



# ***The Variable***

Presented by the Saskatchewan Mathematics Teachers' Society

**Volume 6**

**Issue 1**

**2021**

**Steering a Robot  
to Engage in Number and  
Spatial Sense**

**My Favourite Lesson:  
How Many Steps?**

**Kindling the Fire:  
Why I Do What I Do**

**High-Impact Solutions  
for Struggling Mathematics Students**

**Renaming Mathematical Diseases**





Recipient of the National Council of Teachers of Mathematics  
Affiliate Publication Award (2019)



This work is licensed under the Creative Commons Attribution-NonCommercial 4.0 International License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc/4.0>



Editorial.....	3
My Favourite Lesson: How Many Steps? .....	4
<i>Jared Hamilton and Angela Fuller</i>	
Alternate Angles: Heroes and Villains .....	15
<i>Shawn Godin</i>	
Kindling the Fire: Why I Do What I Do .....	19
<i>Alessandra King</i>	
High-Impact Solutions for Struggling Mathematics Students.....	25
<i>Karine S. Ptak</i>	
Steering a Robot to Engage in Number and Spatial Sense .....	30
<i>Krista Francis, Stefan Rothschuh, Sarah Hamilton, &amp; Graham Diehl</i>	
Intersections .....	45
Tangents: Extracurricular Opportunities for Students .....	47
Math Ed Matters by Matthew Maddux	
Extremely Amateur Math Ed Morphology: Renaming Mathematical Diseases .....	51
<i>Egan J Chernoff</i>	
Call for Contributions .....	57

## Cover Image

Image source: Malík, T. (2020). Aerial photography of snow covered trees [Digital image]. Retrieved from <https://www.pexels.com/photo/aerial-photography-of-snow-covered-trees-3509971/>

## SMTS Executive (2020-2022)

### President

Chad Williams

[chad@beyondthealgorithm.ca](mailto:chad@beyondthealgorithm.ca)

### Vice-President

Ilona Vashchyshyn

[ilona@smts.ca](mailto:ilona@smts.ca)

### Treasurer

Cam McRae

[cam.mcrae@gmail.com](mailto:cam.mcrae@gmail.com)

### Secretary

Lana Steiner

[lanarae\\_6@yahoo.com](mailto:lanarae_6@yahoo.com)

### Directors

Anita Hamm

Robin Dubiel

Judson Trenholm

Heather Morin

Randa Marcus

### Past President

Michelle Naidu

### Liaisons

Naomi Loyola

(University of  
Saskatchewan Student  
Representative)

Egan Chernoff

(University of  
Saskatchewan)

[egan.chernoff@usask.ca](mailto:egan.chernoff@usask.ca)

Gale Russell

(University of Regina)

[gale@smts.ca](mailto:gale@smts.ca)

### NCTM Regional Services Representative

Chad Williams

[chad@beyondthealgorithm.ca](mailto:chad@beyondthealgorithm.ca)

### Members at Large

Kirsten Dyck

Stacey Harvey

Michael Graham

### *The Variable* Editors

Ilona Vashchyshyn

Nat Banting

## Notice to Readers

The views expressed in *The Variable* are those of the author(s), and not necessarily those of the Saskatchewan Mathematics Teachers' Society. In the publishing of any works herein, the editorial board has sought confirmation from all authors that personal quotes and examples have been included with the permission of those to whom they are attributable.

## Notice to Contributors

*The Variable* welcomes a variety of submissions for consideration from all members of the mathematics education community, including classroom teachers, consultants, teacher educators, researchers, and students of all ages. Submitted material is assessed for interest, presentation, and suitability for the target audience.

Please submit articles by email to [thevariable@smts.ca](mailto:thevariable@smts.ca) in Microsoft Word format. Authors should aim to limit submissions to 3000 words or less and include a photo and a short biographical statement of 75 words or less with their submission. Accepted contributions are subject to revision. Editors reserve the right to edit manuscripts for clarity, brevity, grammar, and style.





The co-editors of *The Variable* are inviting you to a scheduled Zoom meeting.

Topic: New Edition and a Quick Wellness Check

Join Zoom Meeting

<https://us02web.zoom.us/j/3141592653>

Meeting ID: 112 358 1321

One tap mobile

+1306 149162536496481100# (Regina)

+1306 13610152128364555# (Saskatoon)

Dial by your location

+1 306 248 1632 (Yorkton)

+1 306 392 7812 (Lloydminster)

+1 306 416 6425 (Prince Albert)

+1 306 525 1256 (Moose Jaw)

+1 306 636 2161 (Swift Current)

+1 306 749 3432 (North Battleford)

Meeting ID: 112 358 1321

Find your local number: <https://us02web.zoom.us/j/3141592653>

At the time of writing, we are still teaching in a historically disconnected time. Divisions across the province are operating under a variety of delivery models, and necessary health and safety measures jeopardize collaboration in our new, distanced reality. Of course, as math teachers, we may appreciate the fact that graphical literacy and probabilistic nuance are finally getting their day in the sun, and might even welcome the opportunity to talk statistics and data modelling over the dinner table. Still, the fact of the matter is that, after almost a year of the “new normal,” we’re all exhausted.

Here at *The Variable*, we are keenly aware that this edition may be viewed as yet *another* thing that claims to show you “how to teach” during these unprecedented times. However, if an editorial is intended to convince you to read on, know that that this is not our motive.

In our estimation, the greatest opportunity that the pandemic has offered is the chance to forefront care: care for our students, care for our communities, and care for ourselves. And although we may be physically disconnected, digital media and technologies have allowed us to continue to connect and care for each other in new and inspiring ways. We therefore humbly offer this periodical as another way in which our readers might connect, support each other, and continue to learn from one another, six (or more) feet apart.

Nat & Ilona  
Co-Editors



# My Favourite Lesson

## How Many Steps?

*Jared Hamilton and Angela Fuller*

**J**ared Hamilton: Recently, I had the opportunity to work as a District Numeracy and Technology Coach in my school division. My main objective in the role was to support classroom teachers in implementing mathematical instruction that utilized principles of Inclusion (Moore, 2016), Mindset Research (Dweck, 2007) and Thinking Classrooms (Liljedahl, 2020) while building numeracy skills and incorporating elements of technology (3D printing, coding, and robotics). My role allowed me to collaborate and co-teach with a variety of educators in the school district, including Angela Fuller. This opportunity allowed us to combine our shared experiences to build lessons that met the needs of her students. We designed the lessons with a focus on equity, providing multiple access points to learning, and opportunities to reflect on the learning process.

As our school district motto puts it, “Together we learn.” And indeed, working with Angela strengthened my belief that, whether it takes place digitally or in person, collaboration is an enriching experience for educators and students alike. Through the process of sharing our experiences, ideas, culture, and values, we were able design learning opportunities that better served the needs of our community of learners. Collaboration also allowed us to share the joy and satisfaction of building lessons we were proud of, and sharing the workload gave us additional time for ourselves to spend with our families and on personal hobbies.

Through the process of sharing, we were able design learning opportunities that better served the needs of our community of learners.

**Angela Fuller:** Jared Hamilton and I have been collaborating for a few years on incorporating innovative math pedagogy and technology into my classroom. He has introduced me to many concepts and strategies that I still use today. The incorporation of technology education (coding) and tools (3D printing and Spheros) has helped students remain engaged and has allowed them to develop skills and use tools they can use in their future careers.

Last year, during remote learning, I was starting my Grade 7 math unit on linear expressions and graphing coordinates and struggling coming up with authentic ways for my students to learn and understand this concept. Jared and I met via Google Hangouts



and devised a lesson that all students, regardless of their math background, could engage in: walk and record their steps. If they did not have a step counter, I provided them with one. It was an incredible success. Not only did the students enjoy the lesson, they were also able to make meaningful connections to the ideas of linear relations, linear equations, interpolation, extrapolation, and more.

### **Lesson Introduction**

This activity focuses on data collection and representation, setting the stage for an investigation of linear relationships. Students will go for a walk while recording how many steps they take during regular, timed intervals. They will represent this data using a graph, then use their graph to make various predictions involving the linear relationship.

All students can access this activity, as long as they have a safe space to travel for 15 minutes. Adult supervision is encouraged for the data collection portion of this activity, especially for younger students. While this lesson was developed to be used during remote learning, it can easily be used in a traditional face-to-face classroom setting. This is a wonderful opportunity to take learning outside the classroom, following the pedagogy and practice of Gillian Judson's book, *A Walking Curriculum: Evoking Wonder and Developing Sense of Place (K-12)*.

The task involves four stages:

- data collection;
- investigation of linear relations;
- comparing results with other students;
- personal reflection.

### **Curricular Competencies and Content**

#### *Curricular competencies*

- collaboration and problem solving;
- reasoning and modelling;
- understanding and solving;
- communicating and representing;
- connecting and reflecting; and
- incorporating elements of technology.

#### *Curricular content*

- building mathematical vocabulary;
- using rational numbers (N9.1);
- data collection and representation with graphs (P9.1);
- solving linear equations (P9.2).
- modelling discrete and continuous linear relationships (FPC10.8, FPC10.9);
- making predictions from data and graphs (interpolation and extrapolation) using a line of best fit (P9.1, FPC10.8, FPC10.9).

#### *Extensions*

There are many potential follow-up activities involving the data students collect during this activity, or that involve creating new data. These include:

- using technology (such as a spreadsheet or Desmos) to represent the linear relationship;
- representing the data as a distance vs time graph;
- examining the effect of changing speed on the graph and the equation (changing slope);
- developing piecewise functions to represent motion involving rest, constant velocity, and acceleration.

### Materials

- pedometer, Fitbit, or smartphone with a step-counting app
- pencil
- ruler
- 0.5-1 cm graph paper and ruled note paper *or* task sheet (p. 7)

### Data Collection

Students will go for a walk with their pedometer, pencil, and task sheet (see p. 7). They will section the walk into five 3-minute chunks (15 minutes total). After each chunk, students will record the number of steps they took during that chunk on the data collection chart provided.

### After the Walk

After the walk, students are tasked with analyzing the data they collected. Using their data, students will:

- create a graph representing their results;
- identify their data as discrete or continuous;
- draw a line of best fit on their graph and use it to make two predictions: one between the data points (interpolation) and another beyond the data points (extrapolation);
- use the line of best fit to model the data using a linear equation,  $y = mx + b$ , where  $m$  represents the slope of the line and  $b$  is the  $y$ -intercept;
- compare their equation with those of other students and provide reasons for any similarities and differences ;
- use their equations to validate the predictions they made using their line of best fit and explain any differences between the values;
- use their equations to make predictions;
- answer personal reflection questions.



## How Many Steps?



It's time to take our learning outside! Your task is to collect some data while you take a walk.

Before we start, however, we need to talk about a few words and what they mean.

- **constant** - a situation or value that does not change (stays the same)
- **t-chart** - a chart used to organize information and calculate values
- **linear** - arranged in a straight or nearly straight line
- **relation** - a connection between two or more things, where the value of one often depends on the value(s) of the other(s).

Make sure you have access to a pedometer, Fitbit or smartphone with a step-counting app.\* If you don't have one, let your teacher know right away so they can sign one out for you.



\*Free pedometer app for Android: **Pedometer-Step Counter**  
**Free**



\*Free pedometer app for iOS (Apple): **Pedometer++**

### Task

Let's collect some data! Go for a walk, taking along your step counter, this task sheet, and a pencil.

You will chunk your walk into five 3-minute chunks (so the walk will be 15 minutes total). After each chunk, you will record the total number of steps you have taken. For example, if after 3 minutes you take 436 steps, record that number in the third row of the second column [*1 (3 minutes)*]. You will then continue to walk for another 3 minutes and record your new total number of steps (e.g. 855), repeating this until you have walked for 15 minutes in total.

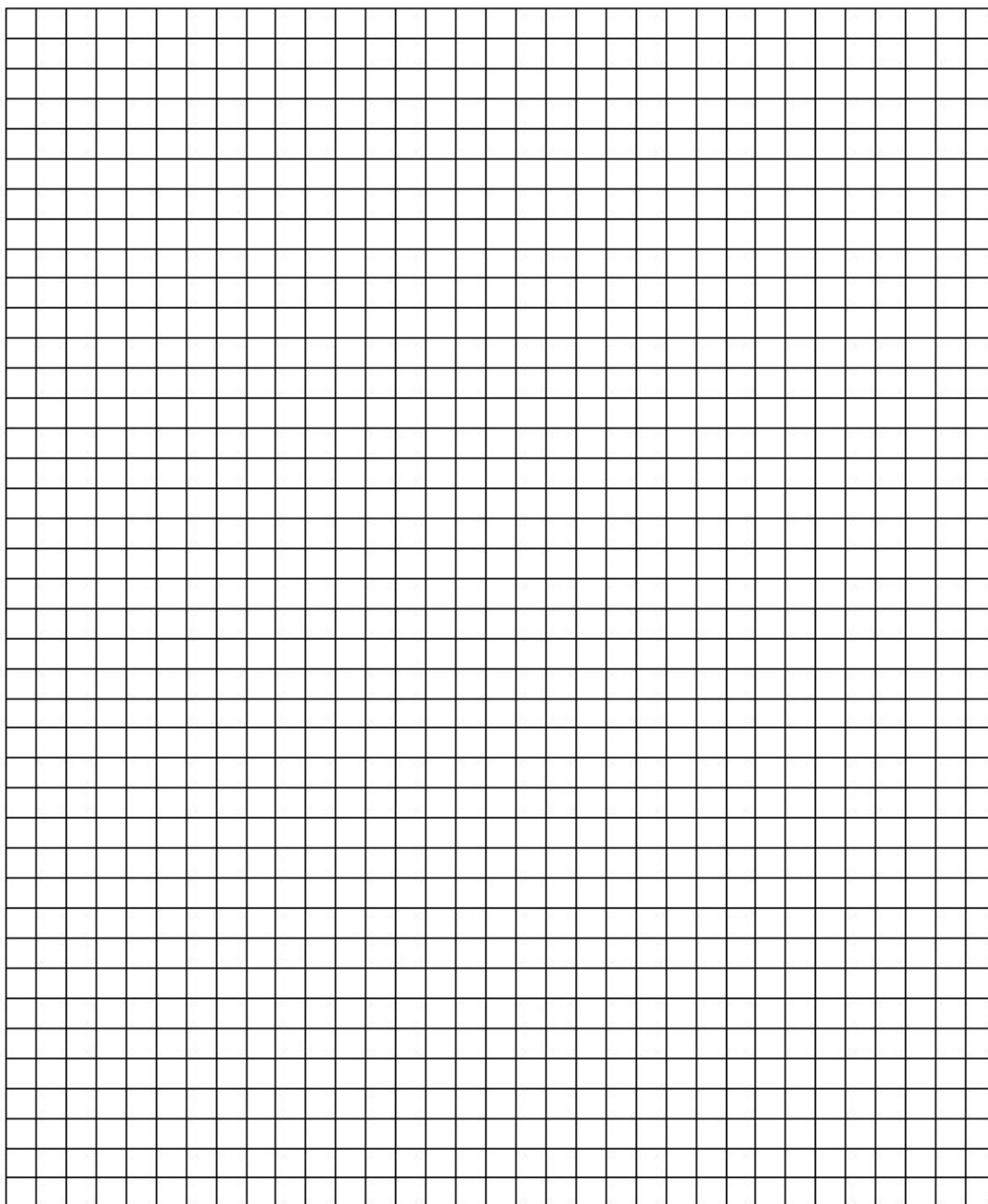
Try your best to maintain **a constant** (the same) **speed** during the entire walk. This means no speeding up, running, slowing down, or stopping.

Chunks Walked (Minutes) <b>X</b>	Total Steps <b>Y</b>
<b>0</b>	0
<b>1</b> (3 minutes)	
<b>2</b> (6 minutes)	
<b>3</b> (9 minutes)	
<b>4</b> (12 minutes)	
<b>5</b> (15 minutes)	

After you have filled out the T-chart, use the grid paper on the next page to sketch a graph using the data you collected. Remember to:

- label the axes;
- divide axes into equal intervals;
- title your graph.





### After the Walk Task

1. Is your data discrete or continuous? Provide reasons for your answer.
  
2. Draw a line of best fit on your graph. Then, use it to answer the following questions:
  - a. How many minutes would it take you to reach 1000 steps? (*When we predict values within the range of our data, we call it **interpolation**.*)
  
  - b. How many steps would you take if you walk for 18 minutes? (*When we predict values outside the range of our data, we call it **extrapolation**.*)
  
3. Recall that a linear equation can be written in the form  $y = mx + b$ , where  $m$  represents the slope of the line and  $b$  is the y-intercept.
  - a. Write a linear equation,  $y = mx + b$ , for your line of best fit.
  
  - b. Compare your equation with the equations of 1-2 other students. What differences and similarities do you notice? Propose reasons for these similarities and differences.

- c. Use your equation to test the predictions you made in question 2. What might account for the differences between the predicted values, if there are any?
  
- 4. Use your equation to predict:
  - a. How many steps would you take in 2 hours?
  
  
  
  
  
  
  
  
  
  - b. How many minutes would it take you to reach 10 000 steps?
  
  
  
  
  
  
  
  
  
- 5. Are the predictions you made in question 4 realistic? Explain why or why not.
  
  
  
  
  
  
  
  
  
- 6. Either on a separate piece of paper or on a digital document, respond to the following personal reflection questions:
  - a. What part of this project am I most proud of? Which part did I enjoy the most?
  - b. Do I feel pride and accomplishment for my work and contributions to this project?
  - c. How did my understanding of linear relations change during this project?
  - d. How might these new understandings be useful to me now, or in the future?
  - e. What would I do differently if I could do this project again? What would I do the same?
  - f. What would I like to investigate or learn about next?



**Assessment: One-Point Rubric****Name:** \_\_\_\_\_

Emerging/Developing	Criteria/Competency	Proficient
	Organize and represent data	
	Model and explain mathematical thinking in contextualized experiences	
	Communicate data collection with other students and identify similarities and differences between student models	
	Reflect on the activity and make personal connections to mathematical competencies and content	

## Student Work Sample

Data Collection

Chunks walked (minutes)	Total Steps
X	Y
0 (0 min)	109
1 (3 min)	487
2 (6 min)	758
3 (9 min)	297
4 (12 min)	1633
5 (15 min)	1960

Investigation

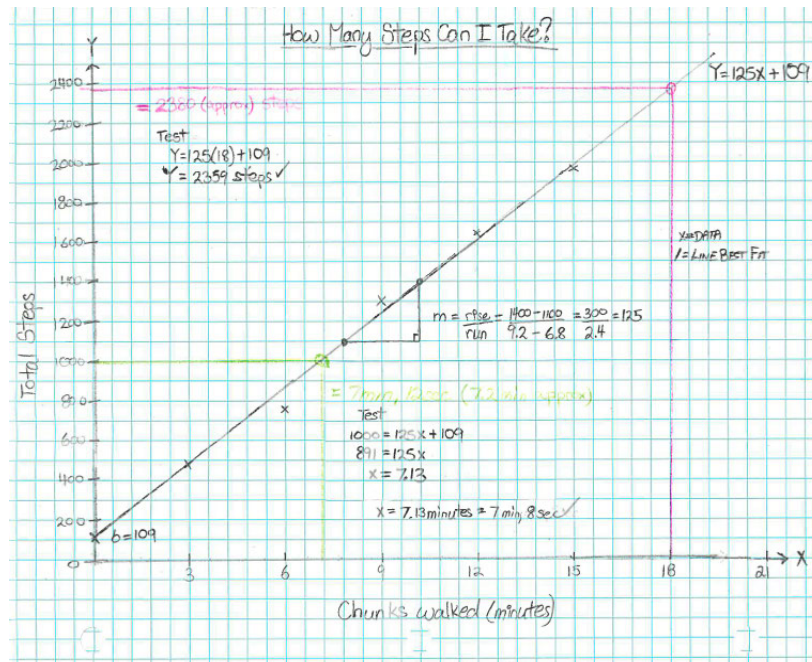
- The number of steps is discrete, since we count steps in whole numbers. Time is continuous since we can use any decimal number to measure time.
- see graph for line of best fit, interpolation, extrapolation, model of equation  $Y = mx + b$ , and tests.
- My predictions using the line of best fit and equation agree within the margin of error. This helps confirm the accuracy of my representation of the data.
- $Y = 125X + 109$   
 $X = 2 \text{ hours} = 120 \text{ minutes}$   
 $Y = 125(120) + 109$   
 $Y = 15,109 \text{ steps}$
- $Y = 125X + 109$   
 $Y = 10,000 \text{ steps}$   
 $10,000 = 125X + 109$   
 $X = 79.13 \text{ minutes} = 79 \text{ minutes, } 8 \text{ sec}$

- My equation  $Y = 125X + 109$  but my friend got  $Y = 134X$ . I feel they have a larger slope because they took more steps than I did for a 3 minute interval. I think this is because I am taller, so they would need more steps to match my leg stride.

My y-intercept is 109 and theirs is 0. This is because I started my data collection after taking 109 steps and then started collecting data at 0 steps.

### Reflection Questions

- My favourite part of this project was the data collection as I love walking and being outdoors.
- I received a lot of comments from my parents, teacher and my friends in math class telling me my work was neat and organized, and that my calculations were correct.
- I learned how to make a line of best fit, to make a linear equation with slope and y-intercept, and to solve for missing numbers.
- This activity was great because I love walking and it helped me think about how many steps I'm walking through the day without looking at my phone.
- I want to do this more and average my data. I also want to try it while running. Next time I won't use 5 squares for 3 minutes since it was hard to stop count by 36 seconds.

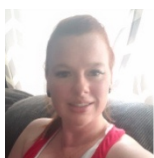


## References

- Dweck, C. (2007). *Mindset: The new psychology of success*. New York, NY: Ballantine Books.
- Liljedahl, P. (2021). *Building thinking classrooms in mathematics, Grades K-12: 14 practices for enhancing learning*. Thousand Oaks, CA: Corwin Press.
- Moore, S. (2016). *One without the other: Stories of unity through diversity and inclusion*. Winnipeg, MB: Portage & Main Press.



Jared Hamilton is a mathematics specialist in Northern British Columbia. He is currently working as a Grade 5 teacher at Anne Roberts Young Elementary School. Jared uses various educational models, such as Universal Design for Learning (UDL), Tribes Learning Communities, Arts Integration, The Walking Curriculum, Thinking Classrooms, and Gradeless Assessment. He has worked all over Western Canada and the United Kingdom and has led several digital professional development sessions worldwide. Currently, he is working towards his PhD in Education. He enjoys spending time with his family, playing video games, watching the Edmonton Oilers, walking, drinking coffee, and eating donuts.



Angela Fuller began her teaching career overseas in Thailand. Currently, she is a Grade 7 teacher at Bert Bowes Middle School in Northern British Columbia, where she has worked since 2007. Angela strives to use technology in her classroom in creative ways to enhance student engagement and to make learning more authentic and meaningful. Angela is also currently working towards her Masters in Education. She enjoys traveling, hiking, reading, and spending time with her family.

---

## Contribute to this column!

*The Variable* exists to amplify the work of Saskatchewan teachers and to facilitate the exchange of ideas in our community of educators. We invite you to share a favorite lesson that you have created or adapted for your students that other teachers might adapt for their own classroom. In addition to the lesson or task description, we suggest including the following:

- Curriculum connections
- Student action (strategies, misconceptions, examples of student work, etc.)
- Wrap-up, next steps

To submit your favourite lesson, please contact us at [thevariable@smts.ca](mailto:thevariable@smts.ca). We look forward to hearing from you!





*Alternate Angles is a column on problems from multiple perspectives: various methods that could be used to solve them, insights we get from their solution, the new paths that they can lead us to once they have been solved, and how they can be used in the classroom.*



## Heroes and Villains

Shawn Godin

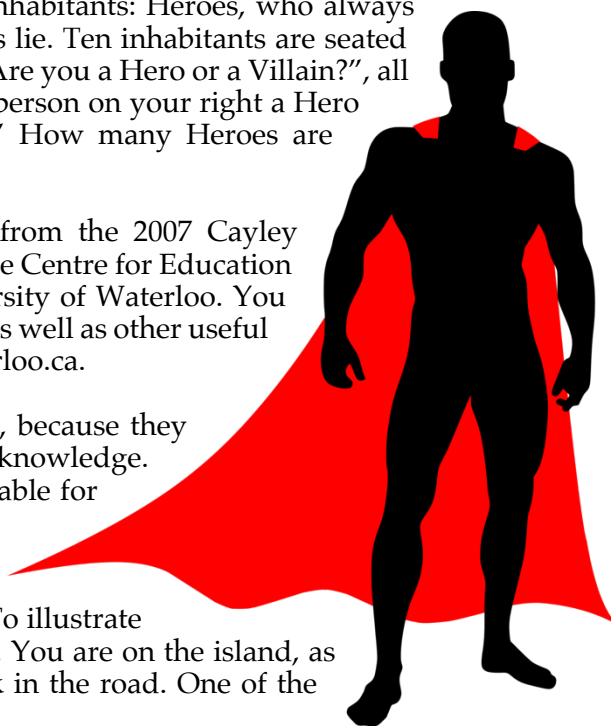
Welcome back, problem solvers! I hope 2021 will be a wonderful year for all. Last issue, I left you with the following problem:

On an island, there are two types of inhabitants: Heroes, who always tell the truth, and Villains, who always lie. Ten inhabitants are seated around a table. When they are asked “Are you a Hero or a Villain?”, all ten reply “Hero.” When asked “Is the person on your right a Hero or a Villain?”, all ten reply “Villain.” How many Heroes are present?

This problem was inspired by question 17 from the 2007 Cayley Contest for Grade 10 students, organized by the Centre for Education in Mathematics and Computing at the University of Waterloo. You can check out their collection of past contests, as well as other useful resources, on their website: [www.cemc.uwaterloo.ca](http://www.cemc.uwaterloo.ca).

Logic puzzles like this are fun and accessible, because they don’t need specific formulas or background knowledge. As such, problems like the one above are suitable for students across the grade levels.

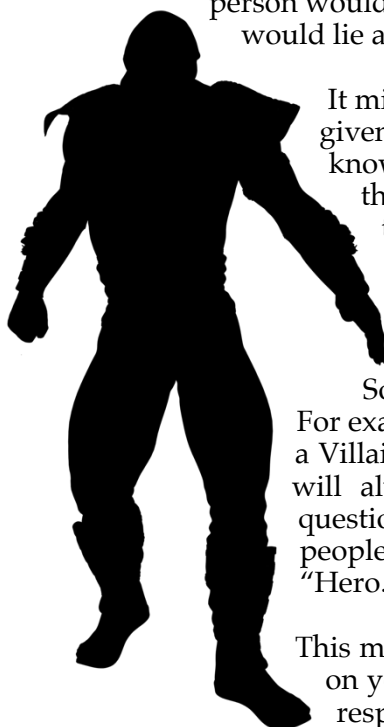
With problems of this type, it is sometimes useful to consider several possible scenarios. To illustrate the process, let’s consider a different problem. You are on the island, as in the given problem, and you come to a fork in the road. One of the



roads leads to the village of Heroes and one to the village of Villains. Standing at the fork is one of the inhabitants of the island, but you are unsure if they are a Hero or Villain. You wish to go to the village of Heroes. Is there a question you can ask the person that will allow you to deduce which way to go? Take a couple of minutes to see if you can come up with a solution.

If you ask “Which path leads to the village of Heroes?” or “Is the path on the right the correct way to the village of Heroes?”, you will get different answers from Heroes and Villains. Therefore, since you don’t know if you are talking to a Hero or Villain, you cannot trust the answer you will get. We must use the fact that a Villain will lie to our advantage.

Suppose that instead, we ask “Which way would someone from the other village say is the way to the village of Heroes?” If the person is a Hero, they know a person from the other village would lie and point towards the Village of Villains, so they would answer truthfully and point towards the village of Villains. If the person is a Villain, they know the other person would tell the truth and point towards the village of Heroes, so they would lie and point towards the village of Villains.



It might seem that we have failed, until we see that they have both given the same answer! Even though that answer is not correct, we know that whoever it is will point the wrong way, so we can take the opposite road. In other words, there are questions we can ask that will yield the same answer from both Heroes and Villains. Can you devise a question that, when posed, will result in both Heroes and Villains pointing towards the village of Heroes? Some answers will be provided later in the column.

Sometimes asking a question is useful, and sometimes it is not. For example, is there any use to asking someone if they are a Hero or a Villain? A Hero will always answer truthfully as “Hero”; a Villain will always answer falsely, also as “Hero.” Therefore, the first question from the given problem is useless. No matter how many people are gathered, when asked this question they will all answer “Hero.”

This means that the only useful question of the two is “Is the person on your right a Hero or a Villain?” Let’s look at all of the possible responses a villager could give:

Type of Villager	To their Right	Response
Hero	Hero	Hero
<i>Hero</i>	<i>Villain</i>	<i>Villain</i>
<i>Villain</i>	<i>Hero</i>	<i>Villain</i>
Villain	Villain	Hero

From the above table, the only way a person would respond “Villain” is if the person to their right is from the opposite village (hmm, does this make you think of adding or multiplying integers?). Thus, the only way all ten could respond “Villain” is if the Heroes and Villains alternate around the table, with 5 Heroes and 5 Villains.

The original version of this problem had four people sitting around a table, with the same responses given by the villagers. Hopefully, you can convince yourself that if we had any even number of inhabitants seated around the table, the solution would be the same (an equal number of Heroes and Villains alternating around the table). On the other hand, if we started with 11 people, or any odd number of people, around our table, it would be impossible to come up with a configuration so that everyone answers “Villain” to the second question (why?).

Some students may benefit from using manipulatives to investigate this problem. Since there are only two types of villagers, we could use different coloured tiles or tiles of different shapes to represent our Heroes and Villains. Another option would be to use coins, so one manipulative could represent either a Hero or a Villain, depending on which side is facing up. This way, using heads for Heroes and tails for Villains, we could check our answer to the original problem as shown in Figure 1 below. On the left, we have ten coins in a circle representing our conjectured set-up of alternating Heroes and Villains, and on the right, we have their responses. We could do this with 20 coins, or 10 if we are careful. Using just the original 10 coins, we could flip, or leave, each coin successively to represent the response. You must be careful when we reach the last coin, however, as you may have flipped the one to its right at the start of the process. Therefore, you need to keep track of the first coin’s original state. It could also be fun to explore what happens if we continue the process around the circle, always making our decision based on what is to the right *at this moment*. Will the pattern continue to change, or will we eventually settle down into some steady state? I leave this for you to explore.

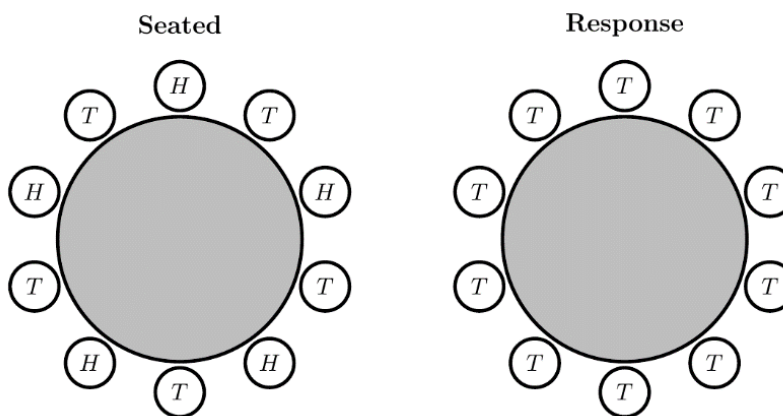


Figure 1: Checking our answer with coins.

What now? It is always a good idea when you have solved a problem to think about it a little more. Could we have solved it another way? Does the idea generalize, as in our case from four people to ten? What else can we learn from this situation? Returning to our fork-in-the-road situation from earlier, another way to get the answer we want is to ask the unknown villager “What would a person from the other village say is the way to the village of the Villains?” Using the same logic as earlier, we would see that both Heroes and Villains

would answer falsely and point towards the village of the Heroes. In asking both questions, we were using the fact that the villager's response would always be a lie. Is that our only option?

Consider the question "Which is the road to *your* village?" A Hero would answer truthfully and point towards the village of the Heroes. A Villain would lie, also pointing towards the village of the Heroes. So, we see we can get the same result using different mechanisms. Yet another avenue that you could explore for the original problem is: Can you have a different configuration of people around the table, ask different questions, yet still get the same responses given in the original problem? I leave this problem, too, for you to explore.

And now for your homework:

Alice places a coin heads up on a table, then turns off the light and leaves the room. Bill enters the room with two coins, puts them onto the table, then leaves. Carl enters the dark room and removes a coin at random. Alice re-enters the room, turns on the light, and notices that both coins are heads up. What is the probability that the coin Carl removed was also heads up?

Until next time, stay healthy, and happy problem solving!



*Shawn Godin teaches at Cairine Wilson Secondary School in Orleans, Ontario. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: leading workshops, writing articles, working on local projects and helping create mathematics contests.*

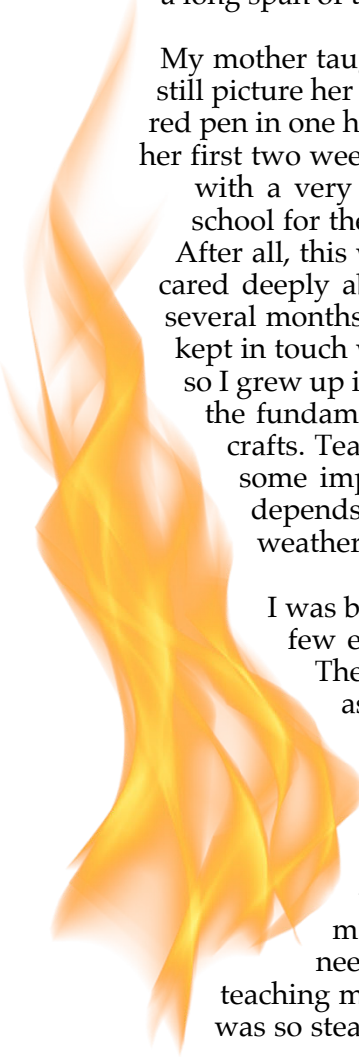


## Kindling the Fire: Why I Do What I Do <sup>1</sup>

Alessandra King

The more I reflect about my journey as a teacher, the more I realize how many people and events have inspired, guided, and shaped my experience—a finely woven web of connections of which I can start to unravel just a few strands, and just barely. Yet all of these converge to explain where I am now and why I do what I do: teach mathematics in an all-girls school.

I am a third-generation teacher. My maternal grandmother was a teacher in a one-room school that served several villages in a hilly region of the central part of Italy. Simply called la maestra, she was "the teacher" for quite a number of children and families over a long span of time.



My mother taught mathematics and science in public middle school, and I can still picture her in the early morning hours checking her students' papers with a red pen in one hand and a cup of steaming espresso in the other. Year after year, her first two weeks of summer always involved spending huge amounts of time with a very large sheet of paper, constructing the schedule for the entire school for the following academic year. She was not paid for this extra duty: After all, this was just what a math teacher would do for her school. And she cared deeply about her students—once we even took one into our home for several months, until his grandparents could step up and take him in, and we kept in touch with him until he graduated, found a job, and got married. And so I grew up in the "shop" of master craftswomen and could observe firsthand the fundamental tools of the most important, most uniquely human of all crafts. Teaching, I found out, is a little bit like wine making, in that it has some important methods and best practices but no hard, set recipe; it depends on so many imponderables, just as wine making depends on the weather, the crop, the soil, and the water.

I was blessed with a succession of very good teachers, and more than a few exceptional ones, each extraordinary in his or her unique way. There was Suor Pierina, who made skip counting so much fun and asked us to write mathematics answers in sentences and keep our work neat, structured, and looking good, like a work of art. Mr. Branchesi, a boisterous professional painter and children's enthusiast, made each day of school a joyful intellectual adventure and never required us to memorize the multiplication tables. To this day, it is a mystery to me how and when I learned them—perhaps by pattern recognition, mental math, and frequent use? All I can say is that I knew them when I needed them. Mr. Castagnari was a wiry, reserved man for whom teaching math was serious business and whose faith in our ability to learn was so steady and deep-seated that he boosted our confidence and pushed

---

<sup>1</sup> Reprinted with permission from "Kindling the Fire: Why I Do What I Do," *Mathematics Teacher*, 112(7), copyright 2019 by the National Council of the Teachers of Mathematics (NCTM). All rights reserved.

us to excel in those critical middle school years. Mrs. Galdenzi, a southerner with a strong Neapolitan accent, taught math in a Milanese school. Her content knowledge, huge heart, and impressive teaching abilities won everybody over, starting with her students. Ms. Santagata, our high school math and physics teacher, ended each and every day of school covered in chalk dust from head to toe. Her precise language; high expectations of abstract reasoning; and profound, multifaceted, open-ended questions astound me to this day. Each of these individuals, and the others I do not have time to mention here, embodied and modeled one or more aspects of great math teaching. They gave the gift of inspiring "joy and wonder" (NCTM, 2018) with passion for their subject, dedication to their profession and their students, creativity and playfulness, deep and thoughtful content knowledge, ability to tell a story, and pedagogical suppleness.

As far as I knew,  
success in  
mathematics  
depended on the  
individual's  
personal talents,  
not his or her  
gender.

Boy, was I wrong.

Teaching and learning—and specifically mathematics teaching and learning—is "part of a complex system of . . . traditions and societal expectations" (NCTM, 2018). Not until I moved out of my sheltered experience and went to live and teach math and physics overseas, in the Philippines first, and then in the United States, did I discover that girls are not supposed to be that good at mathematics. This realization came as a surprise, as math had always been my favorite subject, and consequently I had always done well in it. In my family, academic expectations were the same for the daughters as for the son. I had not noticed any difference in my school years. When I was a physics major in my hometown college, many (although not most) of the students in my mathematics and physics classes were women. My thesis adviser was also a woman; I had asked her to advise me because she was the only professor who studied extragalactic active nuclei, not because she was a woman. And I am sure she accepted me because I was a fair bet for a paper. As far as I knew, success in mathematics depended on the individual's personal talents, not his or her gender.

Boy, was I wrong. The news was finally delivered in full by the new Barbie™ doll who declared in no uncertain terms, "I hate math." At that time I had two daughters, a toddler and a baby, and decided on the spot to ban Barbie from our house. The embargo caused many a conversation and some arguments—with my daughters, their young friends, and their parents—and more than a few awkward moments. And it was ferociously applied for many years, until the high cost of its enforcement in time and effort made it, like all embargos, untenable.

That was not the only way we fought what later became known as the "stereotype threat" (Steele, 2011). We built Lego® structures, snow forts, and marble sand tracks; played Blokus® and Mastermind®; looked for patterns in the maps of the cities we visited; rode the simulator and touched the moon rock at the National Air and Space Museum; and raised tadpoles in a bucket. We sorted, counted, and classified; transformed recipes to feed our growing family; admired the symmetry and proportions of paintings, monuments, and architectural landmarks; saw interesting buildings as complex shapes; recognized patterns and created new ones as we learned to quilt; measured, calculated and converted three-dimensional (3-D) objects in 2-D when we tried sewing. And with the inspiration and guidance of some phenomenal mathematics and science teachers (Lappin-Scott, 2017), my daughters created budgets with their future first salary, played the stock market, designed and tested rollercoasters, assembled a Foucault pendulum (a sand-filled soda bottle hanging from the ceiling of our garage), and built their dream school with plywood and

Styrofoam® in their geometry class. No wonder one of them, in her application to a magnet high school, wrote how much fun she had in third grade when she "first learned how to add, subtract, multiply, and divide fractions." I cannot wait to play some math with my soon-to-be-born granddaughter!

The Barbie doll incident was just the tip of the iceberg. The environment in which our girls grow and learn mathematics can affect the way they see themselves (Boaler 2015) as students of mathematics (or of any hard science, such as physics, often called the gateway for STEM careers). Although we have made progress, the same condescending attitude toward women in math, science, and technology resurfaces every now and then, in unexpected places or occasions. It appeared when my daughter, one of only two girls in her fully enrolled Advanced Placement Physics class, heard her class addressed as "Gentlemen." Or when, as a mathematics major in college, she was asked again and again if she intended to go into math education. That question was never asked of her male classmates. It reemerges in the far too common habit of steering our girls to dad for help

Although we have made progress, the same condescending attitude toward women in math, science, and technology resurfaces every now and then, in unexpected places.

with the math homework. It materializes in many a conversation when too many women publicly state, "I was always bad at math," (Eccles & Jacobs, 1986) whereas an equivalent statement about reading would hardly be socially acceptable. It shows in "the soft bigotry of low expectations" (Bush, 2000) with which sometimes we as educators are tempted to lower the standards—even unintentionally—to make our girls "feel good" about their mathematical achievement.

Girls do not need extra help to learn and enjoy mathematics. They need only a level playing field and a culture that, instead of subtly undermining their confidence, bolsters their efforts. We have made and are

continuing to make progress, slowly but surely. The movie *Hidden Figures* (Shetterly, 2016) sends a very different message and presents inspiring female role models engaged in great mathematics and exciting work. In the last few years some children's books on women mathematicians, such as Sophie Germain, Ada Lovelace, and Hypatia, have been published, and there is an effort to make more gender-neutral STEM toys. And the world also celebrates mathematics in the work of Maryam Mirzakhani, the doodles of Vi Hart, the books of Eugenia Cheng, and the talks of Hannah Fry.

As for teaching mathematics, I take my cue from a statement attributed to the poet de Saint-Exupéry:

If you want to build a ship, don't drum up people to collect wood and don't assign them tasks...; rather, teach them to long for the endless immensity of the sea.

To me this means seizing all opportunities to engage my girls with the beauty, excitement, and "unreasonable effectiveness of mathematics" (Wigner, 1960). And so we collaborate on interdisciplinary projects (King, 2014c, d, e, f, h; 2015a, d; 2016; 2017d; 2018a), challenging problem-solving tasks, and hands-on activities. We explore unusual, intriguing topics like fractals and taxicab geometry (King, 2014a, g). We work together on Fermi questions (King, 2014b, 2015c); we celebrate numbers any time possible; we play Sudoku, 2048, and FlowFree (King, 2017c). We contemplate the Pythagorean theorem and its many extensions and connections, sharing its story all the way to Fermat's Last theorem. We develop the

quadratic formula the historical way, as an extension of completing the square and program our calculators to do it for us. We place the concepts we study within the rich tapestry of the history of mathematics, which helps us realize that such concepts were developed through years and sometimes centuries of hard work, partial success, sacrifices, trials, excitement, adversities, and delight; and at the same time appreciate that we are taking part in one of the most creative human endeavors (King, 2013; 2017a, b). We use Instagram to highlight the mathematics around us (King, 2018c); we blog about contemporary women mathematicians and their work on International Women's Day (Algebra 2 Class, 2018); and we write about the innumerable applications of math in our daily life (Algebra Classes, 2018). We connect concepts, ideas, theories, and models; solving problems in a variety of ways and rejoicing in the diversity of our thinking processes; we organize the Problem of the Cycle (King, 2015b) and the Math Squad; and we go on math trails (King, 2018b).

I expect my girls—whether my daughters or my students—to tackle challenging problems and be creative problem solvers. I expect them to face engineering projects, solve puzzles, and read articles and magazines related to math. I expect them to compete in math meets. I expect them to talk about math, ask questions, and take intellectual risks. I expect them to work hard, persevere, and learn from the mistakes they may make. I expect them to enjoy math, to be successful in math class, and to carry their quantitative thinking for life. I expect that because I. Know. They. Can.

## References

- Algebra Classes, Holton Arms. (2018). Mathematics applications. *LucidPress*.  
[www.lucidpress.com/documents/edit/ed3942b2-e90f-47bb-8a45-76c14a8f57f5](http://www.lucidpress.com/documents/edit/ed3942b2-e90f-47bb-8a45-76c14a8f57f5)
- Algebra 2 Class, Holton-Arms. (2018, March). Celebrating women in mathematics. *Celebrating Women in Mathematics*. [womeninmath.edublogs.org/](http://womeninmath.edublogs.org/)
- Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco, CA: Jossey-Bass.
- Bush, G. W. (2000, July 10). George W. Bush Speech to the NAACP. *The Washington Post*.  
[www.washingtonpost.com/wp-srv/onpolitics/elections/bushtext071000.htm](http://www.washingtonpost.com/wp-srv/onpolitics/elections/bushtext071000.htm)
- Eccles, J. S., & Jacobs, J. E. (1986). Social forces shape math attitudes and performance. *Signs: Journal of Women in Culture and Society*, 11(2), 367–80.
- King, A. (2013). Mathematical Explorations: Finding pi with archimedes' exhaustion method. *Mathematics Teaching in the Middle School*, 19(2), 116–23.
- King, A. (2014a, December 8). Enjoying math in the middle grades. *MTMS Blog: Blogarithm*.  
<https://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Enjoying-Math-in-the-Middle-Grades/>
- King, A. (2014b, November 12). Fermi questions—A STEM activity for middle school. *Journal of The Association of Independent Maryland and DC Schools*.  
<https://aimsmddc.wordpress.com/2014/11/12/fermi-questions-a-stem-activityfor-middle-school/>

- King, A. (2014c). Math for real: Space and time. *Mathematics Teaching in the Middle School*, 20(2), 128–30.
- King, A. (2014d). Math for real: The top speed for humans. *Mathematics Teaching in the Middle School*, 20(1), 64–66.
- King, A. (2014e). Mathematical Explorations: Freshwater Scarcity: A Proportional Representation. *Mathematics Teaching in the Middle School*, 20(3), 178–84.
- King, A. (2014f). The perpendicular bisector: A closer look. *The Australian Mathematics Teacher*, 70(3), 36–39.
- King, A. (2014g, December 12). Teamwork and diversity. *MTMS Blog: Blogarithm*. <https://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Teamwork-and-Diversity/>
- King, A. (2014h). Quick reads: MathMania: A middle school puzzle book. *Mathematics Teaching in the Middle School*, 19(7), 438–42.
- King, A. (2015a). Mathematical explorations: Leafing through irregular shapes. *Mathematics Teaching in the Middle School*, 21(1), 53–60.
- King, A. (2015b). Quick reads: A historical twist on problems of the week." *Mathematics Teaching in the Middle School*, 21(3), 176–79.
- King, A. (2015c, January 5). Making sense with Fermi questions. *MTMS Blog: Blogarithm* <https://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Making-Sense-with-Fermi-Problems/>
- King, A. (2015d). Math for real: Ramp it up. *Mathematics Teaching in the Middle School*, 21(5), 320–22.
- King, A. (2016). Movies and math help make global connections. *AMLE Magazine*, November.
- King, A. (2017a). A brief history of numerical systems—Alessandra King. *Lessons Worth Sharing: TED-Ed Animations*, January. [ed.ted.com/lessons/a-brief-history-of-numerical-systems-alessandra-king](http://ed.ted.com/lessons/a-brief-history-of-numerical-systems-alessandra-king)
- King, A. (2017b). A brief history of banned numbers— Alessandra King. *Lessons Worth Sharing: TED-Ed Animations*, September. [ed.ted.com/lessons/a-brief-history-of-banned-numbers-alessandra-king](http://ed.ted.com/lessons/a-brief-history-of-banned-numbers-alessandra-king)
- King, A. (2017c, October 13). Building a sense of community—with math. *Edutopia*. <https://www.edutopia.org/article/building-sense-community-math>
- King, A. (2017d). Understanding the election process through math in middle school. *Association for Middle Level Education*. [www.amle.org/BrowsebyTopic/WhatsNew/WNDet/TabId/270/ArtMID/888/ArticleID/706/Understanding-the-Election-Process-through-Math-in-Middle-School.aspx](http://www.amle.org/BrowsebyTopic/WhatsNew/WNDet/TabId/270/ArtMID/888/ArticleID/706/Understanding-the-Election-Process-through-Math-in-Middle-School.aspx)



- King, A. (2018a). Integrating global education in the middle school math classroom. *AMLE Magazine*, 6(1), 18–20.
- King, A. (2018b, January 9). Finding the beauty of math outside of class. *Edutopia*. <https://www.edutopia.org/article/finding-beauty-math-outside-class>
- King, A. (2018c). Instagram for maths. *Mathematics in School*, 47(4), 25–29.
- Lappin-Scott, H. (2017, February 3). To get more women in STEM little girls need better role models. *The Conversation*. [theconversation.com/to-get-more-women-in-stem-little-girls-need-betterrole-models-70763](http://theconversation.com/to-get-more-women-in-stem-little-girls-need-betterrole-models-70763)
- Lee Shetterly, M. (2016). *Hidden figures: The American dream and the untold story of the black women mathematicians who helped win the space race* [DVD]. Directed by Theodore Melfi.
- National Council of Teachers of Mathematics (NCTM). (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: NCTM.
- Steele, C. (2011). *Whistling Vivaldi: And other clues to how stereotypes affect us*. New York: W.W. Norton and Company.
- Szyborska, W., & Cavanagh, C. (2007). *Nonrequired reading*. New York: Harcourt.
- Wigner, E. P. (1960). The unreasonable effectiveness of mathematics in the natural sciences. Richard Courant lecture in mathematical sciences delivered at New York University, May 11, 1959. *Communications on Pure and Applied Mathematics*, 13, 1–14.



Alessandra King, [Alessandra.King@holton-arms.edu](mailto:Alessandra.King@holton-arms.edu), explores mathematics with her students at the Holton-Arms School in Bethesda, Maryland. An astrophysicist by training, she has taught physics and mathematics at the middle and high school levels on three continents—in Milano (Italy), Manila (Philippines), and the greater Washington, D.C. area. She particularly enjoys creative problem solving and reading about the history of mathematics.

## High-Impact Solutions for Struggling Mathematics Students<sup>2</sup>

Karine S. Ptak

During the 2013–2014 school year, the Maryland State High School Assessment in Algebra with Data Analysis (HSA) was a requirement for high school graduation in Maryland. At Frederick High School, only 27 percent of students who took the test that year passed. To respond to such an alarming failure rate, three teachers and I developed and piloted a new approach to teaching the curriculum, along with other skills necessary for improving students' success—not only on the HSA but also in the class, their other classes, and in life.

The Frederick County Public School System has two levels of classes: (1) The merit level is for students who struggle with the content and need more time and resources to learn; (2) the honors level is for students who quickly absorb the content and do well on assessments. At Frederick High School, the day is organized into four 80-minute blocks, plus one 40-minute block in the middle of the day that is used as an intervention/enrichment period. Our pilot program had four sections of twenty preselected students each, with a mix of ability, grade level, and language comprehension (27 percent of the pilot students were classified as English language learners [ELLs]). All our students had failed the HSA at least once; several had done so multiple times (it was still a requirement for graduation at that time), and all students were considered likely to fail either the assessment again or the class itself.

### Breaking the Instructional Routine

These sobering facts led us to suspect that the more traditional approaches to teaching mathematics were not reaching these students. Therefore, while keeping the Common Core mathematics standards and practices in mind (CCSSI, 2010) and concerned with students' success with the curriculum and standardized assessments, we carefully examined traditional teaching approaches, and we deliberately chose to redefine our instructional methods. One of the controversial decisions we made was to refrain from giving homework to our students, focusing instead on maximizing their time in the classroom by having them practice skills and discover concepts.

*Practicing in Class.* Our main motivation was to free up time in class so students could develop their understanding of concepts and practice independently, in pairs, or in groups. We believe that practicing in class helped our students gain confidence in their abilities and reduce their stress, which in turn may have alleviated behavioral issues, off-task activity, and resistance to learning. A benefit of doing most of the work in class is that students receive immediate feedback, which helps them correct course immediately if needed and learn it "right the first time," or seek enrichment when they are finished. Immediate feedback from teachers, as well as reinforcing the use of resources in class, helps students build confidence and self-reliance. Collaborative work in class fosters the practices of

All our students  
had failed the HSA  
at least once...

The more  
traditional  
approaches to  
teaching  
mathematics were  
not reaching these  
students.

<sup>2</sup> Reprinted with permission from "High-Impact Solutions for Struggling Mathematics Students," *Mathematics Teacher*, 112(5), copyright 2019 by the National Council of the Teachers of Mathematics (NCTM). All rights reserved.

mathematical discourse, perseverance, and the strategic use of appropriate tools (CCSSI 2010).

Practicing in class also allows students to have access to the technology they need and deserve but cannot always get at home. About 30 percent of our students had little to no access to the Internet at home; more than 90 percent of them did not have access to a calculator outside of school. If we intend on leveling the playing field for all students, we must ensure they are able to use the technology where they can find it, and that is in school. Assigning technology-heavy homework every night can result in low participation from students, which means they have fewer opportunities to practice the skills they need to succeed on assessments. Although it is possible to set aside some time for in-class practice using the traditional teaching model, maximizing the time may require letting go of some, or all, traditional practices.

Many effective teachers will say that a coherent math class session follows this routine: warm-up, homework check, lecture, in-class practice (if time allows), exit slip, outside-of-

We believed that we could increase the rate of success in the course if we dedicated most of the class time to practicing skills that would be assessed either on standardized or in-class tests.

class practice, and start all over the next day. Although we realize that this structure works in many honors-level classes, at our school, we find that this approach is ineffective for our merit-level students. We believed that we could increase the rate of success in the course if we dedicated most of the class time to practicing skills that would be assessed either on state standardized tests or tests in class. When students struggle with the material, it becomes essential to make sure they have access to resources (teachers, classmates, notes) that will allow them to make progress. We maximized practice time in class so students would be able to work collaboratively or alone, would have access to resources in class that were unavailable to them at home (Internet, peers, teachers), and

would receive feedback as soon as they needed it instead of the next day or not at all. To have more in-class practice time, we had to make cuts in class routines. Removing ineffective, time-consuming activities is one way to ensure more time is spent on practicing in class.

*Assigning No Homework.* Let's take the example of a traditional, merit-level student's experience of a mathematics class. The warm-up is typically a repeat of the homework from the night before. While students are working independently on their warm-up activity, the teacher walks around the room to check on homework. When polled, unfortunately, 73 percent of the pilot students stated that the reason they do not do homework is because they "do not know what to do." Only 21 percent said they "do not have time." From the minute they sit down in class, they are staring at a set of questions that they still do not know how to answer, and they have not completed their homework either. When students who cannot do homework are placed in a situation of being judged every day on something they were unable to do, they often shut down. Learning under these conditions is difficult at best, impossible at worst. Teachers also struggle with repeatedly having to encourage students to do their homework and can often become distraught and/or irritated when they perceive their students as disrespectful for not doing the homework. These emotional responses from students and teachers contribute to a negative environment, which increases the likelihood of a less productive classroom (Kohn, 2006).

Moreover, some of our students do not have homes to return to after school, and the term homework may carry negative connotations for them. Additionally, the results of a survey of our students highlighted that 58 percent of them worked after school at least three days per week. Some chose to work to earn spending money, but some had to work to help pay bills at home. Our students had already put in a full day's work by the time they left school, only to face putting in more time working outside of school. Some did not get home until late at night, often working a full eight-hour shift and arriving home at about midnight. The idea of giving them homework on top of everything else they had to do seemed unfair to us. Even for our nonworking students, after asking them to work in school for eight hours, we wondered if we had "the right to dictate how our students spend their time after school" (Kohn, 2006).

One of the modifications we made was not to assign traditional at-home practice of any kind. Instead, we used the concept of an "optional daily practice."

Therefore, one of the modifications we made was not to assign traditional at-home practice of any kind. Instead, we used the concept of an "optional daily practice." These practices were composed of ten to fifteen exercises that highlighted essential skills for students to master each week, such as solving equations of any type, graphing, analyzing graphs, and modeling various functions. On the first day of each week, students were given access to the daily practice (either electronically or on paper, as needed). Students were to complete it by the last day of the week, and we encouraged them to sign up for the midday 40-minute intervention/enrichment block, form study groups, work as a team, rely on one another, and use their resources (notes and prescribed websites). In return, if the review was completed by the end of the week, they would receive one extra percentage point on their next assessment, but they were not penalized if they could not or chose not to complete it. Many students quickly realized that doing the daily practice, in addition to giving them an extra point, gave them the advantage of knowing some of the types of questions that would be on their tests and helped them retain information better. Finally, the daily practice reinforced the idea that although a unit of study has ended, the skills and concepts are still useful for application in later units. Over time, the number of students doing the daily practice increased; thus many students learned the value of extra practice outside of class.

Placing direct instruction and note taking toward the end of class was another deliberate choice of ours, after researching the concepts of "primacy and recency."

In addition to encouraging students to practice daily, we encouraged students to review their notes daily, redo examples as needed to build confidence and fluency, and generate questions to ask the next day, since they often worked on their notes at the end of a block. Placing direct instruction and note taking toward the end of class was another deliberate choice of ours, after researching the concepts of "primacy and recency." These concepts refer to the brain's ability to retain information in chunks; the first topic and the last topic covered in class are most likely to

be remembered (Morrison, 2015). Not having to go over homework in class (an activity that can take up to one-third of class time) gave us more time to help students practice skills in class. Moreover, it freed us to do other activities to better support and enrich our students. For example, we had more time available to spend on specific vocabulary instruction using a modified Frayer model, weekly or biweekly, which helped our students with varied literacy and language skills. We also used that time to introduce structured and organized

Complex Instruction activities (Stanford University 2017) to build our students' ability to work collaboratively, use mathematical discourse, and critique one another's work. We registered our students with Code.org, a website designed to introduce high school students to computer coding, and we gave students time in class to practice applying mathematical logic to create small computer programs. We provided opportunities to learn keyboarding during that time as well as to become more familiar with Google applications such as Google Docs™, Sheets™, and Slides™. We asked students to enter their thoughts on many different topics into a journal to help them gain confidence in their writing and to help us get to know them better. We fostered their natural curiosity using Notice and Wonder activities (Fetter, 2015). Finally, we used some of that time to do intensive HSA testing review to help students understand the questions and become more familiar with the format of the assessment.

### The Results

Our students took the Algebra High School Assessment in November 2014, and some took it again in January 2015. The pass rate went from 0 percent at the beginning of the school year to 47 percent after the first administration and 67 percent after the second administration (see Table 1). Although we are proud of our students' accomplishments on the assessment, those results pale in comparison to what they accomplished in class, which was that 93 percent passed the class, with 53 percent earning a grade of B or better. Remember that all students had previously failed the class at least once. Because 20 percent of them did so well, we recommended that they be placed into an honors-level geometry class for the next academic year.

Student behavior also improved over time. Student satisfaction increased (as shown in their journals and on surveys). Our students learned to become more self-reliant, to be self-starters, and to depend more on one another and their resources and less on us. By the end of the year, students would automatically log onto their classroom website and start working on their own. They no longer relied on prompts provided by us during journaling activities or group activities, and they were able to ask thoughtful questions of themselves and others.

**Table 1 Students' Rate of Success on HSA by Demographic Criteria (in Decreasing Order)**

Demographic Criterion	Total No. of Students	No. of Students Who Passed the HSA	Rate of Success (%)
Multiple Races	3	3	100
Asian	6	5	83
African American/Black	25	20	80
Caucasian	12	9	75
Students on an Individualized Education Plan (Special Education)	9	6	67
Students Receiving Free and Reduced-Price Meals (FARM)	47	26	55
English Language Learners (ELL)	21	9	43
Hispanic	33	13	39



## Conclusion

As Maria Montessori said, "The greatest sign of success for a teacher is to be able to say, 'The children are now working as if I didn't exist.'" I am proud to report that we were able to say so by the end of the year. When presented with a new approach, our students showed many areas of growth, including achievement, engagement, motivation, and self-reliance. They became organized—a skill they will certainly need in geometry, the following course.

Much more progress, however, remains to be made if we want to level the playing field for all students, not just our small pilot program. To help us in this task, we have recruited other teachers to join us. We have a plan to refine our process even more by including targeted instruction for our ELL students (using resources in their native language), increasing the number of Complex Instruction activities, and reinforcing soft skills, such as organization and study skills, earlier in the year. We are hopeful that our struggling students will continue to show gains and increasing success.

## Bibliography

Common Core State Standards Initiative (CCSSI). (2010). *Common Core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Available at [http://www.corestandards.org/wp-content/uploads/Math\\_Standards.pdf](http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf)

Complex Instruction Program. (n.d.). Achieving equity in the classroom. *Stanford University School of Education*. Retrieved May 1, 2017, from <http://cgi.stanford.edu/group/pci/cgi-bin/site.cgi?page=index.html>

Fetter, A. (n.d.). Beginning to problem solve with 'I notice/I wonder.' *The Problem Solving and Communication Activity Series*. National Council of Teachers of Mathematics Math Forum. <http://mathforum.org/pow/support/activityseries/understandtheproblem.html>

Framer model. (n.d.). *Reading Educator*. Retrieved May 1, 2017, from <http://www.readingeducator.com/strategies/framer.htm>

Kohn, A. (2006). *The homework myth: Why our kids get too much of a bad thing*. Cambridge, MA: DeCapo Press.

Morrison, M. (2015, March 17). Primacy and recency effects in learning. *RapidBi*. Retrieved May 1, 2017, from <https://rapidbi.com/primacy-and-recency-effects-in-learning/>



Karine S. Ptak, [ksptak@jravery.com](mailto:ksptak@jravery.com), teaches mathematics at Frederick High School in Frederick, Maryland. She enjoys sharing her love of mathematics with students who struggle with concepts, are English language learners, or view the content as irrelevant.

## Steering a Robot to Engage in Number and Spatial Sense

*Krista Francis, Stefan Rothschuh, Sarah Hamilton, & Graham Diehl*

This “hard fun” (Papert, 2002) task mathematically models how the <Move> Programming Block on the Lego Mindstorms Education software works. The EV3 Lego Mindstorms is an integrated robot and programming software platform. The software utilizes drag and drop object-based programming that is easily accessible to learners. This task could certainly be done with other wheel-based robots and software. However, we prefer the integration of the EV3 Lego robots, software, and tablet for classroom use, as the technical troubleshooting is minimal compared to many others we have tried.

Learning how to model the robot’s turning provides students with contextualized and spatialized experiences of measurement, geometry, fractions, percentages, data collection, data interpretation, and more. Consistent with the front matter of the Saskatchewan curriculum (Saskatchewan Ministry of Education, 2007a, 2007b, 2008, 2009), this robotics task provides different forms of representation for “exploring mathematical ideas, solving problems and communicating understandings” (Saskatchewan Ministry of Education, 2008, p. 5). This task promotes student engagement, reasoning, mathematical modeling, collaboration, communication, use of appropriate tools, finding patterns and regularity, and persistence in problem solving. It is also highly spatial and provides opportunities for developing spatial skills and spatial representations of mathematical concepts. Spatial reasoning is an important consideration for teaching, as it can improve mathematics achievement (Mix et al., 2016; Newcombe, 2010) and enjoyment.

Students will engage with logical thinking and number sense while gathering and interpreting data to predict how the robot moves and turns. For example, programming the robot to move specific distances requires applying decimal numbers as a proportion of wheel rotations. Exploring the relationship between the <Move> steering input and the resulting robot turn requires observing, measuring, and eventually predicting the consequential distance around the circle’s edge and the width of the circle.

### Overview

We have implemented this task several times in Grade 4-7 classrooms, and each time found the children highly engaged and amazed at how the robot steering works. There are three parts to this task: (1) determining how the robot’s wheels rotate according to the steering setting, (2) describing and modeling the robot’s turns according to the steering setting, and (3) modeling the steering differential with fractions and percentages. Each part delves into important mathematical concepts and takes about 45 minutes (total time: 2 hours and 15 minutes). For example, mathematical concepts associated with measurement in this task utilize an implicit number line to help students understand number as measurement, and

as position or motion along a path. These spatial conceptualizations of number are essential for understanding rational numbers and algebra (Norton & Alibabi, 2019).

For teaching this task, we offer some handy tips to keep in mind. First, terminology matters. For consistency, we adopted the convention that the *wheels rotate* and the *robot turns*. Second, to promote collaboration, we found pairs of students to work best. However, equipment limitations may necessitate larger groups of 3 or 4. Finally, a little coaching about how to work in pairs or groups goes a long way to ensure productive participation for all individuals. For instance, you may assign roles such as observer/feedback provider, recorder, programmer, and measurement taker. It is best if everyone in the group gets a turn with all of the different roles.

## Robotics Tasks

### Part 1: Determining how the Wheels Rotate According to the Steering Setting

#### *Robotics Task Considerations*

*Learning Goal and Materials.* The learning goal of Part 1 is to understand how the steering in the <Move> Programming Block varies the left and right wheel rotations of the robot. The directionality of the turn helps students gain a spatial sense of positive and negative numbers.

Students will need a basic Lego Mindstorms EV3 robot that is built according to the instructions provided, the Lego Mindstorms Education EV3 classroom software installed on a laptop or tablet, and a printable recording sheet that is available at [http://stem-education.ca/files/SteeringRecordingsheet\\_2020Part1.pdf](http://stem-education.ca/files/SteeringRecordingsheet_2020Part1.pdf) for recording observations. Also, a version with the older EV3 Education program is available here: [http://stem-education.ca/files/SteeringRecordingsheet-oldEV3\\_2020.pdf](http://stem-education.ca/files/SteeringRecordingsheet-oldEV3_2020.pdf).

*Student Mathematical Engagement.* Part 1 involves observation and data collection, interpretation of data, as well as the use of negative numbers and fractions. Although Grade 4 students have likely not yet been introduced to negative integers, Canada's cold weather provides prior experiences with negative numbers for even the youngest children (e.g., many children know that  $-20^{\circ}$  Celsius means cold). As well, consistent with Dehaene's (1997) observations, the robot's motion is coherent with an intuitive understanding of the number line, where increasing positive numbers signify moving forward and increasing negative numbers signify moving backwards.

Positive wheel rotations in the <Move> Programming Block result in forward motion, and vice versa for negative numbers. In our experience, an initial brief explanation of positive and negative numbers with a number line example is grasped quickly. To reinforce and scaffold spatial understandings of numbers, it is useful to provide a vertical and horizontal number line representation of the fractions to help students visualize number as position or location (see Figure 1).

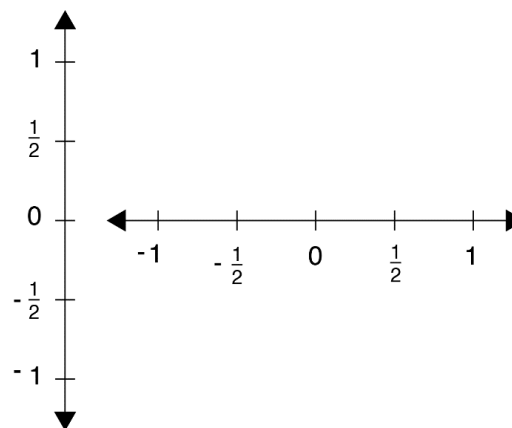


Figure 1: Example of a vertical and horizontal number line to scaffold and reinforce spatial understandings of number

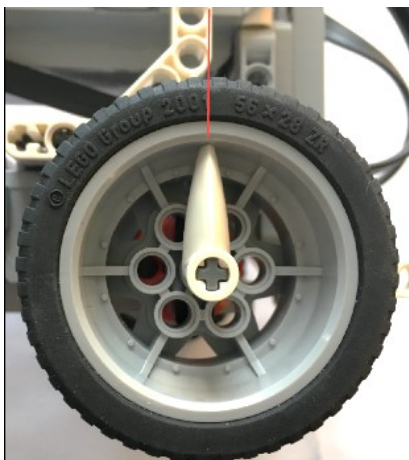


Figure 2: Wheel holder

wheel holder in an upwards position will help facilitate accurate observations (see Figure 2).

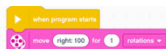
*Teaching Suggestions.* When getting started, make sure all the students have named their robots differently. This makes the Bluetooth connections of multiple robots in a classroom possible. It is also prudent to show students how to save their files. We encountered a few tears when students lost their programs on their iPad or computer after closing unsaved files.

The recording sheet provides detailed instructions about the task. In short, students observe and record the direction and amount of wheel rotations according to variations in the steering setting of the <Move> Programming Block, i.e., -100, -75, -50, -25, 0, 25, 50, 75, and 100. Note that in this part of the task, the robot is held above the ground so that the wheels do not touch the floor or surface. Encouraging students to place the arrow-like

### Instructions for Part 1

1. Drag and drop a <Move> block onto the programming chain.
2. Enter “right 100” in the <Move> bubble. **Leave the number of wheel rotations as “1”.** Leaving the number of wheel rotations at 1 is necessary for consistent results to help

students to identify the pattern.



3. Download the program to the robot.
4. Watch the right wheel. Run the program. Hint: Line up the right white wheel holder with the top support.
5. Which direction does the right wheel travel? Draw an arrow on the worksheet to indicate the direction.
6. How many rotations did the right wheel make? Enter the number of rotations on worksheet.
7. Repeat Steps 5 and 6 for the left wheel.
8. Draw the direction the wheel travels on the worksheet.
9. Repeat all of the above steps for 75, 50, 25, 0, -25, -50, -75, and -100.

### Observations of Student Engagement

*Evidence of Student Understanding.* Figure 3 (below) provides an example of a completed Part 1 recording sheet. As you can see from the example of the student’s work in Figure 3, students are measuring the rotation of the wheels, recording direction along with positive and negative numbers (spatial understanding of number), ordering fractions and whole numbers (extending understanding of fraction equivalence and ordering), representing data, and finding patterns and structures for interpretation. Your students may start to see the pattern quite quickly. The pattern is symmetrical around 0, where the wheels rotate clockwise with positive steering and counterclockwise with negative steering. Encourage students to make predictions before testing their ideas.

Slider Steering Setting	Left Wheel Direction*	# of Left Wheel Rotations	Right Wheel Direction	# of Right Wheel Rotations*	Draw the direction the robot travels (clockwise or counterclockwise)
100	↑	-1	↓	-1	clockwise
75	↑	$\frac{1}{2}$	↓	$-\frac{1}{2}$	clockwise
50	↑	1		0	CLOCKWISE
25	↑	1	↑	$\frac{1}{2}$	CLOCKWISE
0	↑	1	↑	1	Forward
-25	↑	$+\frac{1}{2}$	↑	$\frac{1}{2}$	counterclockwise
-50		0	↑	1	counterclockwise
-75	↓	$-\frac{1}{2}$	↑	1	counterclockwise
-100	↓	-1	↑	1	COUNTERCLOCKWISE

Figure 3: Example of a Grade 5 student's completed recording sheet for part 1

*Assessment of Understanding.* Students demonstrate understanding of Part 1 when they can articulate/explain the relationship between negative and positive steering setting and subsequent positive and negative left and right wheel rotations: E.g., a <Move> steering setting of 25 moves the right wheel  $\frac{1}{2}$  rotation forward and the left wheel 1 rotation forward.

### Part 1 Summary of Curriculum Outcomes

The mathematical connections in Part 1 is align with several Saskatchewan Curriculum Outcomes (Saskatchewan Ministry of Education, 2007a, 2007b, 2008, 2009) including Number – fractions (N4.6, N5.5, and N7.3), Number – integers (N6.6), Patterns – represent, analyze, relationships (P4.1, P5.1, P6.1, and P7.1), and Patterns – Variables and equations (P4.2, P5.2, P6.3, and P7.2). Appendix A provides a more detailed curriculum mapping.

### Part 2: Describing and Modeling Robot Turns According to the Steering Setting

Part 2 contains an unplugged activity and a plugged activity.

#### Unplugged Task Considerations

*Learning Goal.* The learning goal of the unplugged activity is to understand that outer wheel of the robot travels further around a circle. Starting with an unplugged embodied activity can really help strengthen understandings of the robot's actions (Sengupta et al., 2018).

*Student Engagement.* Have a group of four students link arms and walk around a circle while each student counts their steps (see Figure 4). Watch a video of children being instructed by their teacher to walk around a circle here <https://vimeo.com/392986435>. In the video, the students are trying to all take the same number of steps. The teacher interferes by explaining that it is more important to stay in a straight line. When asked how many steps each student took, they responded 13, 10, 13, and 17. While they each had different numbers of steps, the innermost student had more steps than the second, indicating the group's misunderstanding. This video highlights how it may not be intuitive to learners that the



outer part of the circle requires a larger distance around (and thus more steps) than the inner part of the circle.

*Teaching Suggestions.* To summarize this lesson (after the video), the teacher gathered the groups and each group explained their observations. The group with the best understanding demonstrated that an increasing number of steps needs to be taken for each student further removed from the center of the circle. With prompting, the students should recognize that the outside person needs to travel further than the inside person. The same is true for the robot. To travel along a circle, the outside wheel needs to travel further than the inside wheel.



Figure 4: Students walking around a circle

### Robotics Task Considerations

*Learning Goal.* The learning goal of the plugged robotics task is to understand how larger steering settings correspond to tighter robot turns (i.e., turns with smaller radii).

*Student Engagement.* In this part of the task, students observe and record the direction and distance the robot travels according to variations in the steering setting of the <Move> Programming Block, i.e., -100, -75, -50, -25, 0, 25, 50, 75, and 100. Students also need to determine which blue circle of the vinyl mat (or circles drawn on the floor) the outer wheel of the robot travels along. An additional task that was not included in the recordings sheets but that would scaffold learning about angles is measuring the angle of the arc that the robot travels with one-wheel rotation for each of the circles of radius 6 cm, 8 cm, 12 cm, and 24 cm.

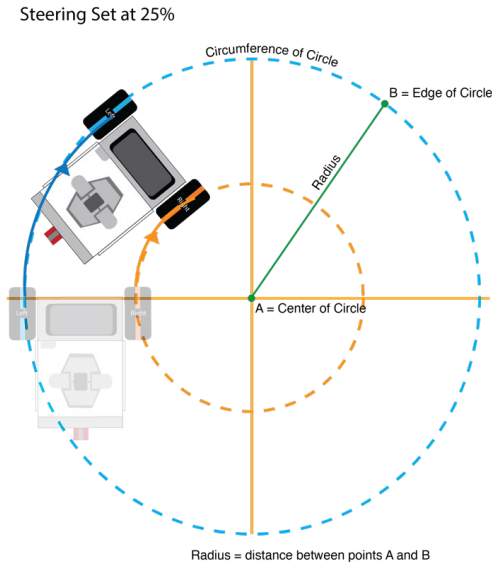


Figure 5: Diagram illustrating terminology for steering of <Move> programming block set to 25

length (i.e., 24 cm, 12 cm, 8 cm, and 6 cm) attached to pen or washable marker (or chalk), you can draw the circles on the floor or on poster boards.

*Teaching Suggestions.* Students will need the same robot, a ruler for measuring radii, a vinyl steering mat (or circles drawn on laminated poster boards or the floor), and a copy of the recording sheet (available at [http://stem-education.ca/files/SteeringRecordingsheet\\_2020Part2.pdf](http://stem-education.ca/files/SteeringRecordingsheet_2020Part2.pdf)). We used vinyl printouts of a mat available at <http://stem-education.ca/files/SteeringMat.pdf>. These mats cost about \$50 CAD each to print. We find that the vinyl mats hold up well after repeated use. However, they are not necessary. To avoid this cost, you can prepare for the task by drawing four concentric circles for each group. The circles need to have diameters of 48 cm, 24 cm, 16 cm, and 12 cm. Using an anchored string of the appropriate

Students may need to first learn some terminology before embarking on this part, including circumference of a circle, diameter, and radius. Part 2 uses these terms in context, which helps students understand their meaning. See Figure 5 for an example of this terminology in context.

### ***Instructions for Part 2***

1. Drag and drop a <Move> block onto the programming chain.
2. Drag the arrow for the steering until 25 is entered.
3. Which blue circle on the steering mat does the outer robot wheel follow?
4. Measure the radius of the blue circle. Record the radius in the table.
5. How many wheel rotations does it take for the robot to make one full circumference of the blue outer circle? Record the number of wheel rotations in the table.
6. Record which robot wheel follows the outer circle.
7. Repeat all the steps for 100, 75, 50, 0, -25, -50, -75, and -100.

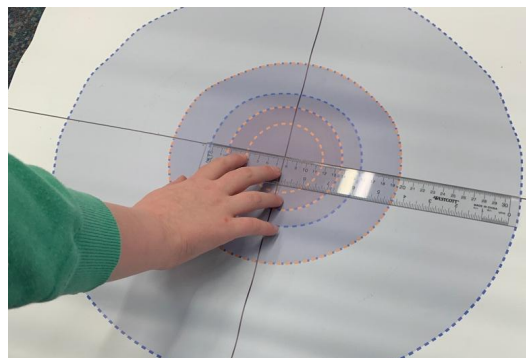


Figure 6: Student measuring radius

### ***Observations of Student Engagement***

*Evidence of Student Understanding.* Figure 6 shows how students working to find the radius, and Figure 7 shows two students with their teacher finding the circumference of the circle on the steering mat associated with the steering on the <Move> Programming Block set to 75.

The following dialog from the video (available at <https://vimeo.com/430192409>, dialog beginning at 00:24) illustrates how students are working with decimal numbers on the <Move> Programming Block to find the precise distance the robot travels around the circle with a radius of 8 cm (i.e., the circumference expressed in wheel rotations). The robot had previously made it half-way around the circle. The teacher reminded the students to line up the center of the ball bearing wheel of the robot with the axis and observe the turn of the robot again.

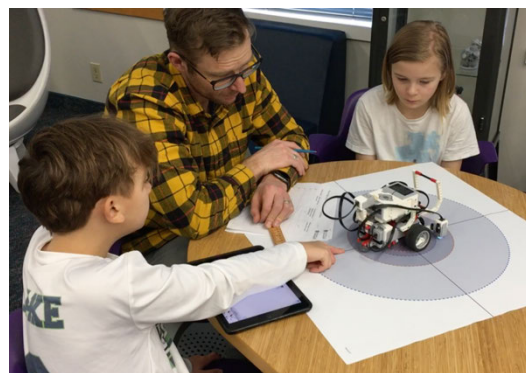


Figure 7: Teacher with students finding the circumference of the circle with wheel rotations and steering set to 75. See video at <https://vimeo.com/430192409>

- Graham: So, it's kind of like a 75 [percent of a full turn], so maybe add a little bit more.  
 Okay, give that a shot... Let's line it up properly (repositions robot on mat).
- Brennan: (Points finger to back wheel, downloads program to the robot and observes the turn.)
- Graham: That was pretty close... okay, let's say 2 point...
- Brennan: ...631 (enters the numbers into the <Move> Programming Block).

Graham: Mmhh... we could have said 2.7.

Brennan: (Downloads program to the robot and observes the turn) ...oh, 2.68.

Graham: Yeah, or maybe 2.65. You are quite precise. Give that a shot.

In the exchange, the students use decimals numbers to hundredths to find the circumference in wheel rotations of the circle with a radius of 8 cm. The example of the student's work in Figure 8 illustrates how students record their measured radii of the circles in cm, the circumference of the circles in wheel rotations (geometric measurement of circles), and which wheel of the robot is on the outer circle. Recall that in the unplugged activity, students learned that the outer wheel travels further around a circle. Noting which wheel travels furthest can also help students predict the direction of turn.

Steering Setting	Radius of robot turn (blue circle)	# of wheel rotations to complete one circle (circumference)	Which wheel is on the outside of the circle?
100	8 cm	2	Left
75	9 cm	2.65	Left
50	12 cm	3.9	Left
25	24 cm	8.6 rotations	Left
0	○	○	○
-25	24 cm	8.6	Right
-50	12 cm	3.9	Right
-75	9 cm	2.65	Right
-100	6	2	Right

Figure 8: Example of a student's completed recording sheet for Part 2

The exchange with the teacher continued as the students filled in the recording sheet. The dialog below (starting at 2:34 in the video, available at <https://vimeo.com/430192409>) is an extract of a conversation where the students are beginning to consolidate their observations. Recall that the goal of Part 2 was to understand how larger steering settings correspond to tighter robot turns, or turns with smaller radii. Notice how the student articulated his emerging understanding of how the steering input changes the radius of the circle the robot travels around.

Graham: Last time, because it was at 100, our radius was tighter, we started from about there... (points with pen).

And it just spun in place (makes a circular motion above the robot).

Brennan: So every time we go on with one [steering input] we just come out a circle more?

Graham: That's a very good observation...

So, every time you are changing your percentage of steering (points to student's program on iPad), you are probably moving to a different spot (points away from the center of the circle).

And it'll be either a larger radius or a smaller radius depending on the number (points to student's program on iPad).

*Assessment of Understanding.* After completing Part 2, students should be able to interpret and explain (i) the relationship between positive and negative steering inputs to the direction of robot turn, (ii) the relationship between the steering input and the radius and size of the circle the robot travels around, and (iii) the relationship between the number of wheel rotations and the distance the robot travels around the circle.

### **Part 2 Summary of Curriculum Outcomes**

Like Part 1, Part 2 aligns well with the strands of Number and Patterns and Relations, as well as Shape and Space in Grades 5, 6 and 7 (Saskatchewan Ministry of Education, 2007a, 2007b, 2008, 2009). Appendix B shows a more detailed mapping of Part 2 to the curriculum. Students are measuring and recording lengths (SS5.2), using decimal numbers (N4.7, N5.6, and N7.3), multiplication (N4.3 and N4.4), estimation (N5.4), percentages and ratios (N6.5 and N6.8), identifying patterns and structures for interpretation (P4.1., P5.1, P6.1, and P7.1), understanding equations and expressions (P4.2, P5.2, P6.3, and P7.2), 2D relations (SS6.1 and SS7.3), and circles (SS7.1).

*Extension:* A suggested extension for students in Grades 7 and above is to calculate the circumference ( $c$ ) of the circle based on the radius ( $r$ ) using the formula  $c = 2\pi r$ . In the example and the dialog associated with Figure 7, the recorded circumference of 2.65 wheel rotations is pretty close to the actual circumference, as the following calculations illustrate:  $c = 2\pi r = 2\pi \cdot 8 \text{ cm} = 50.27 \text{ cm}$ , and there are 17.6 cm per wheel rotation, so  $50.27 \text{ cm} \div 17.6 \frac{\text{cm}}{\text{wheel rotation}} = 2.86$  wheel rotations. In an extension to the task, students could convert the measured circumference in wheel rotations into centimeters and compare their answers to explain similarities and differences. They should notice that their answers are close, though slight variations may occur due to rounding or measurement errors, as well as mechanical friction when the robot moves. This extension is available on the recording sheet at [http://stem-education.ca/files/SteeringRecordingsheet\\_2020Part2.pdf](http://stem-education.ca/files/SteeringRecordingsheet_2020Part2.pdf).

### **Part 3: Modeling Steering Differential with Fractions and Percentages**

#### ***Robotics Task Considerations***

*Learning Goal.* The learning goal of Part 3 is to understand that the number entered in the <Move> steering determines a steering differential.

*Student Engagement.* The steering differential is the difference in power that is delivered to each wheel. When the differential is at its maximum, each wheel is going in different directions for the same amount of rotations. For example, as found in Part 1 above, when the <Move> steering is set to 100, the left wheel moves forward and the right wheel moves equally backward, and from Part 2 above, the robot turns around a circle of radius 6 cm. This is the tightest turn the robot can make, and it essentially pivots on the spot.

*Teaching Suggestions.* One of the teachers that we worked with demonstrated the concept of a steering differential using the YouTube video "Around the Corner – How Steering Differential Works" (<https://youtu.be/yYAw79386WI>). A student noted that an excavator



uses the same differential when moving. At this point, there is a natural opportunity to transition to a science inquiry about gears. However, our focus remains on the mathematics.

For Part 3, students will need the same robot and a copy of the recording sheet, which is available at [http://stem-education.ca/files/SteeringRecordingsheet\\_2020Part3.pdf](http://stem-education.ca/files/SteeringRecordingsheet_2020Part3.pdf). The recording sheet contains the instructions for the task. To reinforce and scaffold spatial understanding of numbers, it may be useful to refer back to the vertical and horizontal number line representation of the fractions that was provided in Figure 3.

### Instructions for Part 3

1. Draw bars to represent how much each wheel rotates (when the steering block is set to 1-wheel rotation) for 75, 50, 25, and 0.
2. Colour in the bars to indicate the amount of power going to the wheel.
3. Describe the robot's turn.

### Observations of Student Engagement

*Evidence of Student Understanding.* Figure 9 shows a) how the steering is programmed – the steering on the <Move> Programming Block set to 100 means 100%, b) the direction in which the wheels rotate, and c) the movement of the robot. The right wheel travels one wheel rotation backward and the left wheel travels one wheel rotation forward. This can be represented by equal sized bars or rectangles (see Figures 9 a) and 9 b). The illustrations in Figure 9 (and in Part 3 of the recording sheet) are useful for spatially representing proportional relationships in Grades 4-6, and spatially reinforcing algebraic ideas about opposite quantities with distance from a center in Grade 7.

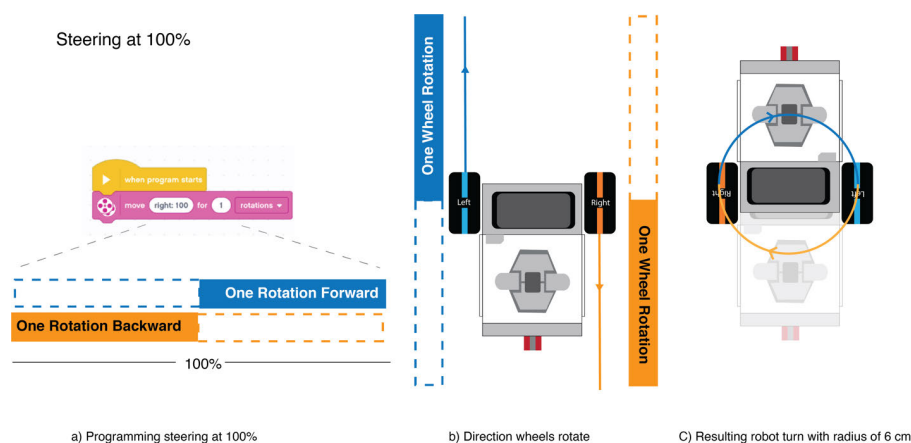


Figure 9: Steering differential at 100

Figure 10 below illustrates that students are measuring the rotation of the wheels (geometric measurement of circles), recording direction along with positive and negative numbers (spatial understanding of number), ordering fractions and whole numbers (extending understanding of fraction equivalence and ordering), representing data, and finding patterns and structures for interpretation.



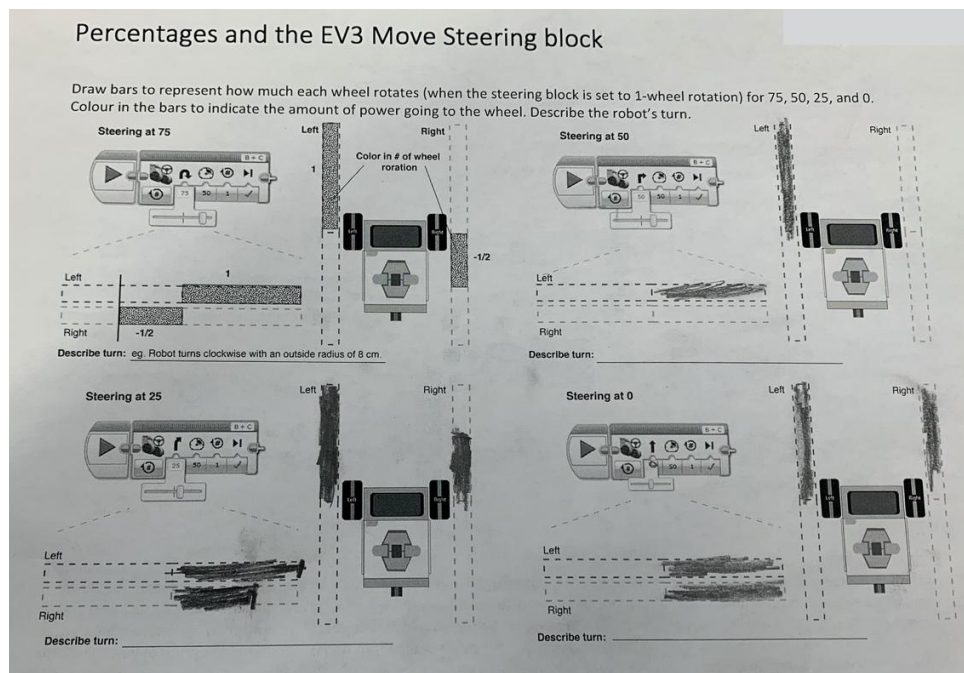


Figure 10: Example of student's work on Part 3 with an older version of EV3 Education

*Assessment of Understanding.* Students demonstrate understanding of Part 3 when they can articulate/explain the relationship between steering differential percentage and the amount and direction of left- and right-wheel rotations.

### Part 3 Summary of Outcomes

The mathematical connections in Part 3 align with several Saskatchewan Curriculum Outcomes (Saskatchewan Ministry of Education, 2007a, 2007b, 2008, 2009) including Number – fractions (N4.6, N5.5, N6.6, and N7.3), Number – percentage (N6.5), Number ratio (N6.8), Patterns – represent, analyze, relationships (P4.1, P5.1, P6.1, and P7.1), Patterns – variables and equations (P4.2, P5.2, P6.3, and P7.2), Statistics and Probability – many-one correspondence (SP4.1). Appendix C provides a more detailed curriculum mapping.

### Summary

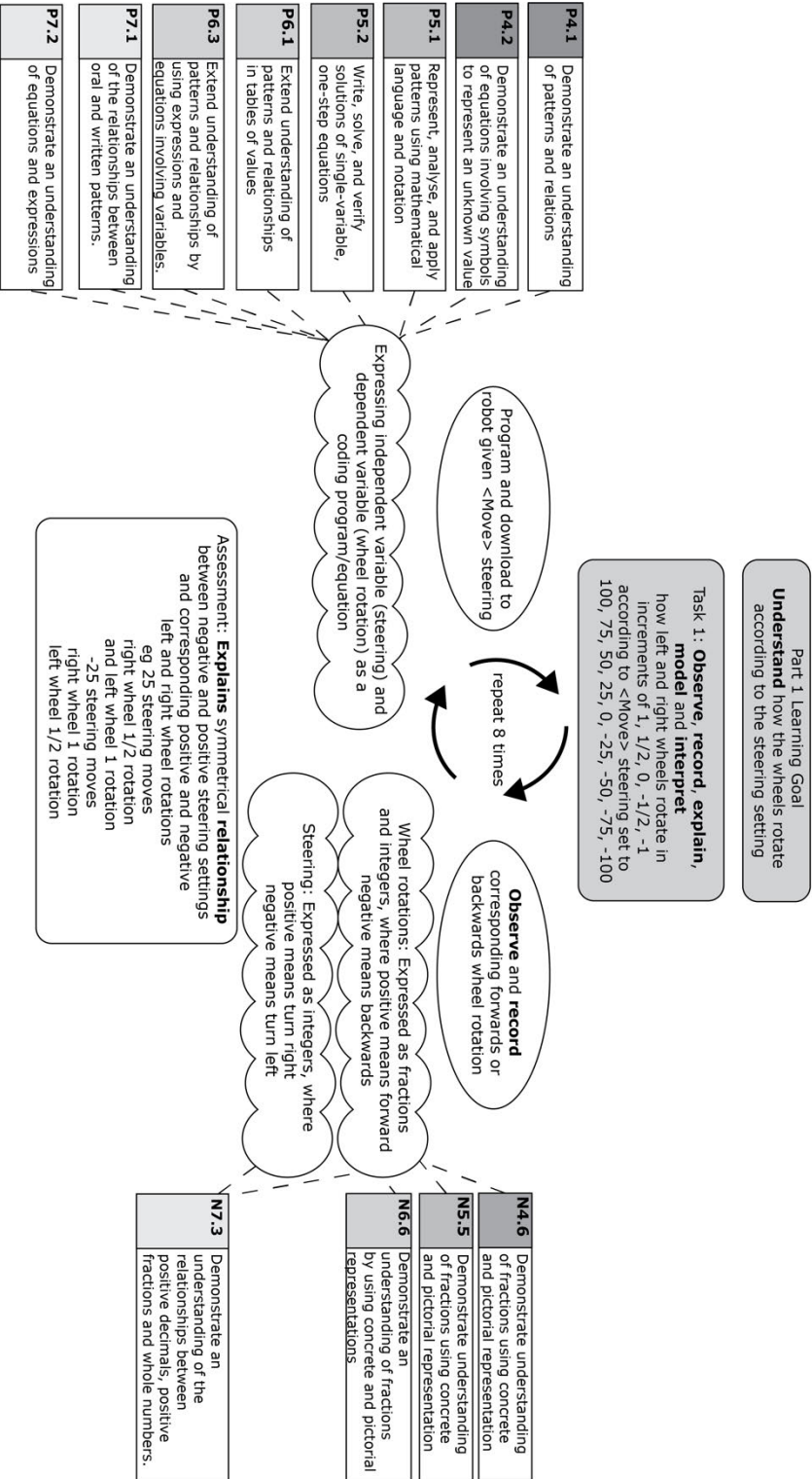
In our many years of working in classrooms with robot tasks, we have noticed how completing Parts 1, 2 and 3 of the <Move> Steering task improves students' precision in maneuvering the robot in more advanced /complex robotics challenges. For instance, in one observed class, the next task was the Pine Beetle Challenge ([http://stem-education.ca/?page\\_id=1071](http://stem-education.ca/?page_id=1071)). The students immediately recognized that they needed the tightest turn (100% steering) to maneuver the robot around the 90° angle corners. They were also able to closely estimate the number of wheel rotations for these turns, making the trial-and-error testing quite short. These two videos were taken on the first day the class tried the Pine Beetle Challenge: <https://vimeo.com/415697745> and <https://vimeo.com/415696291>. Note how precisely each robot moves forward and turns. In previous classes, where the <Move> Steering task presented in this paper was not introduced, we noticed that many students struggled to move their robot precisely in complex tasks. Overall, we have found that the experience of learning how the robot turns

enables a shift to more complex computational thinking and sequencing of programming blocks.

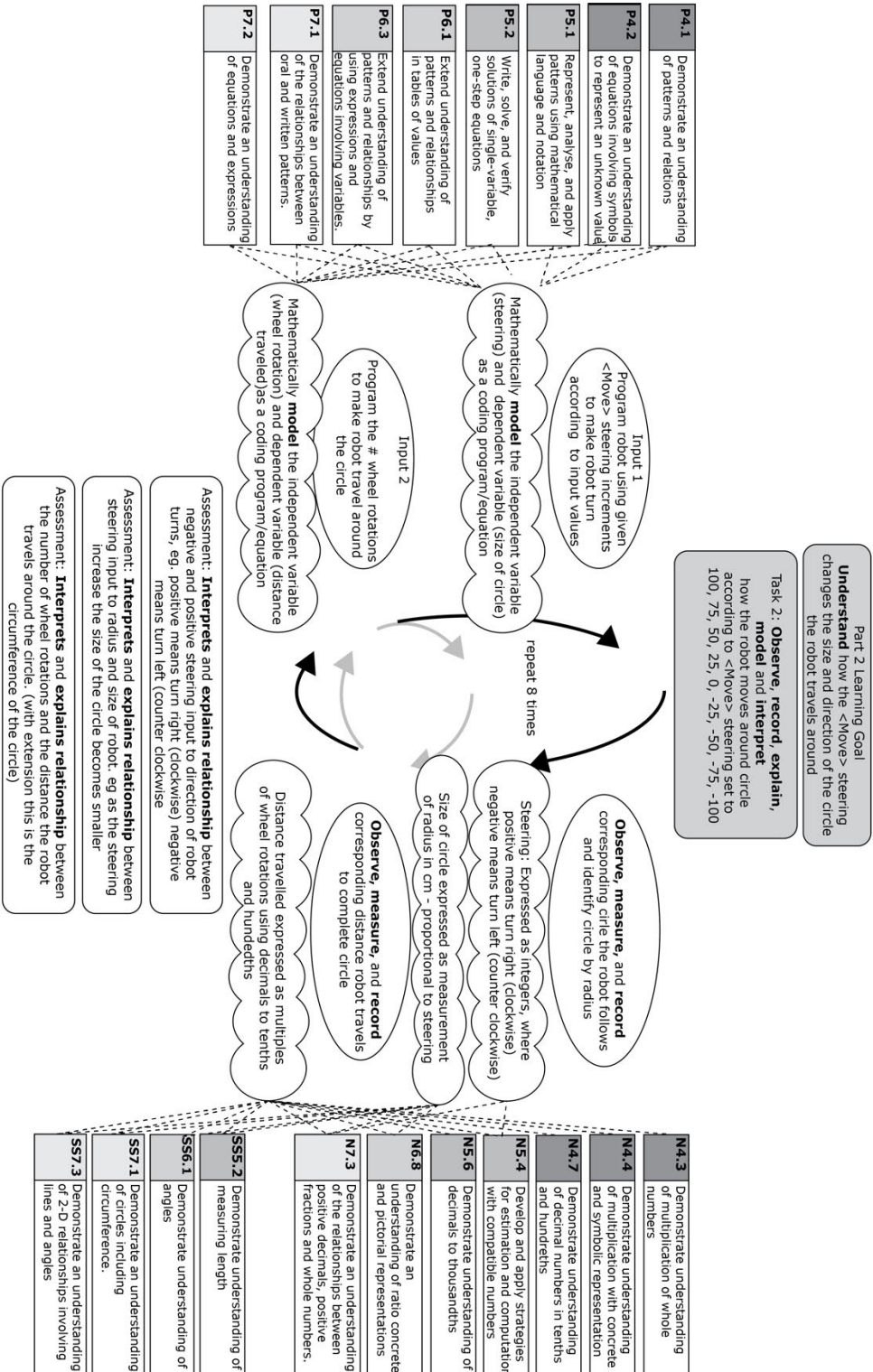
Mathematically modeling how the <Move> Steering programs the robot to turn is a fun, rigorously structured inquiry that incorporates concepts related to number, patterns and relations, shape and space, and data interpretation. Students will encounter these concepts in embedded and spatial manners that will contribute to meaningful experiences. The constant interplay of the students actively experiencing the task, the robot performing actions, the programming interface that determines robot movements, and the accompanying/complementary graphic and symbolic representations on the recording sheets supports students' conceptual understanding of the embedded mathematics. The task also simultaneously facilitates students' self-paced problem-solving, collaboration, and mathematical reasoning skills. After completing the tasks, students will have a greater understanding of how the steering programming of the robot works, enabling them to have greater precision with any future robotics tasks.

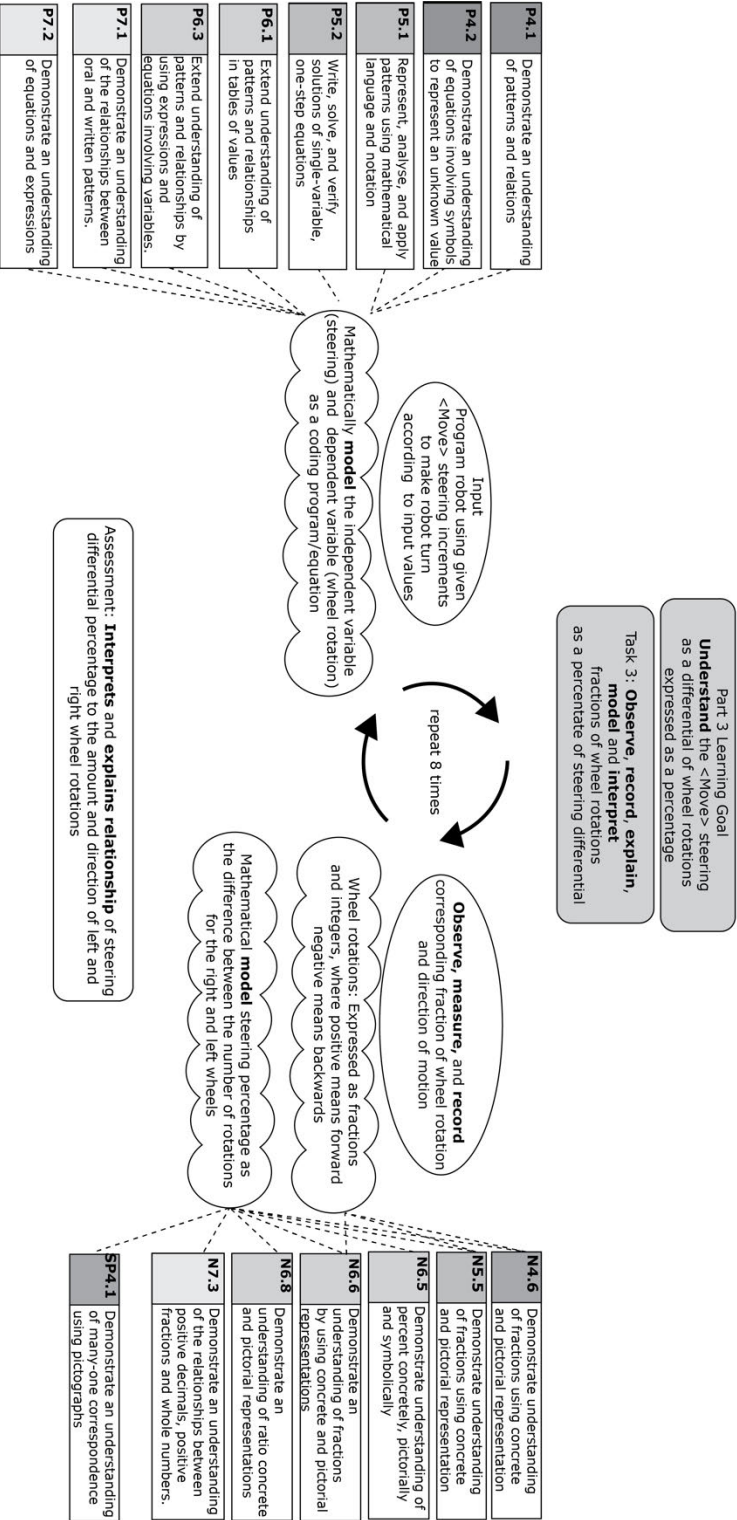
## References

- Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. Oxford University Press.
- Mix, K. S., Levine, S. C., Cheng, Y.-L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General*, 145(9), 1206–1227. <https://doi.org/10.1037/xge0000182>
- Newcombe, N. S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator*, 34(2), 29-35. <https://link-gale-com.ezproxy.lib.ualgary.ca/apps/doc/A233291449/AONE?u=ucalgary&sid=AONE&xid=8574ce5a>
- Norton, A. & Alibali, M. W. (Eds.) (2019). *Constructing number: Merging perspectives from psychology and mathematics education*. Springer.
- Papert, S. (2002). *Hard fun*. <http://www.papert.org/articles/HardFun.html>
- Saskatchewan Ministry of Education. (2007a). *Mathematics 4*. [www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics\\_4\\_2007.pdf](http://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics_4_2007.pdf)
- Saskatchewan Ministry of Education. (2007b). *Mathematics 7*. [www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics\\_7\\_2007.pdf](http://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics_7_2007.pdf)
- Saskatchewan Ministry of Education. (2008). *Mathematics 5*. [www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics\\_5\\_2008.pdf](http://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics_5_2008.pdf)
- Saskatchewan Ministry of Education. (2009). *Mathematics 6*. [www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics\\_6\\_2009.pdf](http://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics_6_2009.pdf)
- Sengupta, P., Dickes, A. C., & Farris, A. F. (2018). Toward a phenomenology of computational thinking in STEM education. In M. S. Khine (Ed.), *Computational thinking in the STEM disciplines: Foundations and research highlights* (pp. 49-72). <https://doi.org/10.1007/978-3-319-93566-9>



## Appendix B









*Dr. Krista Francis, kfrancis@ucalgary.ca, is an Assistant Professor of STEM Education and at the Werklund School of Education at the University of Calgary in Canada. Her passion is finding innovative ways for teaching and learning mathematics concepts.*



*Stefan Rothschuh is a PhD student in Mathematics Education at Werklund School of Education at the University of Calgary. Stefan taught mathematics from Grade 5 to high school. His research explores how embodied learning can be promoted in mathematics classrooms.*



*Sarah Hamilton is a doctoral student in the Learning Sciences Werklund School of Education at the University of Calgary. She is a Research Lead at Calgary Academy. Her interests include mathematics education, teacher professional learning, and innovative teaching practices.*



*Graham Diehl is a teacher at Calgary Academy (12 years). Graham has taught in the regular classroom (Grade 5) as well as in the elective course setting including Multimedia, Computer Science, and Grade 9 Cycle (shop class).*

*Calgary Academy is a non-profit, independent K-12 school in Calgary, Alberta that specializes in working with students with learning differences.*



*In this column, you'll find information about upcoming math education-related workshops, conferences, and other events. Some events fill up fast, so don't delay signing up! For more information about a particular event or to register, follow the link provided below the description. If you know about an upcoming event that should be on our list, please contact us at [thevariable@smts.ca](mailto:thevariable@smts.ca).*

### **NCTM Virtual Annual Meeting**

*April 21-24 & April 28-May 1, 2021*

*Presented by the National Council for Mathematics Teachers*

The NCTM Annual Meeting & Exposition is the premier event for mathematics educators, from PreK-12 teachers to university professors. Due to the ongoing pandemic, the 2021 Annual Meeting & Exposition—originally scheduled for St. Louis—has been reimagined into a fully virtual experience. Featuring 600+ education sessions, the NCTM 2021 Virtual Annual Meeting will provide the full range of program content, learning opportunities, and collaboration typical of major NCTM events. The event will be held from Wednesday-Saturday across a two-week period - April 21-24 and April 28-May 1. There will be one registration rate for the entire event experience. Look for registration rates to be available in January.

More information at <https://www.nctm.org/annual/>

### **NCTM Annual Meeting and Exposition: From Critical Conversations to Internal Actions**

*September 22-25, 2021*

*Atlanta, GA*

*Presented by the National Council for Mathematics Teachers*

Join your mathematics education peers at the premier math education event of the year! Network and exchange ideas, engage with innovation in the field, and discover new learning practices that will drive student success. Classroom teachers, administrators, math coaches, supervisors, college professors, and preservice teachers can all benefit from the sessions and learning at this event. The NCTM Annual Meeting and Exposition in Atlanta content strands are:

- Broadening the Purposes of Learning and Teaching Mathematics
- Advocacy To Make an Impact in Mathematics Education

- Equitable Mathematics Through Agency, Identity, and Access
- Building and Fostering a Sense of Belonging in the Mathematics Community
- Effective Mathematics Teaching Practices

More information at <https://www.nctm.org/annual/>

### **OAME/AO EM 2021 Virtual Conference**

*May 17-21, 2021*

*Presented by the Ontario Association for Mathematics Education*

The main focus of OAME/AOEM 2021 - Equity Counts is to address equity in mathematics education and promote best classroom practices. This means to strive to have students attain proficiency in mathematics, regardless of race, gender, language, socioeconomic status, or learning style. Due to the COVID-19 pandemic, OAME/AOEM 2021: Equity Counts will be a fully virtual conference. Even though we will not be meeting in-person, we will still have a full program. Please visit the website below to see all that will be available at the conference.

More information at <https://sites.google.com/oame.on.ca/oame2021>

### **Education Week Webinars**

A collection of free and premium virtual broadcasts, including upcoming and on-demand webinars. These virtual broadcasts cover teaching and learning and include webinars on differentiated instruction and the common-core standards. All webinars are accessible for a limited time after the original live streaming date. For all webinars broadcast by Education Week after August 1, 2019, Certificates of Completion are available to all registered live attendees who attend 46 minutes or more of any webinar.

Available at [www.edweek.org/ew/marketplace/webinars/webinars.html](http://www.edweek.org/ew/marketplace/webinars/webinars.html)

### **Global Math Department Webinar Conferences**

The Global Math Department is a group of math teachers that organizes weekly webinars and a weekly newsletter to let people know about the great stuff happening in the math-Twitter-blogsphere and in other places. Webinar Conferences are presented every Tuesday evening at 9 pm Eastern. In addition to watching the weekly live stream, you can check the topic of next week's conference and watch any recording from the archive.

Available at [www.bigmarker.com/communities/GlobalMathDept/conferences](http://www.bigmarker.com/communities/GlobalMathDept/conferences)

### **NCTM E-Seminars and Webcasts**

*Presented by the National Council for Mathematics Teachers*

E-seminars are recorded professional development webinars with facilitator guide and handouts. E-seminars are free for NCTM members. Webcasts of Annual Meeting Keynote Sessions offer notable and thought provoking leaders in math education and related fields as they inspire attendees at NCTM Conferences.

Available at <https://www.nctm.org/NCTM/templates/ektron/two-column-right.aspx?pageid=75420>



# Tangents

## Extracurricular Opportunities for Students

**T**his column highlights local and national extracurricular opportunities for K-12 students interested in mathematics, including collaborative, individual, online, and in-person challenges, contests, and camps. For dates, registration procedures and applications, and other information about the contests listed, please head to the contest websites, included below the descriptions. If we have missed an event that should be on our list, please contact us at [thevariable@smts.ca](mailto:thevariable@smts.ca).

If you are looking for contests available in other provinces, head to the Canadian Mathematical Society website ([cms.math.ca/Competitions/othercanadian](https://cms.math.ca/Competitions/othercanadian)). The CMS also maintains a list of resources for students who are looking to build their problem-solving skills and succeed in competitions: see [cms.math.ca/Competitions/problemsolving](https://cms.math.ca/Competitions/problemsolving).



### Canadian Computing Competition

February 17, 2020

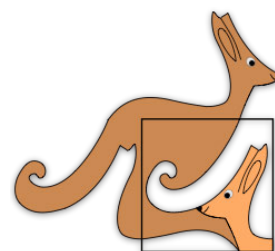
The Canadian Computing Competition (CCC) is a fun challenge for secondary school students with an interest in programming. It is an opportunity for students to test their ability in designing, understanding and implementing algorithms. Students are encouraged to prepare; see suggestions on contest website. The contest is held online in schools.

More information at <https://www.cemc.uwaterloo.ca/contests/computing.html>

### Canadian Math Kangaroo Contest

Written in March

The purpose of this competition is to introduce youngsters from Grade 1 to Grade 12 to math challenges in a fun and enjoyable way, thus inspiring their further interest and advancement in mathematics. The competition is held yearly in more than 50 Canadian cities. Students may choose to participate in English or in French.



More information at <https://mathkangaroo.ca>

### **Canadian Team Mathematics Contest**

*April 8 & 9 2021*

*Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)*

The Canadian Team Mathematics Contest (CTMC) is a fun and challenging competition for teams of 6 secondary school students in any combination of grades. One teacher and groups of six students participate at their own school over 3 consecutive hours. The curriculum and level of difficulty of the questions will vary. Junior students will be able to make significant contributions but teams without any senior students may have difficulty completing all the problems. Written in April at the University of Waterloo; teams may participate unofficially in their school on any day after the official contest date.

More information at [www.cemc.uwaterloo.ca/contests/ctmc.html](http://www.cemc.uwaterloo.ca/contests/ctmc.html)

### **Caribou Mathematics Competition**

*Held six times throughout the school year*



The Caribou Mathematics Competition is a worldwide online contest that is held six times throughout the school year. Each of these days, five contests are offered, one for each of the grade levels 3/4, 5/6, 7/8, 9/10 and 11/12 and each one in English, French and Persian. The Caribou Cup is the series of all Caribou Contests in one school year. Each student's ranking in the Caribou Cup is determined by their performance in their best 5 of 6 contests through the school year.

More information at [cariboutests.com](http://cariboutests.com)

### **Euclid Mathematics Contest**

*April 7, 2021*

*Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)*

The Euclid Mathematics Contest is an opportunity for students in their final year of secondary school and motivated students in lower grades to have fun and to develop their mathematical problem solving ability. Most of the problems are based on the mathematical curriculum up to and including the final year of secondary school. Most of the problems are based on curricula up to and including the final year of secondary school. Some content might require students to extend their knowledge and the best way to familiarize oneself with commonly appearing topics is to practice using past contests. The contest is written by individuals in schools.

More information at [www.cemc.uwaterloo.ca/contests/euclid.html](http://www.cemc.uwaterloo.ca/contests/euclid.html)

### **Fryer, Galois, and Hypatia Mathematics Contests**

*April 14, 2021*

*Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)*

The Fryer, Galois and Hypatia Math Contests are an opportunity for students to write a full-solution contest. They are fun way to develop mathematical problem solving skills through a written mathematical activity. For students in Grades 9 (Fryer), 10 (Galois) and



11 (Hypatia). Questions are based on curriculum common to all Canadian provinces. Rather than testing content, most of the contest problems test logical thinking and mathematical problem solving. The contest is written by individuals in schools.

More information at [www.cemc.uwaterloo.ca/contests/fgh.html](http://www.cemc.uwaterloo.ca/contests/fgh.html)

### **Gauss Mathematics Contests**

May 12, 2021

*Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)*

The Gauss Contests are an opportunity for students in Grades 7 and 8, and interested students from lower grades, to have fun and to develop their mathematical problem solving ability. Questions are based on curriculum common to all Canadian provinces. The Grade 7 contest and Grade 8 contest is written by individuals and may be organized and run by an individual school, by a secondary school for feeder schools, or on a board-wide basis.

More information at [www.cemc.uwaterloo.ca/contests/gauss.html](http://www.cemc.uwaterloo.ca/contests/gauss.html)

### **Opti-Math**

Written in March

*Presented by the Groupe des responsables en mathématique au secondaire*

A French-language mathematics challenge for secondary students who would like to exercise and develop their problem-solving skills.



*Les Concours Opti-Math et Opti-Math + sont des Concours nationaux de mathématique qui s'adressent à tous les élèves du niveau secondaire (12 à 18 ans) provenant des écoles du Québec et du Canada francophone. Ils visent à encourager la pratique de la résolution de problèmes dans un esprit ludique et à démystifier, auprès des jeunes, les modes de pensée qui caractérisent la mathématique. Le principal objectif des Concours est de favoriser la participation bien avant la performance. La devise n'est pas : « que le meilleur gagne » mais bien « que le plus grand nombre participe et s'améliore en résolution de problèmes ».*

More information at [www.optimath.ca/index.html](http://www.optimath.ca/index.html)

### **Pascal, Cayley, and Fermat Contests**

February 23, 2021

*Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)*

The Pascal, Cayley and Fermat Contests are an opportunity for students in Grades 9 (Fryer), 10 (Galois)m and 11 (Hypatia) to have fun and to develop their mathematical problem solving ability. Early questions require only concepts found in the curriculum common to all provinces. The last few questions are designed to test ingenuity and insight. Rather than testing content, most of the contest problems test logical thinking and mathematical problem solving. The contest is written by individuals in schools.

More information at [www.cemc.uwaterloo.ca/contests/pcf.html](http://www.cemc.uwaterloo.ca/contests/pcf.html)

### **The Virtual Mathematical Marathon**

*Supported by the Canadian National Science and Engineering Research Council*



**VIRTUAL MATHEMATICAL MARATHON**  
Let's enjoy mathematics together!

The virtual Mathematical Marathon has been developed by an international team of mathematicians, mathematics educators, and computer science specialists with the help of the Canadian National Science and Engineering Research Council.

The main activity is a competition allowing students to enjoy solving challenging mathematical problems all year around. Students can join the game at any time and at no cost, simply creating an individual profile with an individual username and a password. Available in French.

More information at [www8.umoncton.ca/umcm-mmiv/index.php](http://www8.umoncton.ca/umcm-mmiv/index.php)

### **Yang Math League**

*Levels:* Grade 8 and under; Grades 9 to 12

*Frequency:* Weekly

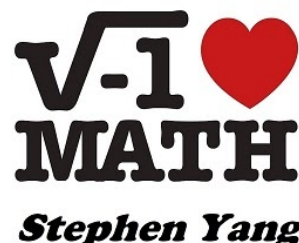
*Time:* 30 minutes at any convenient time on Saturday or Sunday

*Questions:* 6

*Format:* Google Forms

*Topics:* Full range of school mathematics

*Goal:* To help students become more interested in math by problem solving and assist them in growing conceptually

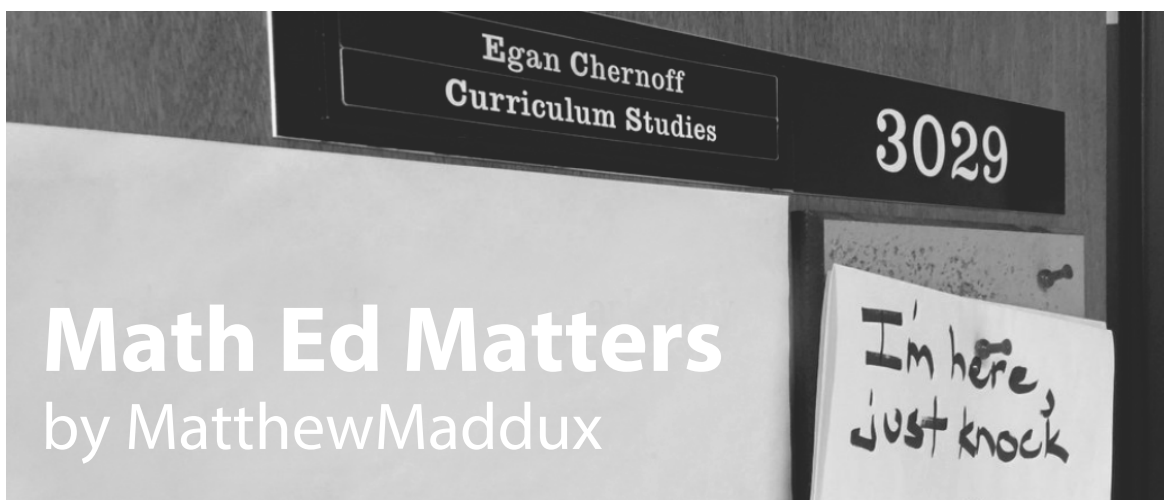


The Yang Math League (YML) is entirely organized and run by Saskatchewan's own Stephen Yang, a talented and passionate Grade 10 math student in Saskatoon. Students receive the six weekly questions through email each Saturday morning at 9 am and can choose when they do them that weekend. They submit their answers on a Google form that is scored automatically, and receive their scores back on Monday evening along with their cumulative score and the names of the perfect scorers. When over 20% of the students ask for a solution to a question, Stephen posts a YouTube video within a week.

Students can participate for as many or few weeks as they want and take a break for one or several weeks. Students who have participated consistently see a growth in their ability to solve tough mathematical problems.

To register, use the following link: <https://bit.ly/2KpRAmX>

Also check out Stephen's YouTube channel, which includes solutions to a variety of tough math questions from contests: <https://bit.ly/39E2C0t>



*Math Ed Matters by MatthewMaddux is a column telling slightly bent, untold, true stories of mathematics teaching and learning.*

## Extremely Amateur Math Ed Morphology: Renaming Mathematical Diseases

Egan J Chernoff  
[egan.chernoff@usask.ca](mailto:egan.chernoff@usask.ca)

Self-isolation during the COVID-19 pandemic became a necessary way of life for months. And, while being stuck at home has undoubtedly tested the human spirit, it has also inspired some spectacularly silly pastimes (e.g., running full marathons on very small balconies, marble racing, and many others). Looking to contribute to the coronavirus corpus of distractions myself, this article continues the silliness that began in my last article, where I renamed certain mathematical diseases associated with common mathematical mistakes (see, Chernoff 2020). So, as we look forward to eventually saying goodbye to the global pandemic, and having just said goodbye to *sinusitis* and *functionitis* in my last article, it's now time to say goodbye to *squaranoia*, *sumonia*, and perhaps even *logarrhea*, which is where we begin.

### **Logarrhea:** $\log(a + b) = \log(a) + \log(b)$

As every single person whom I've asked over the years has confirmed, the association with the term *logarrhea* is, of course, diarrhea. Unfortunately, all of us are familiar with diarrhea in some way or another, so there's no need to elaborate on that condition here. The word *diarrhea*, if you didn't know, is the combination of the prefix *dia-* (meaning through, throughout, or completely) and the suffix *-rrhea* (meaning flow, discharge, or secretion). Stemming from diarrhea, then, *logarrhea* implies a flow, discharge, or secretion of logs (logarithms). I must admit, given the familiarity of the term diarrhea, I am a little hesitant about changing the name for mistakenly thinking  $\log(a + b) = \log(a) + \log(b)$ . Maybe, just maybe, there's a better option.

Arguably, the term *logarrhea* works because, in the mistake in question, it appears that the logarithm on the left side of the equation,  $\log(a + b)$ , flows (or discharges, or secretes) throughout or completely through the brackets to each of the terms in the bracket, resulting

in  $\log(a) + \log(b)$ . However, although both notions found in the term diarrhea ('throughout' from *dia*- and 'flow' from *-rrhea*) are both used in logarrhea, only the suffix, *-rrhea*, makes its way into the term *logarrhea*. Any attempts at a quick fix, such as *dialogarrhea*, *logadiarrhea*, or some other combination, simply don't work from a phonaesthetics perspective.

Looking more closely at the mathematics of the mistake currently associated with this term, one could argue that students are distributing the logarithm through the brackets, much in the same way that they were earlier taught to distribute both numbers and variables in front of a bracket, such as in  $2(x + 2) = 2(x) + 2(2)$  or in  $x(x + 3) = x^2 + 3x$ . If this is the case, then they are applying what they've already learned to a novel scenario. The only problem is that in this case, applying what they've learned doesn't work. In line with this reasoning, then, the root of the problem has less to do with logarithms and more to do with a misunderstanding of the notion of distribution or the distributive property.

Full disclosure: While isolating at home during COVID-19, I spent a lot of time attempting to rename logarrhea. I tried to, somehow, integrate the prefix *sinistr* / *sinistro*- (meaning left or left side) and *dextr* / *dextro*- (meaning right or on the right side), to no avail. I focus-grouped a bunch of terms only slightly different from logarrhea; however, *logarrhage* (utilizing the suffix *-rrhage*, meaning to burst forth) and *logarrhexis* (using the suffix *-rrhexis*, meaning rupture) remained ripe for the same critique I have just presented for the original term. And then, one day, on a long walk I took in an attempt to kill time, it hit me: As *Crystal Pepsi*, *New Coke*, and other infamous rebrands have taught us, don't mess with a classic. And so I realized, after much deliberation, that logarrhea must remain logarrhea.

As I continued walking, I also realized that my earlier critique regarding logarrhea might not hold as much water as I initially thought. Yes, logarrhea, which stems from *log*- (denoting logarithms) and *-rrhea* (denoting flow), does not integrate *dia*- (meaning through, throughout, or completely). However, the accepted term is logarrhea, and not *logrrhea* or *logorrhea*. In other words, and remembering that this is extremely amateur math ed morphology taking place here, it could be argued that the "a" between *log*- for logarithms and *-rrhea* for flow is actually a remnant of the *dia*- prefix. A piece of a prefix, if you will, which I'm sure has its own term. And there you have it: The word logarrhea stays, and there's good enough reason (in my mind) for it staying. Logarrhea, then, becomes the first of the named infamous math mistakes that stays. Classic!

### **Squaranoia:** $(x + y)^2 = x^2 + y^2$

Ask any upper-years math teacher: Squaranoia is a thing. Initially, they might not know what you're talking about, because squaranoia is also known as *brackephobia*, or *brackaphobia* (portmanteaus of 'bracket' and 'phobia'). Both terms work, and both have a rather nice ring to them. However, as we dig a little deeper, we'll see that perhaps, it's time for a change.

There are several issues with the term *squaranoia*. To the best of my knowledge, *squaranoia* is rooted in the term *paranoia*. In a general sense, then, the notion of paranoia (i.e., anxiety- and fear-riddled thoughts that can lead to irrational decisions) is referring to what *should* happen versus what *does* happen when students are confronted with an expression such as  $(x + y)^2$ . Instead of FOILING (another issue for another time), which would result in  $x^2 + 2xy + y^2$ , students, apparently riddled with fear, instead apply the square to each of the terms

Ask any upper-years math teacher: Squaranoia, also known as *brackephobia*, or *brackaphobia*, is a thing.

in the brackets, resulting in  $x^2 + y^2$ . In other words, the term *squaranoia* implies a fear of properly squaring. Phonaesthetically speaking, *squaranoia* is pretty good, but from an extremely amateur mathematical morphology perspective, perhaps things could be even better.

*Paranoia*, to the best of my knowledge, is derived from the ancient Greek words *para* (irregular) and *nous* (thought, mind). As for *squaranoia*, the previously undocumented prefix of *squara-* would have to mean something along the lines of 'of or related to squares or squaring.' However, there's the issue of increasing powers. For example, when a student rewrites  $(x + y)^3$  as  $x^3 + y^3$ , is the student suffering from *cubanoia*? Similarly, then, students might also suffer from *quatranoia*, *quintanoia*, or *pentanoia*, and so on. However, as seen in each of the examples, no matter the exponent outside of the bracket, the error in question is essentially the same. In other words, *squaranoia*, *cubanoia*, *quatranoia*, and *pentanoia* are really one and the same issue, which is why *brackaphobia* or *brackephobia* is arguably a more accurate term (and a great term, phonaesthetically speaking). However, that doesn't mean we need to stop at *brackephobia*. Let's now look at things more closely from a morphological angle.

The term *phobia*, as you are undoubtedly aware, refers to an intense fear or aversion to something. Akin to how *arachnophobia* describes an intense fear of spiders (or *arachnids*), *brackephobia* would, similarly, indicate an intense fear of brackets. Now, *phobia* concurrently exists as a standalone word and as (what I'll call) a loose suffix, and because of this, many people take artistic license. For example, I have a phobia of being buried alive, and so I might say that I have *being-buried-alive-phobia* (in fact, the established term to describe this fear is *taphophobia*). To give another example, someone who is afraid of golf might think that they suffer from *golfophobia* (or *golfaphobia* or *golfephobia*), when they actually suffer from *golffophobia*. Based on the former and latter then, *brackephobia* or *brackaphobia* (or even *bracketphobia*) would work better as a general mistake descriptor than, say, *squaranoia* or *cubanoia*.

By changing terms, we no longer need to be concerned with the particular exponent outside the brackets, as the issue now lies with the brackets, and not the exponent. And the higher the power, the more brackets: For example,  $(x + y)^5$  is equivalent to  $(x + y)(x + y)(x + y)(x + y)(x + y)$ , which is akin to encountering more and more spiders, which should result in a greater degree of fear. At the same time, though, the mathematical errors resulting from ignorance of necessary brackets, leading to, for example,  $(x + y)^{10}$  being simplified to  $(x^{10} + y^{10})$ , would only freak someone out if they realized how many brackets are actually involved. And so, while there may be fear, even for me, associated with correctly simplifying an expression such as  $(x + y)(x + y)(x + y)(x + y)(x + y)(x + y)(x + y)(x + y)(x + y)(x + y)$ , perhaps there's an alternative that will address issues raised.

Without getting into the larger question of whether you can be afraid of something you don't know exists, I am recommending *brackephobia* over *squaranoia*. (I should point out that, yes, you could fear that a monster exists under your bed, but perhaps the notion of a monster under your bed had to be planted in your head before you could start worrying about Gary. And, yes, I guess that we do often fear the unknown). My point is that the notions of *paranoia* and *phobia* aren't the best descriptors of what is going on when a student makes the mistake that  $(x + y)^2 = x^2 + y^2$ . After all, many who make the error don't even know that  $(x + y)^2$  is equivalent to  $(x + y)(x + y)$ .

Essentially, what we need is a way to describe not a fear or a phobia, but rather what's happening when expanding an expression such as  $(x + y)^2$  (and, if possible, to use a medical



term to keep with the theme of diseases). I spent a considerable amount of time searching medical terms to describe the fear of not being aware of, or not understanding the unknown (e.g., *panphobia*), to no avail. Almost on the brink of having to accept *brackephobia*, while scanning one last time through a list of suffixes and prefixes, I found a few items that I had previously dismissed too quickly. With many terms available to build upon (for example, *brackets*, *parentheses*, *powers*, and *exponents*), with suffixes such as *-staxis* (dripping or trickling) and *-ptosis* (falling, downward placement), and with prefixes such as *cata-* (down) and *acr/acro-* (extremity or topmost), I found myself with renewed confidence in my ability to rename *squaranoia* and *brackephobia*.

Without further ado, I contend that *expoptosis* (rhymes with *halitosis*; the second *p* is silent), defined as the falling or downward placement of exponents, should replace *squaranoia* and *brackephobia* on the list of mathematical diseases. Two key elements are captured in the word *expoptosis*: First, for the reasons detailed above, the notions of phobia or paranoia are removed from the term. Second, the term describes what is actually taking place during the mistake. In other words, the exponent “drips” or “trickles down” to each of the terms inside the bracket.

As we’ll now see, a focus on the exponents will also be at play when I attempt to rename *sumonia*.

**Sumonia:**  $(a^x)^y = a^{(x+y)}$

*Sumonia* was a prevalent problem in the math classes that I used to teach, and I was able to get a peek behind the curtain of what some students might be thinking in the *sumonia* scenario thanks to a tutoring session I had one the same topic. Having just tore through the topic in class at school one day, I thought I would draw on the same material I had used earlier that day to “inform” my tutee that same evening. Not so fast, I would learn.

After a session about simplifying powers, I didn’t think twice about extending my lesson to what was presented in that particular textbook as the “power of a power rule.” Having just finished several examples involving adding and subtracting exponents (for example,  $a^4a^7 = a^{11}$ ), naturally (to me), I extended the session by asking the student to simplify  $(a^2)^4$ . And there it was, *sumonia*, in all its glory:  $(a^2)^4 = a^{(2+4)} = a^6$ . This time, however, in what many consider to be the highly coveted one-on-one setting, I was sure I’d be able to help my tutee deal with their mathematical disease. Again, not so fast.

In what many consider to be the highly coveted one-on-one setting, I was sure I’d be able to help my tutee deal with their mathematical disease. Not so fast.

My tutee had a pretty solid argument for why they did what they did. “Well,” they began, “when you have the same base, you add the exponents.” To this, I had no immediate retort. “There’s only one base in this question, which means that you add the exponents,” they continued. In response, I fumbled around with different bases to make a point. Flailing, I used different letters, different numbers, different symbols, and even pens from the pencil case on the table. Nothing landed. No matter what I did, my tutee could not see that  $(x^2)^3 = x^2x^2x^2 = x^{(2+2+2)}$ . Sure, they knew that  $x^2x^2x^2 = x^6$ , but they had known that from the beginning. The issue I was facing, then, was how to explain that  $(x^2)^3 = x^2x^2x^2$ . As is customary, I attempted to draw upon what they already knew. So, I asked them to explain what an expression such as  $c^4$  meant. They just sat there. With the tutee’s parents within earshot, uncomfortable with the silence that

was filling the room, I said, "Well, I know that you know that  $c^4$  is the same as  $c \cdot c \cdot c \cdot c$ ." My tutee replied, "Yeah, I never really got that." After another awkward pause on my part, I wrote exponents of 1 above the  $c$ 's to show that  $c^1 c^1 c^1 c^1 = c^4$ . He replied, "Yeah, I get the

I was clambering for how to explain something that, to me, was so obvious. After the session was over, I began to question the term *sumonia*.

whole imaginary 1 thing, but I just don't see why  $c^4$  is  $c \cdot c \cdot c \cdot c$ ." In my head, I was clambering for how to explain something that, to me, was so obvious. After the session was over and I skulked out of my tutee's house (cash in hand), I began to question the term *sumonia*.

I believe, and stand to be corrected, that the mathematical disease called *sumonia* is derived from the term *pneumonia*. Pneumonia, as we've become more acquainted with during the COVID-19 pandemic, is an infection of one (or worse, both) of the lungs caused by a bacteria, virus, or fungus. In

the corresponding mathematical mistake, one adds (or *sums*) the exponents, as opposed to multiplying them. *Summing* and *pneumonia*, then, becomes *sumonia*. Keeping with other names thus far, let's examine the quasi-medical morphology of *sumonia*. Speaking again as a morphology amateur, *pneumonia* is derived from *pneumon-* (lungs) and *-ia*. However, the term *sumonia* suggests that *pneumonia* is actually a combination of *pneu-*, which is not necessarily a prefix, and *-monia*. It should be pointed out that, yes, there are many words that end with *-onia*, but this is not, from what I've gathered, a medical suffix. As such, I contend that the term *sumonia* is ripe for replacement.

When looking to replace *sumonia*, it is important to call attention to the fact that the mistake is occurring at, for lack of a better descriptor, the level of exponents. As a result, I propose tacking on a medical prefix or a suffix to the word *exponentiation* (loosely defined as the mathematical operation involving raising a base to an exponent, which is really what should be going on when simplifying) to replace *sumonia*. When going through possibilities, I considered some adequate prefixes, such as *iso-* (same), *ite-* (resembling), and *peri-* (surrounding or around another), but two fixes immediately came to the forefront as leading contenders.

Before declaring *sumonia*'s potential replacement, I had to make a difficult decision between using the prefix *psuedo-* (false or fake) and the suffix *-oid* (resemblance to). *Pseudoexponentiation*—that is, fake exponentiation—works rather well because *pseudo-* is such a well-known prefix and, with respect to the mistake, it could be argued that fake exponentiation is just what is taking place. Another possible alternative, *exponentiatoid*, focuses not on the process but on the product that results from making the mistake. For example, should an individual mistakenly simplify  $(a^4)^2$  as  $a^6$ , then that person's *pseudoexponentiation* has resulted in an *exponentiatoid*. A little hard to pronounce at first, *expo-nen-she-a-she-oid* rolls off the tongue, eventually.

### Changing Terminology

To borrow a tired, but true phrase: Change is the only constant. During the time of COVID-19, for example, the term *social distancing* was replaced with *physical distancing* to stress the fact that social connections could and should still be maintained, even at a distance. Along a similar vein, and also during the time of COVID-19, in this article and in Chernoff (2020), I have proposed a rather radical renaming of mathematical diseases, mathematical diseases that are infamous enough to have their own monikers. Gone, but not forgotten, are the terms *sinustitis*, *functionitis*, *squaranoia*, and *sumonia*; in their place, respectively, I propose

that we adopt the more quasi-medically and quasi-mathematically accurate terms of *lateralparantheticsinucentesis*, *endoparentheticfunctionostomy*, *expoptosis*, and *exponentiatoid*. Of course, as we all know, change begets change, which leads us to shine the spotlight on the notion of *mathematical diseases* itself.

As I also recently argued, the diseases on the list of infamous *mathematical diseases* should, rather, be called *mathematical conditions*. First, as our society increasingly strives to use more precise and more inclusive (or, as some might say, “politically correct”) language, “the days of declaring that students are riddled with various mathematical diseases are probably over” (Chernoff, 2020, p. 43). Second, parsing diseases and conditions can lead us to consider the notions of mathematical symptoms and syndromes (i.e., a set of signs and symptoms, correlated with each other, and often associated with a particular disease or disorder).

Long, long ago, when the poster of mathematical diseases hung prominently on the front wall of my math classroom, I would not hesitate to let a student that they were riddled with a mathematical disease such as, *squaranoia*, *logarrhea*, *sinusitis*, *functionitis*, *cancellitis*, *sumonia*, *rootobia*, *negativitis*, and *moveitis*. Something about the disease angle landed with students, and once I pointed it out, they often never made the mistake again. Some would laugh, but not all, which brings me to present day. I would not recommend singling a student out in your class and telling them they are riddled with a mathematical disease. Things are different now. Alternatively, by embracing new terms, as mathematical conditions and not diseases, you could let a student know that *lateralparantheticsinucentesis* and *endoparentheticfunctionostomy* are mathematical conditions—abnormal states of mathematical health that interfere with usual mathematical activities like simplifying. How far you wish to get into parsing mathematical diseases, symptoms, conditions, disorders, and syndromes—for example, the *paracentesis* syndrome (i.e., the puncturing and draining of brackets) can be found at the root of many mathematical conditions including (but not restricted to) *logarrhea*, *sinusitis* and *functionitis*—is a matter how far you wish to take this COVID-19-inspired silliness into your math class.

## References

Chernoff, E. J. (2020, Summer/Fall). Reducing Inflammation. *The Variable: An SMTS Periodical*, 5(2), 41-44.



Egan J Chernoff (@MatthewMaddux) is a [Professor of Mathematics Education](#), Department of Curriculum Studies and an Associate Member, Department of Educational Administration in the College of Education at the University of Saskatchewan (U of S). He is also an Associate Member, Department of Mathematics and Statistics in the College of Arts and Science at the U of S, and an Adjunct Professor, Faculty of Education at Simon Fraser University (SFU).

# Call for Contributions

*The Variable* is looking for contributions from all members of the mathematics education community, including classroom teachers, consultants, and teacher educators. Consider sharing a favorite lesson, an essay, a book review, or any other work of interest to mathematics teachers in Saskatchewan. Articles may be written in English or French. If accepted for publication, your article will be shared with a wide audience of mathematics educators in Saskatchewan and beyond.

We also welcome student contributions in the form of artwork, stories, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom, and for teachers to recognize students' efforts during their journey of learning mathematics.

All work is published under a Creative Commons license. To submit or propose an article, please contact us at [thevariable@smts.ca](mailto:thevariable@smts.ca). We look forward to hearing from you!

*Ilona & Nat,  
Editors*









