What was the temperature after it dropped?

$$-4 - 7 = -11°$$
(a) Alice

$$-7 + 4 = \cancel{\#} - 1\cancel{0}°$$

$$\cancel{-3} - 4 = 3$$

$$7 - 4 = 3$$
$$-4 + 7 = 3$$
(b) Jace

$$-7 + -4 = -11°$$
(c) Kim

Jace: Oh, I did something wrong [looking at this number sentence, $7 + -4 = 3$].

Kim: [Leaning over and circling "dropped" on Jace's paper] Key words.

Teacher: What do you mean "key words"?

Kim: It is dropped [motioning with hand downward], and it didn't go up [motioning upward with hand].

Alice: [Looking at Kim] Dropped would be minus. Why did you do plus?

Kim: It would still be the same. I just did two negatives.

Jace: [Writes on paper.] Teacher: What did you just write on there? Can you tell me?

Jace: I did –7 + –4 = –11.

Teacher: So, we have –7 + –4 = –11 and –7 – 4 = –11. What do you/we think about these number sentences?

Kim: They work. If you just did 7 + 4, it's 11, but both of the numbers are negatives. So, it's technically the same thing but happening in the negatives.

Teacher: What do you mean, "both of the numbers are negatives"? Kim: Both the 7 and 4 are negative, so it's just like adding.

Teacher: Do we all think that both the numbers are negative?

Alice: [Shaking head no.] Because if this was negative, it would say –7 [pointing at the word problem]. If it was negative, which it isn't, it would say it.

Teacher: Is your number sentence different?

Alice: Yes.

Teacher: Can you explain your number sentence and how it works?

Alice: Because the temperature was –4 and temperature dropped, had dropped 7 degrees, so I did –7, and that's what the temperature is now.

Because I wanted to facilitate their discussion to dig a little deeper about consistency, I reminded Alice and Kim of an earlier conversation. I questioned Alice about why she had asked Kim "why she used plus."

Alice: Because she said dropped, and dropped is minus, then why did you do plus?

Teacher: So what do you all think about that? Do you think *dropped* is minus?

Jace: Well, it is a 7 and doesn't have a negative symbol in front of it. But since it dropped, you could either do –4 + –7 or –4 – 7 and it would still be the same answer.

Teacher: Do you think they both work?

Alice: They both work, but I like mine better [referring to –4 – 7].

Teacher: Do you like yours better just because it's yours or for a different reason?

Alice: I think it makes more sense.

Teacher: What made you choose to not write –4 – 7?

Jace: Well, when I was going through and thinking about it. I did 7 – 4 = 3, and then I changed it to 7 + 4 = 11.

Reflections on Student Thinking in Vignette 2
Wondering why she, Jace, and Kim had obtained the correct solution, Alice questioned other students about how a falling temperature could support a number sentence containing addition. She asked, "Dropped would be minus; why did you do plus?"

Kim also helped leverage a discussion on consistency with her emphasis on thinking about the nature of dropped temperature versus increased temperature. Together they

questioned how a falling temperature could support an addition number sentence, supporting –4 – 7 = –11 over –7 + –4 = –11. In other words, the following questions are related to their queries: How can we incorporate a dropped temperature with adding? and How can we use the positive integer, given that the temperature dropped?

Reflections on Teaching in Vignette 2 In this vignette, I facilitated a focus on consistency by asking students various questions: Is your number sentence different? Can you explain your number sentence and how it works? Do you think they both work?

I also asked students to consider which number sentence was best. This supported student discussion about how to write a number sentence that included a positive temperature and also a dropped temperature. Even though all the students had the correct solution, I supported students in discussing the differences in their number sentences, which afforded them the opportunity to also pose questions that directed attention toward precision.

Vignette 2 Highlight
Students and teachers alike can facilitate discussion about consistency, even when students are focused on obtaining correct answers.

Vignette 3: 5 + 9 = 14 Sounds Right, but Where is the –9°?
Alice, Jace, and Kim solved the following North Pole/South Pole Temperature problem: The warmest recorded temperature of the North Pole is about 5° Celsius.

> The warmest recorded temperature of the South Pole is about –9° Celsius. Which place has the warmest recorded temperature? And how much warmer is it?

The three students worked on the problem individually first and then engaged in a small-group discussion. Their number sentences were all different for vignette 3. Alice's and Kim's answers were 14 degrees (see Figures 4 and 6). Jace determined that the South Pole was warmer by 4 degrees (see Figure 5). This vignette highlights how, if allowed to create solutions and number sentences, students may write an addition number sentence rather than a subtraction number sentence even without knowing a procedure.

Alice constructed tallies that looked similar to finding distance on a number line. However, she made fewer tallies than there would be on a number line that illustrates the distance of –9 to 5. When asked about this, Alice explained that she had not included zero—making a drawing of –9, –8, –7, –6, –5, –4, –3, –2, –1, 1, 2, 3, 4, 5.

After the students worked individually, I asked them to share their thinking. Jace and Alice seemed confident in their solutions that the North Pole is warmer by 14 degrees, but Kim wavered between 14 and 4 degrees as a solution. Although she wrote the number sentence 5 + 9 = 14, she also agreed with Jace's response (–9 + 5 = –4). Jace's response seemed worth consideration to Kim because it included –9 degrees, whereas 9 + 5 = 14 did not include the –9. The following conversation explored this discrepancy:

> *Teacher:* [Looking at Kim] So first you had 4, then you had 14, and then you went back to 4? Alice: And, then you went back to 14.

> *Kim:* [Putting hands on face.] Oh, no.

Teacher: Can you tell me your struggle— like, this is why I thought 4, and this why I thought 14? Can you help me with that?

Kim: After looking, debating, I declare it is 14. Teacher: You think it's 14. . . . OK, tell me why it's 14, no number sentence.

Kim: Um . . . I just added. Without a negative, I added 5 + 9. Because . . . [making a face]. I don't know. I don't know how to explain this.

Teacher: You don't know how to explain it?

Kim: Technical difficulties.

Alice: Technical difficulties.

Teacher: But you think it's 14? Kim: I am positive it is 14.

Teacher: You are positive it's 14 but are struggling to explain it. So, where do you think it's warmer at?

Alice and Kim: North Pole.

Teacher: And, then, so what does the 14 mean to you?

Kim: The 14 is how much warmer it is.

Alice: OK, but how is it warmer if it's not even that warm?

Kim: Isn't 5 warmer than –9?

Alice: Well . . .

Kim: Positive versus negative here.

Alice: OK, is this negative, or is it positive? [Pointing to Kim's number sentence 5 + 9 = 14]

Kim: Positive.

Alice: Well, this is a positive? [Pointing at 9 in Kim's number sentence and then –9 in the temperature problem.]

Reflections on Student Thinking in Vignette 3

Although curricula (e.g., Lappan et al., 2006) connect these types of temperature comparison problems (state-state translation) to integer subtraction, we see that these students initially wrote addition number sentences (Jace wrote –9 + 5 = –4; Alice and Kim wrote 5 + 9 = 14). This temperature problem includes an integer subtraction number sentence, 5 – –9 = 14, but is that how these students, who first encountered this problem type, solved this problem? We saw these fifth graders extend their whole-number reasoning and persevere in making sense of this difference problem by writing addition number sentences, which is a correct mathematical solution process.

Although writing addition number sentences is a productive way to solve this problem, the students wondered how to incorporate the –9 degrees. This vignette ends with Alice stating to Kim, "Well, this is a positive." With this, Alice points to the fact that –9 degrees was in the problem, but the number sentence, 5 + 9 = 14, included only + 9. For this reason, Alice wrote two number sentences: 5 + 9 = 14 and 5 – –9 = 14. When Alice was asked to explain

her thinking about these number sentences and which number sentence worked better, she drew a picture containing 14 tally marks, with –9 and 5 marked on the first and last of those tallies (see Figure 4), attempting to explain to the other students how 5 + 9 = 14 and 5 – –9 = 14 are related.

Students like Alice and Jace wanted to include the –9 in the number sentence. But this became problematic when the students also wanted to use addition (5 + 9 = 14). Jace did this incorrectly with addition (–9 + 5 = –4). Alice, however, made the connection to integer subtraction (5 – –9 = 14). Although Kim recognized that –9 and 5 are 14 units apart, she agreed with Jace's inclusion of the –9 in his number sentence (–9 + 5 = –4). This introduced some uncertainty in her response. Alice's contribution of writing both number sentences 5 + 9 = 14 and 5 – –9 = 14 and Jace's and Kim's desire to include –9 in the number sentences became paramount to focusing on consistency in the temperature problem.

Reflections on Teaching in Vignette 3
Vignette 3 highlights how students, if allowed to create solutions and number sentences, may write addition number sentences rather than subtraction number sentences. Because of the opportunity to write number sentences as she wished, Alice was able to question, "Where is the –9 in 5 + 9 = 14?" This prompted students to consider 5 – –9 as a relevant number sentence equivalent to 5 + 9 = 14. Notably, these students did not have prior experience with any procedures for integer subtraction. Consequently, using this problem type of temperature comparison and allowing students to write the number sentences that they wanted freely empowered them. They were able to invent ways of considering a typical integer subtraction problem. By focusing on whether or not to include –9 in the number sentence, students attended to the consistency between the number sentences and the context.

Vignette 3 Highlight
Without prior instruction on integer subtraction, students are capable of making sense of challenging problems and considering the consistency of the solution methods using the context of the problems.

Concluding Remarks and Suggestions for Teaching
Coordinating both correctness (i.e., the solution to the word problem) and consistency (i.e., writing a

Fig. 4 Alice's work took a circuitous path.
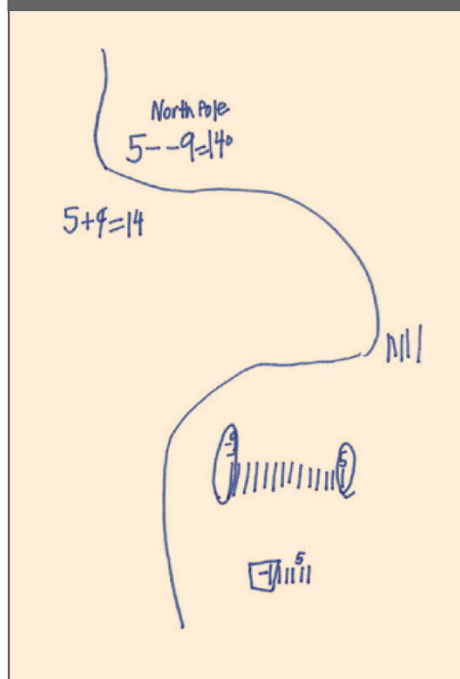
Fig. 5 Jace's written work for the North Pole South Pole problem contained a slight error.
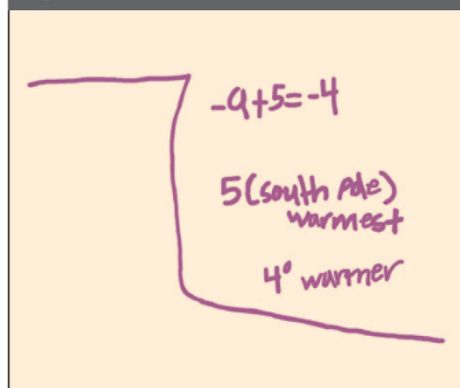
Fig. 6 Kim determined that the North Pole was warmer and wrote the number sentence 5 + 9 = 14.

number sentence that matches the word problem) are important components of attending to precision, the sixth of the Common Core Standards for Mathematical Practice (CCSSI, 2010). When students get correct answers but write different number sentences, they may be imprecise or not represent the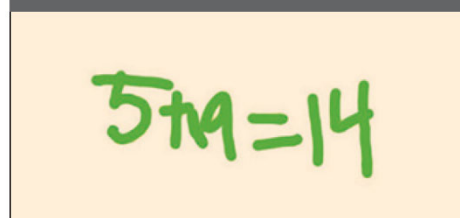 contexts as precisely as possible. We must support our students as they attend to precision and move beyond just a correct answer. Attending to precision in these vignettes did not entail telling students what types of number sentences they needed to write or that they were wrong about the number sentences that they did write. Instead, two important components were included: (1) supporting students in creating their own number sentences and (2) facilitating discussion about why number sentences matched or did not match.

Try it. When giving students temperature problems or any contextual problem for integers, allow students to solve them in their own way and create their own number sentences. Now that you have read these vignettes, try giving your students these temperature problems. Have students share the different number sentences they construct. Facilitate the discourse and highlight questions about consistency. By creating their own number sentences and discussing the solutions, your students will unpack the complexities of matching number sentences to contextual problems with integers.

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Nicole M. Wessman-Enzinger, nicole.enzinger@georgefox.edu, is a mathematics teacher educator at George Fox University in Newberg, Oregon. She is interested in the creative and inventive ways that students engage with mathematics, and particularly with integers.

Family Math: Winning Them Over

Cindy Kroon

"Is it family math this week?"

 "Math is my favorite class now!"

Music to the ears of math educators everywhere! Unfortunately, such exclamations are not the norm. Too many adults have developed a dislike, or even worse, a fear of mathematics. As John Allen Paulos (2001) has pointed out, it has even become socially acceptable to voice this opinion. This is particularly concerning, as negative messages about mathematics can be (inadvertently) passed from parents to students. However, a productive disposition toward mathematics can be similarly passed down through positive family experiences, which could have far-reaching impacts on students. "Someone should do something!", I told myself… and the Family Math project was born!

Involving Students and Families: The Big Idea

In the Montrose family Math project, elementary students work with engaging math games, activities, or puzzles during class, then take home all materials needed to play. Family Math brings parents and children together in a math-supportive environment. Criteria for choosing activities include: easy to learn and fast to play; appropriate mathematical content; and simple and inexpensive materials. Critically, games must also be *fun*. (If it feels like homework, it isn't appropriate.)

Classroom Visit: 2nd Grade Fun with Ladybug Dice

I visit each K-5 classroom monthly with a new activity. It is typical for students to clap and cheer when they realize that it is Family Math day, often exclaiming, "I love math!" The class gathers in a circle on the floor to observe a sample game between their teacher and me. Students quickly begin to cheer for their teacher, but allegiances often change several times as the game continues. Over time, the circle draws inward as student enthusiasm grows.

Once everybody understands the game, each student receives a take-home packet containing instructions and all needed materials. Pairs or small groups briefly practice playing (Figures 1 and 2). Students take their packet home for one week, "just like a library book," then return it to their classroom teacher.

Figure 1: Students practice playing for a few minutes

A favorite game among the second graders I visit is Ladybug Dice. This fun, fast-moving game focuses on addition.

Ladybug Dice

Materials: 2 dice for each group of 2-4 players

The ones on the dice are called "ladybugs." If a one is rolled, special rules apply.

Directions:
1. Player 1 rolls both dice and announces the sum. Player 1's score for the round is the sum, unless one or more ladybugs are rolled.
2. If a player rolls one ladybug, s/he records no score for the round. The player MUST trade papers with the player having the highest score, even if it is not to their advantage to do so.
3. If a player rolls two ladybugs (doubles), s/he must cross off their score and start over at zero on their next turn.
4. Players alternate turns, adding each new score to their previous sum. The first player to reach 100 points wins.

Family Favorites

Over the course of the school year, 54 different activities went home with students. Games were selected by math standard, as determined by the Common Core State Standards (CCSSM). Strategy games focused on the Standards for Mathematical Practice (SMP) expecting students to "analyze givens, constraints, relationships, and make conjectures." These are also the skills practiced during game play.

Table 1 includes a family favorite from each grade level, as ranked by parent/student feedback. In addition to the family favorite, listed sites proved a particularly rich source of ideas for further exploration. A quick internet search for "Family Math" will yield many additional resources.

Figure 2: Second grade students love playing Family Math games

Grade/Standard	Game	Source
Kindergarten K.OA.A.2	Ladybug Spots Add and subtract within five	North Carolina Department of Public Instruction
1st 1.OA.C.6	Over the Hill Math Roll dice to find sums up to 18	www.zenomath.org
2nd 2.NBT.B.5	Ladybug Dice Fast-moving addition dice game	Author (Figure 1)
3rd 2.NBT.B.5	Oh No! 99! Addition and subtraction to 99	Bresser (1999)

Grade/Standard	Game	Source
4th SMP 1,2,4	Jumpin' Jiminy (Golf Tee Game) Strategy game requiring logical thinking	Author, also www.joenord.com
5th SMP 1,2,4	Ultimate Tic-Tac-Toe Classic tic-tac-toe game with a major twist	www.thegamegal.com

Table 1: Families rated their favorite games on provided feedback forms.

A favorite game among the fourth graders I visited was Jumpin' Jiminy (Figure 3).

Jumpin' Jiminy!
A cooperative game for 2 or more players

The object of the game is to remove all but one of the counters from the game board.
- Place 14 counters on the board (Figure 3), leaving the START space empty.
- Jump one counter over another into an empty space. You may only jump in a straight line.
- Remove the counter that was jumped over.
- Alternate turns and keep jumping until only one counter remains… if you can!

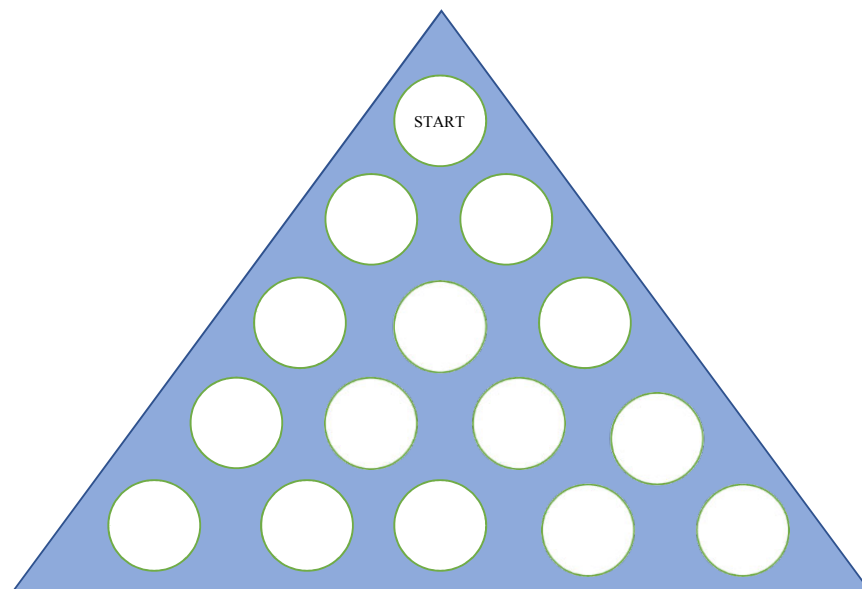
Figure 3: Jumpin' Jiminy! game board

Materials and Implementation
As a Title I school, approximately 20% of Montrose Elementary students come from low income families. These students sometimes experience an opportunity shortfall due to

family circumstances. Providing a packet with all needed materials enables low-income families to participate fully.

A start-up budget of $900 was funded by the Montrose Booster Club. Since loss and timely return of materials was a concern, inexpensive materials (playing cards, dice, colored counters, etc.) were chosen (Figure 4). We decided not to discipline students who lost materials or returned them late. Missing recess, withholding privileges or other consequences might transform Family Math into a negative experience— exactly the opposite of the project's intended results. Despite this, throughout the school year, over 900 individual packets were distributed, of which over 99% were returned. Late/missing materials proved to be much less of a concern than was initially

Figure 4: Supplies were purchased in class sets of 25

anticipated; losses for the year totaled less than $10. Clearly, students also learned lessons about punctuality and responsibility.

Outcomes and Family Feedback
During the school year, each elementary student took home nine games. Approximately half of the families completed and returned the enclosed feedback form (Figure 5). It is not possible to know with certainty whether the others did not play at all or simply did not supply feedback. However, nearly all students, and also many parents mentioned that they enjoyed the games and looked forward to more. Therefore, it seems reasonable to assume the latter. Lacking complete information, some anecdotal conclusions can still be drawn.

- Students frequently sought me out to report that math was now their favorite subject. They gleefully reported winning against a parent or older sibling (frequently one of my own high school students).
- Parents visited my classroom to discuss activities they had particularly enjoyed. They also mentioned appreciating the opportunity to put down their phones, turn off the television, and be fully present with their children.
- High school students frequently mentioned playing with younger siblings, and expressed that they had also enjoyed the games. The high school favorite was Ultimate Tic-Tac-Toe.
- Students challenged one another, and even organized a tournament during their study hall time.

An unexpected result: as the year progressed, I emerged as a minor celebrity among elementary students. Passing through the lunchroom or playground often resulted in students' running over to tell me about their most recent game experience. This sharing was frequently accompanied by a big hug, and an inquiry about when the next activity would be coming to their classroom.

Did you/student/family play the game? (Yes) No
Were the directions clear? Yes Beth No
How about difficulty level? Too easy Just right Pretty hard Too difficult
Who played? ___ Dad Mom
How many times? 4
Did your family enjoy the game? Yes no

Suggestions for a future activity? Yes no

Other comments? I think you should Make a harder game not to be round I think you should thee.

Thank you for participating! Please return this page with all activity packet/materials to your child's classroom teacher. Mr.

Did you/student/family play the game? Yes No
Were the directions clear? Yes No
How about difficulty level? Too easy Just right Pretty hard Too difficult
Who played? Student and Parent
How many times? 3
Did your family enjoy the game? This was a great game! My family liked it the most!(so far) It was challenging for him at first. I enjoyed seeing him catch on quickly and start crunching those numbers. We will play again.
Suggestions for a future activity? X

Other comments? X

Thank you for participating! Please return this page with all activity packet/materials to your child's classroom teacher.

Cindy Kroon
cindy.kroon@k12.sd.us

Did you/student/family play the game? Yes No
Were the directions clear? Yes No
How about difficulty level? Too easy Just right Pretty hard Too difficult
Who played? the entire family played it each a few times with Paisley
How many times? 12+
Did your family enjoy the game? yes
Suggestions for a future activity?

Other comments? Mrs. Kroon— the games you are sending home are great & give kids such confidence—even if its just basic numbers & counting! We love it!

Thank you for participating! Please return this page with all activity packet/materials to your child's classroom teacher. Thanks for doing this for our school!

Cindy Kroon
cindy.kroon@k12.sd.us

Figure 5: Both parents and students provided feedback, often reporting playing multiple times!

Reflections and Next Steps

In retrospect, it would have been best to collect initial baseline data on student and parent attitudes about mathematics, followed by a final end-of-year survey. With this data in hand, stronger conclusions could have been drawn regarding the project's impact. Collecting before/after information will be a goal in future iterations of this project. Others planning implementation of a similar project should consider collecting pre-implementation baseline data to better enable further research.

Once teachers and district leadership are on board, expense is an obvious consideration. Community Foundations, district Booster Clubs, and other non-profit resources are potential sources of funding. Many organizations, both local and national, have donations in hand and are waiting for funding requests. The Montrose Family Math project was funded through donorschoose.org, the Montrose Community Foundation, and the local Booster Club.

Because of the small student population, the Family Math project was a district-wide effort. However, this project could be scaled up or down based on the needs and resources available. Options could include grade-level projects implemented by teacher leaders or math coaches. Family Math also has potential for individual classroom implementation. Teachers could send home a "game of the month" with students as resources and time allow.

Conclusion: Everybody Wins!

Family Math was exciting for students, enjoyable for parents, and fun for teachers. What's not to like? I found it particularly rewarding to experience the new enthusiasm displayed by students. A modest investment of time and money provided a huge return through improved perceptions of mathematics.

Many students and parents have commented that they experienced a "whole new side" of mathematics through the Family Math project. Isn't that exactly what we want?

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Cindy Kroon, cindy.kroon@k12.sd.us, teaches mathematics at Montrose High School in Montrose, South Dakota, USA. She is a frequent presenter at local and regional conferences, and is past-president of South Dakota Council of Teachers of Mathematics.

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Intersections

In this column, you'll find information about upcoming math education-related workshops, conferences, and other events. Some events fill up fast, so don't delay signing up! For more information about a particular event or to register, follow the link provided below the description. If you know about an upcoming event that should be on our list, please contact us at thevariable@smts.ca.

Within Saskatchewan

Accreditation Initial Seminar
October 15-16, October 29-30 2020
Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Accreditation seminars are offered to enable qualified teachers to become accredited. Accreditation is the process by which qualified teachers are granted the responsibility of determining the final mark or standing of the students in a specified Grade 12 (level 30) subject or subjects. The Accreditation seminar provides an opportunity for teachers to challenge, extend, enhance and renew their professional experience with an emphasis on assessment and evaluation. Participation in this seminar results in partial fulfilment of the requirements for accreditation in accordance with the Ministry of Education's publication *Accreditation (Initial and Renewal): Policies and Procedures* (2017).

More information at https://www.stf.sk.ca/professional-resources/events-calendar/accreditation-initial-seminar

Accreditation Renewal/Second Seminar
October 15-16, 2020
Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

The Accreditation seminar provides an opportunity for teachers to challenge, extend, enhance and renew their professional experience with an emphasis on assessment and evaluation. Participation in this seminar results in partial fulfilment of the requirements for accreditation in accordance with the Ministry of Education's publication *Accreditation (Initial and Renewal): Policies and Procedures* (2017).

More information at

Beyond Saskatchewan

NCTM Annual Meeting and Exposition in Atlanta: From Critical Conversations to Intentional Actions
April 21-24, 2021
Atlanta, GA
Presented by the National Council for Mathematics Teachers

Join thousands of your mathematics education peers at the premier math education event of the year! Network and exchange ideas, engage with innovation in the field, and discover new learning practices that will drive student success. Classroom teachers, administrators, math coaches, supervisors, college professors, and preservice teachers can all benefit from the sessions and learning at this event. The NCTM Annual Meeting and Exposition in Atlanta content strands are:

• Broadening the Purposes of Learning and Teaching Mathematics
• Advocacy To Make an Impact in Mathematics Education
• Equitable Mathematics Through Agency, Identity, and Access
• Building and Fostering a Sense of Belonging in the Mathematics Community
• Effective Mathematics Teaching Practices

More information at https://www.nctm.org/annual/

NCTM Annual Meeting and Exposition in St. Louis: Moving Forward - Kicking off NCTM's Next 100 Years
September 22-25, 2021
St. Louis, MO
Presented by the National Council for Mathematics Teachers

As NCTM kicks off its next 100 years, the St. Louis Annual Meeting & Exposition will bring together members of the math education community from around the globe—classroom teachers; school, district, and state mathematics education leaders; administrators; mathematics teacher educators; mathematicians; and researchers—in a setting that encourages conversation, collaboration, and the sharing of knowledge. You'll see and hear new ideas and approaches that will help you move forward and provide more and better mathematics instruction for each and every student. The NCTM Annual Meeting and Exposition in St. Louis content strands are Systemic Change; Agency, Identity, and Access; Professionalism and Advocacy; and Mathematics Teaching and Learning.

More information at https://www.nctm.org/stlouis2021/

MCATA Fall Conference: 2020 Vision – Bringing Mathematical Thinking into Focus
October 23-24, 2020
Red Deer, AB
Presented by the Mathematics Council of the Alberta Teachers Association

Join MCATA in celebrating their annual fall conference with keynote speakers John Orr and Kyle Pearce. The MCATA Fall Conference 2020 guiding questions are:

- How do we make thinking visible in assessments?
- How do we make thinking visible through learning progressions?
- How do we make thinking visible through the use of technology, manipulatives, and mathematical tools?

More information at https://www.mathteachers.ab.ca/fall-conference/

OAME Annual Conference 2021: Equity Counts
May 20-21, 2021
Toronto, ON
Presented by the Ontario Association for Mathematics Education

OAME hosts an annual conference where educators have the opportunity to hear from keynote and featured speakers, attend workshops and networking events, and explore the latest resources available from exhibitors. The main focus of OAME 2021 – Equity Counts is to address equity in mathematics education and promote best classroom practices. This means to strive to have students attain proficiency in mathematics, regardless of race, gender, language, socioeconomic status, or learning style.

More information at https://sites.google.com/oame.on.ca/oame2021

Virtual Professional Development

Education Week Webinars
A collection of free and premium virtual broadcasts, including upcoming and on-demand webinars. These virtual broadcasts cover teaching and learning and include webinars on differentiated instruction and the common-core standards. All webinars are accessible for a limited time after the original live streaming date. For all webinars broadcast by Education Week after August 1, 2019, Certificates of Completion are available to all registered live attendees who attend 46 minutes or more of any webinar.

Available at www.edweek.org/ew/marketplace/webinars/webinars.html

Global Math Department Webinar Conferences
The Global Math Department is a group of math teachers that organizes weekly webinars and a weekly newsletter to let people know about the great stuff happening in the math-Twitter-blogosphere and in other places. Webinar Conferences are presented every Tuesday evening at 9 pm Eastern. In addition to watching the weekly live stream, you can check the topic of next week's conference and watch any recording from the archive.

Available at www.bigmarker.com/communities/GlobalMathDept/conferences

NCTM 100 Days of Professional Learning
Presented by the National Council for Mathematics Teachers

E-seminars are recorded professional development webinars with facilitator guide and handouts. E-seminars are free for NCTM members. Webcasts of Annual Meeting Keynote Sessions offer notable and thought provoking leaders in math education and related fields as they inspire attendees at NCTM Conferences.

Available at https://www.nctm.org/1day-ssp/

NCTM E-Seminars and Webcasts
Presented by the National Council for Mathematics Teachers

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Available at https://www.nctm.org/NCTM/templates/ektron/two-column-right.aspx?pageid=75420

NCTM 2020 Virtual Conference
November 11-14, 2020
Presented by the National Council for Mathematics Teachers

NCTM is committed to bringing the math community together for engaging content that will help transform the learning and teaching of mathematics. Join your colleagues for the NCTM 2020 Virtual Conference and share in the excitement and love of math! Our exciting online platform will provide opportunities for networking, small chat rooms, discussions with exhibitors and much more.

More information at https://www.nctm.org/virtual2020/

Problems to Ponder

Goats
Anne and Michel each have goats. Michel says to Anne : "If you give me one of your goats, I will have double of what you will have remaining." Anne replies: "If you give me one of your goats, we will have exactly the same number of goats".

How many goats do Anne and Michel have?

Source:
    https://www.pedagonet.com/brain/brain47.htm

Tangents
Extracurricular Opportunities for Students
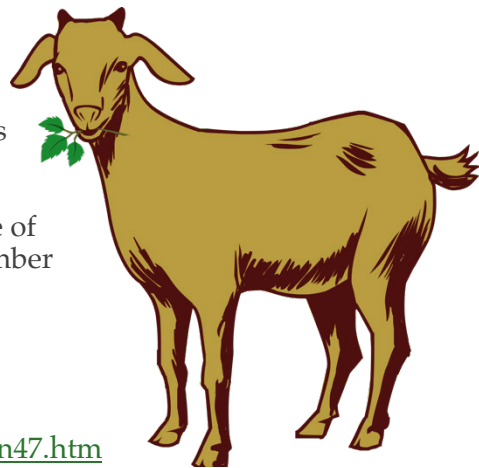
*T*his column highlights local and national extracurricular opportunities for K-12 students interested in mathematics, including collaborative, individual, online, and in-person challenges, contests, and camps. For dates, registration procedures and applications, and other information about the contests listed, please head to the contest websites, included below the descriptions. If we have missed an event that should be on our list, please contact us at *thevariable@smts.ca*.

If you are looking for contests available in other provinces, head to the Canadian Mathematical Society website (cms.math.ca/Competitions/othercanadian). The CMS also maintains a list of resources for students who are looking to build their problem-solving skills and succeed in competitions: see cms.math.ca/Competitions/problemsolving.

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Beaver Computing Challenge
Weeks of November 9 & November 16, 2020

The Beaver Computing Challenge (BCC) is designed to get students from Grade 5-10 with little or no previous experience excited about computing. The BCC is a problem solving contest with a focus on computational and logical thinking, with questions inspired by topics in computer science but which only require comfort with concepts found in mathematics curriculum common to all provinces. Connections to Computer Science are described in the solutions to all past contests. The contest is held online in schools.

More information at https://www.cemc.uwaterloo.ca/contests/bcc.html

Canadian Computing Competition
February 17, 20201

The Canadian Computing Competition (CCC) is a fun challenge for secondary school students with an interest in programming. It is an opportunity for students to test their ability in designing, understanding and implementing algorithms. Students are encouraged to prepare; see suggestions on contest website. The contest is held online in schools.

More information at https://www.cemc.uwaterloo.ca/contests/computing.html

Canadian Math Kangaroo Contest
Written in March

The purpose of this competition is to introduce youngsters from Grade 1 to Grade 12 to math challenges in a fun and enjoyable way, thus inspiring their further interest and advancement in mathematics. The competition is held yearly in more than 50 Canadian cities. Students may choose to participate in English or in French.

More information at https://mathkangaroo.ca

Canadian Mathematical Gray Jay Competition
October 8, 2020
Presented by the Canadian Mathematical Society

The Canadian Mathematical Gray Jay Competition (CMGC) is a new Canadian math competition open to students in grades K-8, with questions based primarily on grade 5-8 curriculum. This competition has been created by mathematicians from across Canada. The problems are meant to be a fun fall activity for students and teachers to complement their math curriculum and build students' problem solving skills. The CMGC will offer engaging problems that will allow for discussion after the competition and get students excited about math. CMS is committed to contributing to a more inclusive environment in the mathematical community. Funding will be made available for students who identify as Black or Indigenous to participate in the 2020 CMGC free of charge.

More information at https://cms.math.ca/canadian-mathematical-gray-jay-competition-cmgc/

Canadian Open Mathematical Challenge
October 29, 2020
Presented by the Canadian Mathematical Society

The Canadian Open Mathematics Challenge (COMC) is Canada's premier national mathematics competition open to any student with an interest in and grasp of high school math. The purpose of the COMC is to encourage students to explore, discover, and learn more about mathematics and problem solving.

Approximately 80 top-ranking students from the COMC and the Canadian Mathematical Olympiad Qualifying Repêchage (CMOQR) will be invited to write the Canadian Mathematical Olympiad (CMO). Based on the results from the COMC, the CMO and other national and international mathematics competitions and camps, the Canadian Mathematical Society IMO Committee will then select six students as part of Math Team Canada to travel to, and compete in, the International Mathematical Olympiad (IMO). Top COMC female contestants will also qualify to be part of Girls' Math Team Canada to represent Canada at the European Girls' Mathematical Olympiad (EGMO). EGMO is an international mathematics competition.

Depending on their grade level and performance, students participating in the COMC will also have the opportunity to be considered for university scholarships, get invited to math camps, garner awards, and win prizes.

More information at https://www2.cms.math.ca/Competitions/COMC/2019/

Canadian Senior and Intermediate Mathematics Contests
November 18, 2020
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Canadian Senior and Intermediate Mathematics Contests (CSMC and CIMC) are two contests designed to give students in Grades 9-12 (and motivated students in lower grades) the opportunity to have fun and to develop their mathematical problem solving ability. Most of the CIMC problems are based on the mathematical curriculum up to and including Grade 10; most of the CSMC problems are based on the mathematical curriculum up to and including the final year of secondary school. The contest is written by individuals in schools.

More information at https://www.cemc.uwaterloo.ca/contests/csimc.html

Canadian Team Mathematics Contest
April 8 & 9 2021
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Canadian Team Mathematics Contest (CTMC) is a fun and challenging competition for teams of 6 secondary school students in any combination of grades. One teacher and groups of six students participate at their own school over 3 consecutive hours. The curriculum and level of difficulty of the questions will vary. Junior students will be able to make significant contributions but teams without any senior students may have difficulty completing all the problems. Written in April at the University of Waterloo; teams may participate unofficially in their school on any day after the official contest date.

More information at www.cemc.uwaterloo.ca/contests/ctmc.html

Caribou Mathematics Competition
Held six times throughout the school year

The Caribou Mathematics Competition is a worldwide online contest that is held six times throughout the school year. Each of these days, five contests are offered, one for each of the grade levels 3/4, 5/6, 7/8, 9/10 and 11/12 and each one in English, French and Persian. The Caribou Cup is the series of all Caribou Contests in one school year. Each student's ranking in the Caribou Cup is determined by their performance in their best 5 of 6 contests through the school year.

More information at cariboutests.com

Euclid Mathematics Contest
April 7, 2021
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Euclid Mathematics Contest is an opportunity for students in their final year of secondary school and motivated students in lower grades to have fun and to develop their mathematical problem solving ability. Most of the problems are based on the mathematical curriculum up to and including the final year of secondary school. Most of the problems are based on curricula up to and including the final year of secondary school. Some content might require students to extend their knowledge and the best way to familiarize oneself with commonly appearing topics is to practice using past contests. The contest is written by individuals in schools.

More information at www.cemc.uwaterloo.ca/contests/euclid.html

Fryer, Galois, and Hypatia Mathematics Contests
April 14, 2021
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Fryer, Galois and Hypatia Math Contests are an opportunity for students to write a full-solution contest. They are fun way to develop mathematical problem solving skills through a written mathematical activity. For students in Grades 9 (Fryer), 10 (Galois) and 11 (Hypatia). Questions are based on curriculum common to all Canadian provinces. Rather than testing content, most of the contest problems test logical thinking and mathematical problem solving. The contest is written by individuals in schools.

More information at www.cemc.uwaterloo.ca/contests/fgh.html

Gauss Mathematics Contests
May 12, 2021
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Gauss Contests are an opportunity for students in Grades 7 and 8, and interested students from lower grades, to have fun and to develop their mathematical problem solving ability. Questions are based on curriculum common to all Canadian provinces. The Grade 7 contest and Grade 8 contest is written by individuals and may be organized and run by an individual school, by a secondary school for feeder schools, or on a board-wide basis.

More information at www.cemc.uwaterloo.ca/contests/gauss.html

Opti-Math
Written in March
Presented by the Groupe des responsables en mathématique au secondaire

A French-language mathematics challenge for secondary students who would like to exercise and develop their problem-solving skills.

Les Concours Opti-Math et Opti-Math + sont des Concours nationaux de mathématique qui s'adressent à tous les élèves du niveau secondaire (12 à 18 ans) provenant des écoles du Québec et du Canada francophone. Ils visent à encourager la pratique de la résolution de problèmes dans un esprit ludique et à démystifier, auprès des jeunes, les modes de pensée qui caractérisent la mathématique. Le principal objectif des Concours est de favoriser la participation bien avant la performance. La devise n'est pas : « que le meilleur gagne » mais bien « que le plus grand nombre participe et s'améliore en résolution de problèmes ».

More information at www.optimath.ca/index.html

Pascal, Cayley, and Fermat Contests
February 23, 2021
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Pascal, Cayley and Fermat Contests are an opportunity for students in Grades 9 (Fryer), 10 (Galois)m and 11 (Hypatia) to have fun and to develop their mathematical problem solving ability. Early questions require only concepts found in the curriculum common to all provinces. The last few questions are designed to test ingenuity and insight. Rather than testing content, most of the contest problems test logical thinking and mathematical problem solving. The contest is written by individuals in schools.

More information at www.cemc.uwaterloo.ca/contests/pcf.html

The Virtual Mathematical Marathon
Supported by the Canadian National Science and Engineering Research Council

The virtual Mathematical Marathon has been developed by an international team of mathematicians, mathematics educators, and computer science specialists with the help of the Canadian National Science and Engineering Research Council.

The main activity is a competition allowing students to enjoy solving challenging mathematical problems all year around. Students can join the game at any time and at no cost, simply creating an individual profile with an individual username and a password. Available in French.

More information at www8.umoncton.ca/umcm-mmv/index.php

Problems to Ponder

The Great Candy Mixup
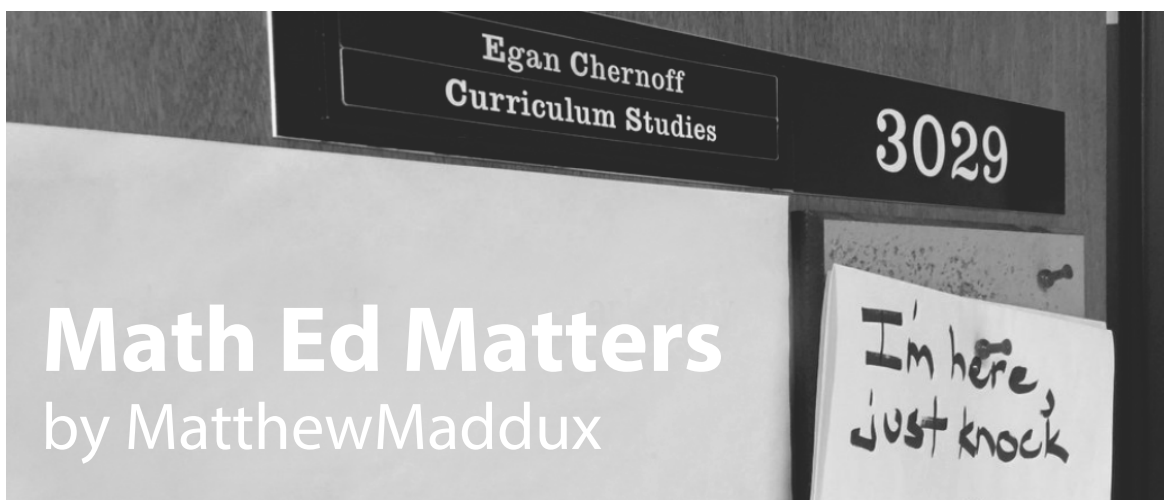Jar A has 800 red candies and jar B has 600 green candies.

If we do the following…

- Move one cup of red candies from jar A to jar B and mix.
- Move one cup of mixed candies from jar B to jar A and mix.
- Move one cup of candies from each of jar A and jar B to an empty bowl and mix.
- Randomly select one candy from the bowl.

…is the candy we select more likely to be red or green?

Source: Brilliant.org. (n.d.). The great candy mixup. Available at https://brilliant.org/daily-problems/candy-mixed/?attempt=Red%20and%20green%20are%20equally%20likely.

Math Ed Matters
by MatthewMaddux

Math Ed Matters by MatthewMaddux *is a column telling slightly bent, untold, true stories of mathematics teaching and learning.*

Reducing Inflammation

Egan J Chernoff
egan.chernoff@usask.ca

Not all math mistakes are created equal. Certain math mistakes are so infamous among math teachers that they get monikers, and for whatever reason, these monikers often sound like mathematical diseases. Take, for example, the mistake $log(a + b) = log(a) + log(b)$, which has sometimes been referred to as *logarrhea*. Technically, though, diarrhea—the inspiration for logarrhea—is a condition, not a disease. And for me, that's the problem with the mathematical diseases: Although the terms are catchy, they fall apart if you don't take them at face value. Actually, I have one other issue with the unofficial list of mathematical diseases, which includes maladies such as *squaranoia*, *logarrhea*, *sinusitis*, *funcitionitis*, *cancellitis*, *sumonia*, *rootobia*, *negativitis,* and *moveitis*: There's too much inflammation.

When it comes to mathematical diseases, the *-itis* suffix is used rather liberally. As mentioned, there's sinusitis, functionitis, and cancellitis, which, respectively, correspond to the mistakes $sin(a + b) = sin(a) + sin(b)$, $f(x + y) = f(x) + f(y)$, and $(a + b) / a = b$. Also, as I've found tossed around in online math teacher message boards and list serves, *-itis* has been used to denote *negativitis* (which corresponds to the inexplicable appearance or disappearance of negative signs) and even *moveitis* or *movitis* (corresponding to errors associated with negative exponents, as well as moving negative numbers from top to bottom or bottom to top). The burr in my bonnet is the accuracy of the medical suffix. To be clear, *-itis* is a perfectly fine suffix, but it denotes inflammation. In other words, while tacking the medical suffix *-itis* onto the end of words creates some great-sounding terms, they don't really describe the mathematical mistake at hand in the way that medical terms are devised to describe physical ailments. Naturally, the notion of renaming the *-itises* came to mind.

Sinusitis: $sin(a + b) = sin(a) + sin(b)$

Knowing the meaning of the *-itis* suffix, we see that the term sinusitis implies inflammation of the sin or the sinus. And, I suppose, it kind of works. One *could* interpret $sin(a + b)$ turning into two sines—that is, $sin(a) + sin(b)$—as a kind of inflammation. Upon closer inspection, the distribution of the log in logarrhea—that is, $log(a + b) = log(a) + log(b)$—is similar in nature to the distribution of the sin in sinusitis—that is, $sin(a + b) = sin(a) + sin(b)$. Given the similarity in the nature of the mistakes, one might propose, in the name of consistency, renaming logarrhea to *logitis* (i.e., an inflammation of logs). However, this simply doesn't sound as catchy as logarrhea. Conversely, one might argue that *sinurrhea* (i.e., the flow secretion or discharge of sinus) also kind of works, but then there's the potential issue of too much *-rrhea* on our hands. Fortunately, I think I've come up with a replacement.

> While tacking the medical suffix *-itis* onto the end of words creates some great-sounding terms, they don't really describe the mathematical mistake at hand.

I contend that *lateralparantheticsinucentesis* should replace sinsusitis as a more accurate descriptor of the mistake that is occurring. My proposed term is comprised of four key elements: *lateral* (to the left or the right side); *parenthetic* (referring to the brackets or parentheses); *sinu* (referring to sin or sinusoidal); and, *centesis* (puncturing and draining). Putting it all together, *lateralparantheticsinucentesis* refers, then, to how the sin (sinu) punctures the brackets and drains (centesis) to each of the terms in the parentheses (parenthetic), with the result on the other side (lateral) of the equals sign. Quite a mouthful, but in a land of many *-itises*, I contend that *lateralparantheticsinucentesis* is a still medically rooted but more accurate descriptor of the mistake in question.

Worthy of note: Lateralparantheticsinucentesis is, really, a general term. Initially, recognizing the two-way nature of the equals sign, I devised two terms for the mistake: *sinistroparantheticsinucentisis* and *dextroparantheticsinucentisis*, which referred to $sin(a + b) = sin(a) + sin(b)$ and $sin(a) + sin(b) = sin(a + b)$, respectively. Full disclosure: I am still unsure whether $sin(a + b) = sin(a) + sin(b)$ would be best described by *sinistroparantheticsinucentisis* or by *dextroparantheticsinucentisis*. Once established, though, the other term would then naturally refer to $sin(a) + sin(b) = sin(a + b)$. The issue is that I was unable to establish was what was being referred to as the left side. In other words, a left side puncturing and draining of the sin to each of the terms in the brackets ends up on the right side. Similarly, the end result of a right-side puncturing and draining of the sin to each term in the brackets is found on the left side of the equation. Given the former and the latter, I was unable to establish whether "left" referred to the side where things happened or the end result. Fortunately, the *lateral-* prefix, meaning left side or right side, allowed me to avoid this particular confusion and focus on the puncturing of brackets and draining of sin to each of the terms on the other side.

Functionitis: $f(x + y) = f(x) + f(y)$

Like lateralparantheticsinucentesis, renaming functionitis would fall prey to the left/right issue described above, which meant that the *sinistro-* or *dextro-* prefixes were ruled out from the beginning. Of course, the effort that was put into lateralparantheticsinucentesis could have easily been applied to functionitis, which would have resulted in *lateralparantheticfunctioncentesis*. However, no matter how I looked at the word, I had issue with the latter part: That is, when joined together, the combination of function and centesis

just didn't sound right to me. I was also looking to differentiate the word from lateralparantheticsinucentesis.

As mentioned earlier, functionitis, meaning and inflammation of the functions, may indeed be an adequate descriptor of what is taking place during the mathematical mistake: That is, an "inflamed" *f* results in two *f*s. However, and now being on a bit or a roll with my amateur use of medical terminology, I established a new word to describe what is taking place during this particular mathematical mistake: *endoparentheticfunctionostomy*. Breaking the term down into its constituent parts, we have: *endo* (denoting something as inside or within); *parenthetic* (relating to brackets or parentheses); *function* (verbatim use of the word function); and, *ostomy* (the creation of an artificial opening, or *stoma*). Put together, then, *endoparentheticfunctionostomy* refers to the *f*'s being distributed inside of the parentheses thanks to an artificial opening of $f(x + y)$.

> Renaming these particular diseases led me to wonder whether or not "disease" was an accurate or appropriate general descriptor for the errors they describe.

In comparison to functionitis, endoparentheticfunctionostomy has two things going for it. First, it reduces the number of *-itis*-based mathematical diseases. Second, breaking the word down helps one get a sense of the mathematical mistake that is taking place. However, renaming these particular diseases led me to wonder whether or not *disease* was an accurate or appropriate general descriptor for the errors they describe.

Mathematical: Diseases, Disorders, Conditions or Syndromes
Initially, a distinction between disease, disorder, condition, or syndrome might not seem like that big of a big deal. As time goes on, however, terminology often gets more and more nuanced. Case in point: What were once known as *venereal diseases* (VDs) were renamed *sexually transmitted diseases* (STDs), which, although often used interchangeably, have since been distinguished from *sexually transmitted infections* (STIs). Similarly, the days of declaring that students are riddled with mathematical diseases are probably over, too. The question remains, then, of how to properly describe scenarios such as the above in math class. Having parsed the notions of disease, disorder, syndrome, and condition (to the best of my ability), allow me to make a suggestion.

Just because a student hands you a paper with $sin(a + b) = sin(a) + sin(b)$—that is, shows you a symptom—does not mean they have the disease now known as *lateralparantheticsinucentesis,* formerly known as *sinusitis*. Rather, I suggest that what were previously known as mathematical diseases be referred to as *mathematical conditions*. In other words, lateralparantheticsinucentesis, endoparentheticfunctionostomy, and others are mathematical conditions: that is, abnormal states of mathematical health that interfere with usual mathematical activities (e.g., simplifying). However, should a student start to exhibit a collection of particular symptoms—for example, falling prey to $log(a + b) = log(a) + log(b)$ AND $sin(a + b) = sin(a) + sin(b)$ AND $f(x + y) = f(x) + f(y)$—then you might have a *syndrome* on your hands. In the instance I've presented, I suspect the syndrome that I will denote as *paracentesis* (the puncturing and draining of brackets, which is another way to describe distribution of the undistributable) might be at play.

Other than a silly exercise, there might not really be much more to renaming mathematical diseases. Take, for example, cancellitis, where the notion of inflammation makes the least amount of sense, especially when compared to other *-itises*. At the same time—ugh!—cancellitis is a such great word and helps draws attention to an egregious mistake (e.g., $16/64 = 1/4$ *because* the 6s cancel). I've tried, and I mean really tried to rename cancellitis, to no avail. In the end, though, silly or not, I've realized that my efforts to rename the mathematical diseases have helped me start to walk a mile in students' shoes. And, silly or not, any exercise that builds empathy for those making common mathematical mistakes can't be in vain.

> Silly or not, I've realized that my efforts to rename the mathematical diseases have helped me start to walk a mile in students' shoes.

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Egan J Chernoff (@MatthewMaddux) is a Professor of Mathematics Education, Department of Curriculum Studies and an Associate Member, Department of Educational Administration in the College of Education at the University of Saskatchewan (U of S). He is also an Associate Member, Department of Mathematics and Statistics in the College of Arts and Science at the U of S, and an Adjunct Professor, Faculty of Education at Simon Fraser University (SFU).

Call for Contributions

The Variable is looking for contributions from all members of the mathematics education community, including classroom teachers, consultants, and teacher educators. Consider sharing a favorite lesson, an essay, a book review, or any other work of interest to mathematics teachers in Saskatchewan. Articles may be written in English or French. If accepted for publication, your article will be shared with a wide audience of mathematics educators in Saskatchewan and beyond.

We also welcome student contributions in the form of artwork, stories, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom, and for teachers to recognize students' efforts during their journey of learning mathematics.

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Ilona & Nat,
Editors