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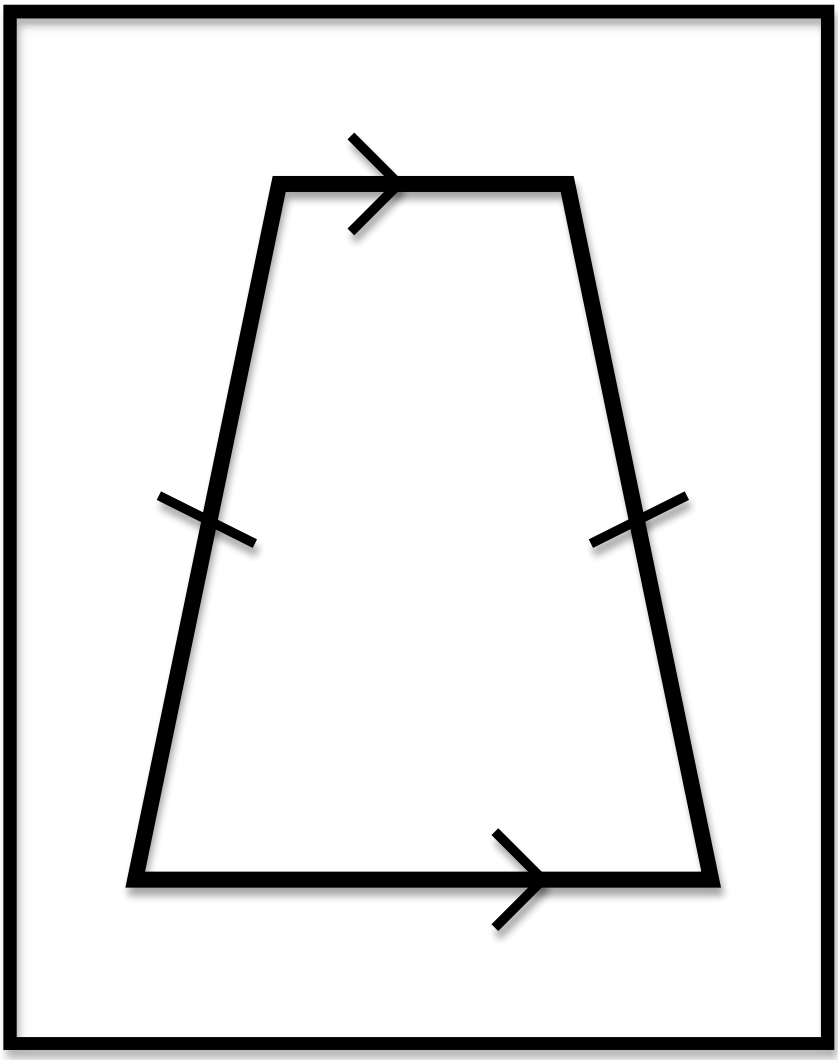
Journal of the Saskatchewan Mathematics Teachers' Society
Volume 1, Number 1 (February 2009)



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Journal of the Saskatchewan Mathematics Teachers' Society

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SMTS objectives—as outlined in the January 1979 SMTS Newsletter—include:

1. To improve practice in mathematics by increasing members' knowledge and understanding.
2. To act as a clearinghouse for ideas and as a source of information of trends and new ideas.
3. To furnish recommendations and advice to the STF executive and to its committees on matters affecting mathematics.

Vinculum's main objective is to provide a venue for SMTS objectives, as mentioned above, to be met. Given the wide range of parties interested in the teaching and learning of mathematics, we invite submissions for consideration from *any persons interested in the teaching and learning of mathematics*. However, and as always, we encourage Saskatchewan's teachers of mathematics as our main contributors. *Vinculum*, which is published twice a year (in February and October) by the Saskatchewan Teachers' Federation, accepts both full-length **Articles** and (a wide range of) shorter **Conversations**. Contributions must be submitted to egan.chernoff@usask.ca by January 1 and September 1 for inclusion in the February and October issues, respectively.

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EDITORIAL: CHANGE(S)

Egan Chernoff

Stated as a proverb, change is the only constant. However, and without getting into a discussion of the derivative, how much change is occurring at a particular point may vary. In other words, while change is the only constant, rate of change is not, necessarily, constant. For example, points in time may experience more or less change than other points in time. At the present point in time, and with respect to the teaching and learning of mathematics in the province of Saskatchewan, we are in the midst of major change.

Saskatchewan's recent adoption of WNCP (Western and Northern Canadian Protocol for collaboration in education) Mathematics' Common Curriculum Framework has introduced new mathematics curricula to the province for grades K-9. Further, the province will adopt new mathematics curricula for grades 10, 11, and 12 in 2010, 2011, and 2012, respectively. Major change to mathematics curricula brings change to related domains (e.g., textbooks, lesson plans, university entrance requirements, etc.). However, and perhaps more importantly, adoptions of the new K-12 curricula this time will, arguably, introduce a new approach to the teaching and learning of mathematics for elementary and secondary school mathematics classrooms.

With the overwhelming change (detailed above) ahead of us, and not wanting to recreate a Western Canadian, nor Saskatchewan, version of the "Math Wars" occurring in the United States, our society, the Saskatchewan Mathematics Teachers' Society (SMTS), is committed to a smooth transition. Our newly elected President, Stephen Vincent, details in his column **PRESIDENT'S POINT** (p. 3) how the SMTS and its executive (also in the midst of major change) is proactively dealing with the change that is and will be occurring.

Change is also occurring here at the Journal of the Saskatchewan Mathematics Teachers' Society. On behalf of the SMTS members, I want to thank Jennifer Von Sprecken, former Editor, for her tireless efforts with *The Numerator*. Personally, I want to thank Jennifer for (1) a smooth transition, and (2) unlimited access to all things related to *The Numerator*. Further, I want to welcome Karen Campbell, Ryan Banow, and Cynthia Sprung as Associate Editors of the journal. Their knowledge, insight, and direction—displayed in our conversations over the past few months—has demonstrated our journal has a strong Editorial Board, further supported by Evan Cole, a member of our (ever growing) Editorial Advisory Board.

Perhaps the most obvious change to our journal is the name: *The Numerator* is now *vinculum*. Vinculum is defined as "a horizontal line placed above multiple quantities to indicate they form a unit" (Weissten, 2009). As such, and as demonstrated on our new cover, *vinculum* (i.e., our journal) symbolizes the horizontal line placed above multiple quantities (i.e., members of the SMTS) to help form a 'unit' (i.e., the SMTS), which, as mentioned, is important in this time of change(s).

References

Weisstein, E. W. (2009) Vinculum. From *MathWorld*—A Wolfram Web Resource. [http://mathworld.wolfram.com/Vinculum.html].

PRESIDENT'S POINT

Stephen Vincent

Increase students' interest and thinking abilities in mathematics. This was a recurring theme of the SMTS as discussed at a recent executive meeting. How are we going to do that? By promoting excellence in the teaching and learning of mathematics. Let me explain.

The SMTS exists to foster excellence in mathematics education in elementary, middle, secondary, and post-secondary education institutions. Practically, this is accomplished by the in-service of teachers in mathematical trends and curricular developments through conferences (such as math conferences and Sciematics), our journal (*Vinculum*), our website (www.smts.ca), and partnering with universities, the Ministry of Education, and the National Council of Teachers of Mathematics. We also provide math enrichment opportunities for students (such as Math Challenge and other experiences).

January 2009 finds us in the middle of one of the largest curriculum renewals in the history of Saskatchewan. We need to take advantage of this unique opportunity to improve the quality of math instruction which will in turn lead to increased student engagement and reasoning. New developments for the SMTS this year include the Saskatchewan Understands Math (SUM) conference held May 8th and 9th in Saskatoon to focus on best practice from K-12. Additionally, our website will undergo a radical change so that it will be an interactive resource and sounding board specifically for Saskatchewan teachers.

Also, this year we are welcoming four new members to our executive. It appears as if the chocolate fondue party at Sciematics was successful in luring in new members. We wish to formally welcome Egan Chernoff (U of S professor), Evan Cole (teacher in Saskatoon), Ryan Banow (teacher in Humboldt) and Cynthia Sprung (teacher in Saskatoon) to the executive committee. We are excited for the vision and innovative ideas these new members bring.

On behalf of the committee, I would also wish to thank former executive members for their years of service. Janet Christ (teacher in Saskatoon and former VP), Cam Milner (teacher in Saskatoon and former Director), Jennifer Von Sprecken (teacher in Estevan and former Editor) and Christina Fonstad (teacher in Prince Albert and former Treasurer) have decided to take a break from the SMTS. We are thankful for all their hard work over the years. Although they will be missed, we greatly appreciate that they are still willing to help out and be consulted as necessary. Further, Karen Campbell deserves recognition as the president of SMTS for the last four years. Karen provided leadership for the executive in an effective and visionary manner. We are grateful that Karen has decided to stay on the executive as past president.

We look forward to the next few years of mathematics education in Saskatchewan, giving our students the best opportunity to learn and be engaged in math. As a volunteer executive, we do our best to foster excellence in math. If you have any ideas, thoughts, questions, or would like to help plan a particular event, please contact me at vincents@spsd.sk.ca. We would love to hear from you. Happy teaching!

MATHEMATICAL SELF-EFFICACY: HOW CONSTRUCTIVIST PHILOSOPHIES IMPROVE SELF-EFFICACY

Susan Wilson

Middle years can to be a turning point in a child's math education; they decide that they either "get" math or they don't; those that do continue on to take high level math electives while those that don't struggle to meet basic graduation requirements in math. This paper suggests that mathematical self-efficacy can be maintained or improved by using constructivist pedagogy instead of the traditional, teacher-centered pedagogy that has been common in many elementary and middle years math classrooms.

By the time students reach middle years (grades 6 to 9), many have developed generalizations about their learning capabilities; "I suck at Math" and its various equivalents are comments familiar to middle years teachers. What transformation occurs between the time a child enters kindergarten, ready to learn anything and everything with zeal, to middle years when they have concluded that they either "get" math or they don't? Children enter school with a powerful urge to find out about things, to figure things out; they question, play, solve puzzles and riddles (Saskatchewan Education, 1994) and they generally believe that they can succeed in school. Could a shift from traditional, teacher-centered philosophies to constructivist philosophies improve, or at least maintain, students' feelings of self-efficacy in mathematics?

According to Bandura (1994), perceived self-efficacy relates to "people's beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect their lives. Self-efficacy beliefs determine how people feel, think, motivate themselves and behave" (para. 1). Bandura elaborates, positing that positive feelings of self-efficacy enhance achievement, assure capabilities, foster intrinsic motivation, and enable learners to set challenging goals and to be committed to them. Failure is seen as avoidable and if it occurs, it can be overcome; the capability is there. Negative feelings of self-efficacy cause learners to avoid challenges, commit weakly, focus on deficiencies and obstacles and prepare for adverse outcomes. Hall and Ponton (2002), believe that "past experiences, often times failures, in mathematics usually dictate student opinions concerning their perception of their ability in mathematics [and that] ...educators...themselves should implement modes of instruction that develop and enhance self-efficacy" (p. 10). They suggest that enhancing mathematics self-efficacy should be a focus of mathematics educators and that this can be done by providing positive experiences for students.

Self-efficacy is a significant factor in determining mathematics achievement but other factors such as self-concept, metacognitive experiences, and level of engagement play a role as well. Self-efficacy differs from self-concept in that it is related to a specific domain whereas self-concept is generally more pervasive (Bandura, 1994). It differs from metacognition as well as metacognition involves beliefs about how one learns (Flavel, 1979).

Educators must provide positive experiences that both engage students and support them in succeeding. If a student expends effort completing a challenging mathematical task and they are successful, then they add to their perceived self-efficacy, but if they put in a lot of effort and they are not successful, then they will harm their feelings of self-efficacy. Bandura points out that "the most effective way of

creating a strong sense of efficacy is through mastery experiences. Successes build a robust belief in one's personal efficacy. Failures undermine it, especially if failures occur before a sense of efficacy is firmly established" (1994, para. 4). The implementation of a constructivist learning philosophy can engage students, motivating them to expend effort and provide supports necessary to attain success thus improving the perceived self-efficacy of students.

Constructivist philosophies are based on learners constructing meaning, both individually and socially, through their interpretations of world experiences (Jonassen, 1999). "A constructivist learning environment promoting community development fosters a social context in which all members, both students and teachers, are participants in the learning process" (Lock, 2007, p. 131). Jonassen's (1999) model for designing constructivist learning environments includes an ill-defined problem or project with a variety of interpretive and intellectual learner supports such as coaching, modeling, and scaffolding.

In *Designing constructivist learning environments* (1999), Jonassen identifies necessary steps to creating constructivist learning environments. The first priority is to present a question, issue, case, problem or project that is interesting, authentic and relevant to the student; one that affords definition and ownership by the learner. Access to related cases and informational resources must be present for the learner as they frame the problem in their own zone of understanding; "without ownership of the problem, learners are less motivated to solve or resolve it" (Jonassen, 1999, p. 219). The problems have unstated goals and solutions follow unpredictable processes. They may have multiple solutions or no real solution at all. Students are required to make judgments about the problem and to defend their decisions.

Replacing the teacher as *sage on the stage* and even as a learning facilitator is the role of the teacher as a coach or simply, as Vygotsky might frame it, a *more knowledgeable other*. The shift from instructor to coach can alleviate the pressure a student may feel to find that one right answer using the teacher's favourite algorithm. The role of a coach is to monitor for success, not watch for mistakes. This shift in pedagogy enables students to engage more freely and to think more creatively. In the absence of the teacher as the all-knowing authority who instils knowledge, children will be motivated to experiment with and explore their learning without the feeling that they will be judged to be right or wrong.

Jonassen (1999) describes teacher or peer coaches as motivators who analyze performance and provide feedback for improvement and opportunities for reflection. The notion of a learning facilitator brings to mind a person who can make the learning easier. Coaches do not make the learning easier; they identify and work with the learner on necessary skills that will enable the learner to succeed at the overarching problem, case or project. Coaches are also to perturb learners' cognitive models as "the mental models that naive learners build to represent problems are often flawed" (p. 234). Just as a coach watches an athlete's performance to assess their strengths, weaknesses and understandings, teachers as coaches must question their students understanding so that misconceptions can be investigated and cleared. Such practices will also promote children to actively construct metacognitive experiences involving self-analysis of learning and self-reflection of process on their own.

To help ensure the success of the learner, Jonassen's (1999) model for designing constructivist learning environments also involves scaffolding as a systematic

approach to support the learner. Scaffolding provides support when the student needs it, where they need it, and only for as long as they need it. Professional and peer tutors provide supports to bridge learner's existing knowledge and skills with those required in the demands of the new mathematical task.

It refers to any type of cognitive support that helps learners who are experiencing difficulty by adjusting the difficulty of the task, restructuring the task to supplant knowledge, and providing alternative assessment to help the learner identify key strategies. Scaffolding may include providing direct instructions and help in the context of the learning activity or engaging the learner in guided participation in similar situations. Scaffolding differs from coaching as it focuses on the task, the environment, the teacher and the learner instead of on the learner's performance. Scaffolds are not permanent however. Effective scaffolding transfers the responsibility of learning and performing from the teacher or more knowledgeable other to the student.

Bandura (1994) identifies social modeling as another method of strengthening beliefs of efficacy. By watching a model (coach, peer; not an expert) perform tasks similar to those expected of the learner, the learner sees the task as possible. "Seeing people similar to oneself succeed by sustained effort raises observers' beliefs that they too possess the capabilities to master comparable activities to succeed" (Bandura, 1994, para. 6). It is important that the students see teachers as learners as well and that the individuals doing the modeling are skilled. Observing the failure of an instructional model will have a negative effect on a learner's feelings of self-efficacy. Heterogeneously grouped students functioning in a cooperative learning community can serve as peer models for each other demonstrating skills and processes in their areas of strength. The more closely the learner identifies with the model, the greater the impact of the model's success or failure on the learner's perceived self-efficacy. Working in a community of practice frees students to learn with and from others; both from those of their same age or ability level and from those at different ages and maturity levels.

Instructional leaders provide behavioural modeling of overt performance and cognitive modeling of covert processes. It is important that activities be modeled by skilled practitioners to provide example of desired performance. As they work through the process, the more knowledgeable other; teacher, student, or other practitioner, should articulate their thought processes, problem solving procedures and reflection so that they can be analysed and understood by the learner. This will also provide example and opportunity for reflection on learning processes that may provide important metacognitive experiences for the students.

Constructivist learning experiences can be presented in a variety of other ways. What is important is that learners are engaged and supported so that they can achieve success and increase their positive feelings of self-efficacy. This is not to say that success should come easily to the students. As Bandura (1994) points out, learners experiencing only easy success will come to expect quick results and will be easily discouraged by failure. When Blumenfeld, Krajcik, Marx and Soloway (1994) presented a teaching-enhancement activity to adult learners; they found that learners expecting to be given the correct answers and processes become passive in their learning. It is important that students be given engaging and challenging learning activities that require critical thought and that they be supported in their knowledge construction. As feelings of self-efficacy improve, students will persevere and will be less affected by setbacks or failures. The

presence of engaging problems and structured supports enable teachers and learners to verbalize their confidence in each other. "It is more difficult to instill high beliefs of personal efficacy by social persuasion alone than to undermine it" (Bandura, 1994, para. 9).

Another significant constructivist ideal that helps develop mathematics self-efficacy is that of cooperative and collaborative group work. As Vygotsky stated, "What a child can do with assistance today she will be able to do by herself tomorrow" (Vygotsky, 1978, p. 87). Cooperative learning experiences involve students working together on a common goal, helping and supporting each other through the knowledge construction process. Knowing that group members "sink or swim" together motivates students to engage, collaborate, and be active learners.

It is important that teachers are skilled at creating and maintaining cooperative learning groups in their classrooms. Pseudo (members assigned to work together with no interest in collaboration) and traditional classroom groups (members who accept that they must work together but see little benefit) will not perform any better or possibly worse than learners working individually (Johnson & Johnson, 1998). Cooperative learning groups consist of members committed to a common goal. They support each other in learning, taking responsibility and accepting accountability for themselves as individuals, for team mates as individuals, and for the group as an entity. "A truly committed cooperative learning group is probably the most productive instructional tool teachers have at their disposal, provided that teachers know what cooperative efforts are and have the discipline to structure them in a systematic way" (Johnson & Johnson, 1998, p. 96).

To improve mathematical self efficacy, educators need to embrace constructivist methodologies involving cooperative and collaborative learning communities. Cooperative learning groups, used in conjunction with constructivist methodologies would result in problem, project, or case-based learning opportunities that engage students. The teacher would function as a coach providing positive social persuasion, lessons and practice opportunity on necessary skills, constructive feedback and encouragement while monitoring student progress and achievement. Teachers and peer members of cooperative learning groups would serve as behavioural and cognitive models. Technology, print resources, adaptations, questioning and worked examples would be used to scaffold learning. Students would expend the necessary effort and would have a probable chance at success. The more realistic success a child achieves, the greater beliefs they have in their own self-efficacy; the greater the feelings of self-efficacy, the more probable their chance of success.

References

Bandura, Albert. (1994). Self-efficacy. In V. S. Ramachaudran (Ed.), *Encyclopedia of human behavior* (Vol. 4, pp. 71-81). New York: Academic Press. (Reprinted in H. Friedman [Ed.], *Encyclopedia of mental health*. San Diego: Academic Press, 1998). Retrieved June 24, 2008 from http://www.des.emory.edu/mfp/BanEncy.html

Blumenfeld, P.C., Krajcik, J.S., Marx, R.W., & Soloway, E. (1994). Lessons learned: a collaborative model for helping teachers learn project based instruction. *Elementary School Journal*. *94*(5). 539-551.

Hall, M. & Ponton, M. (2002). A comparitive analysis of mathematics self-efficacy of developmental and non-

developmental freshman mathematics students. Presented by Mike Hall at the 2002 Meeting of Louisiana/Mississippi Section of the Mathematics Association of America. Retrieved June 23, 2008 from http://www.mc.edu/campus/users/travis/maa/proceedings/spring2002/michael.hall.michael.ponton.pdf

Johnson, D.W., Johnson, R.T. (1998). Chapter 3: Basic Elements of Cooperative Learning. In: *Learning Together and Alone: Cooperative, Competitive, and Individualistic Learning*. Allyn & Bacon.

Jonassen, D. (1999). Designing constructivist learning environments. In C. Reigeluth (Ed.), *Instructional-design theories and models: a new paradigm of instructional theory*, Vol.2. Mahwah, NJ: Lawrence Erlbaum Associates.

Lock, J.V. (2007). Laying the groundwork for the development of learning communities p. 130 to 133. *Online Learning Communities* edited by Rocci Luppicini. Information Age Publishing Inc. Charlotte North Carolina. Retrieved June 25, 2008 from http://tinyurl.com/4sb485

Saskatchewan Education. (April 1994). *Children First: A Curriculum Guide for Kindergarten.* Curriculum and Instruction Branch. Retrieved June 23, 2008 from http://www.sasked.gov.sk.ca/evergreen/kindergarten/index.html

Vygotsky, L. (1978). Interaction between Learning and Development. In *Mind in Society* (pp. 79-91). (Trans. M. Cole). Cambridge, MA: Harvard University Press. Retrieved June 26, 2008 from http://www.comnet.ca/~pballan/Vygotsky(1978).htm

BECOMING PROBLEM SOLVERS

Grayson H. Wheatley

Even with the excellent direction provided by the National Council of Teachers of Mathematics in the *Principles and Standards for School Mathematics*, (NCTM, 2000), there is still much controversy about what type of materials should be used in mathematics classes as well as which instructional strategies are effective. In the vast majority of middle school mathematics classes there is an emphasis on practicing algorithms demonstrated by the teacher and shown in the text. It is my belief that such a procedurally oriented approach dominated by teacher explanation is ineffective. This article describes the positive results from using a contrasting approach.

During the summer of 2008, a three-week program for more able mathematics students was offered in the Midwest. Forty-nine students were enrolled in the program. The students were entering grade seven in the following school year. This program had been offered for more than 20 years, so there was considerable experience to draw upon. The goal of the program was to enhance students' mathematics knowledge in the areas of problem solving, algebra, geometry, spatial reasoning, proportional reasoning, and rational number arithmetic. A conscious effort was made to avoid material they might experience in their schools. Most of the activities were written by the author. The students came from eight school systems in a 40 mile radius. In this region, there were programs for highly able students available and thus they were not in this program. In most cases, students were recommended by their teachers. The students selected for this program can best be described as more able learners but not exceptional students. Some kids who were enrolled by their parents were reluctant at first but quickly became engaged. After a few days, Matt said, "This is neat!"

The program

The 49 students were organized into three classrooms with two teachers per room. Each three-hour morning was partitioned into four periods, one was always a 45 minute time in the computer lab. The other three periods featured, problem solving, spatial activities, and algebra with lessons on fractions, decimals and percents interspersed, usually in a problem-based setting. Many of the activities were taken from *Developing Mathematical Fluency* (Wheatley & Abshire, 2002).

Instructional strategy

The primary method of instruction was Problem Centered Learning (Wheatley, 1991). In this method, the students were arranged in pairs of like ability and, without much prior discussion, a task was presented for the pairs to complete/solve. After about 20 minutes, the students came together as a group and presented their solutions to the class for validation. The teacher served as a facilitator and usually determined the order of pairs to present their work but did not offer opinions about the ideas presented. The class was encouraged to ask questions and express agreement or disagreement. Usually more than one method was presented. The goal was to reach consensus, which was not always possible. Under this method, there is the opportunity for students to make sense of the problem/task and to learn of other approaches. There were learning opportunities first in the pairs and then in the large group discussion.

The staff

The staff consisted of seven teachers plus a coordinator. The teachers were formed into three teams of two with one teacher assigned to the computer lab. Thus, each class had two teachers, usually one experienced and one new to the program. The teachers in this program varied greatly in years of experience from 27 years to beginners. Some of the teachers had been working with the program for a number of years. The program director wanted this to be a learning experience for teachers as well as students. Thus the staff met in the mornings for a week prior to the students arriving. These sessions were led by the author, who has been a part of this program from its inception more than 20 years ago. In the sessions, the Problem Centered Learning Instructional teaching method was discussed and utilized. Particular problems and activities were explored and discussed. In addition to thinking about instruction, teachers were engaged in doing mathematics. Possible activities to be used were discussed. Teachers decided which problems and activities they would use with their class. Many problem sets and activity sheets were available for teachers to select from; there was considerable flexibility for them to choose particular problems and activities. Just as students must construct knowledge for themselves, so must teachers.

The computer lab component

Students had 45-minutes in the computer lab each day. Over the three weeks of the program, two software packages were used: Battista's (2003) *Shape Makers* and the author's *The Distance Game*. Shape Makers is designed to help students develop their knowledge of geometric figures and their properties. We chose to focus on quadrilaterals. *The Distance Game* develops knowledge of plotting points in the coordinate plane, strategic reasoning and the Pythagorean theorem. Normally we had students working two at one computer, even though we had enough computers for each student to work alone. The students were especially enthusiastic about the computer activities. For several years, students have cited *The Distance Game* as their most enjoyable activity of the summer program.

Homework

Each day students left with a challenging mathematics problem to solve. They were encouraged to talk with parents and others about the problem. Most days began with a discussion of the homework problem. Here is one of the problems used. The next day the discussion was usually the first activity.

> During the census a man told the census-taker that he had three children. When asked their ages he replied, "The product of their ages is seventy-two. The sum of their ages is my house number." The census-taker turned, ran outside to look at the house number displayed over the door. He then reentered the house and said, "Using the information you have given me, I cannot tell their ages." The man then said, "I should have told you that the oldest likes angel food cake." Hearing this, the census-taker promptly wrote down the ages of the three children. What did he write?

Data collection

During the first day, students took a 30-minute test of problem solving consisting of seven problems. The test has been refined for use over several years in the program and the characteristics of the tasks are well known. A question from the pretest is shown below:

> A painting measuring 47 inches by 63 inches has a two-inch wide frame around it. If a string is put completely around the frame, how long would it be?

The author graded all the test papers. On the next to last day of the three-week program a posttest that paralleled the pretest was given. That is, the items had the same deep structure and similar difficulty. Again the author scored all papers. Each test item was scored using a rubric of 0 - 3 with zero for no response, a one for some meaningful work, a two if there was work that could lead to a solution but was incomplete or had a computational error. A three was given if the answer was all correct.

Results

The mean score on the pretest was 8.7 out of 21. The mean score on the posttest was 13.4 out of 21. This represents a 54% increase in scores from pre- to posttest. As the scores show, the tests were challenging. While the tests consisted of non-routine problems, good number sense, mathematics knowledge and reasoning was needed to be successful.

Summary and conclusions

During a three-week summer program for more able rising seventh grade students, a mathematics program was offered in the mornings with the goal of enhancing mathematics knowledge, especially mathematical reasoning. There was an emphasis on number sense and mathematical reasoning. The primary instructional strategy used was Problem Centered Learning, which encourages students to become active learners and comfortable being challenged with tasks/problems where no solution method had been demonstrated.

It should be recognized that classes were small and that the teacher-pupil ratio was higher than might be found in a typical public school. Nevertheless, this study demonstrates that with appropriate activities and instructional approaches, impressive gains in mathematics problem solving can be obtained. At other sites (public school), using similar activities and the same instructional approach, impressive gains have been obtained.

References

Battista, M. T. (2003). *Shape Makers: Developing Geometric Reasoning in the Middle School with the Geometer's Sketchpad*. Berkeley, CA: Key Curriculum Press.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Wheatley, G. H. (1991). Constructivist perspectives on science and mathematics learning. *Science Education 75*(1). 9-21.

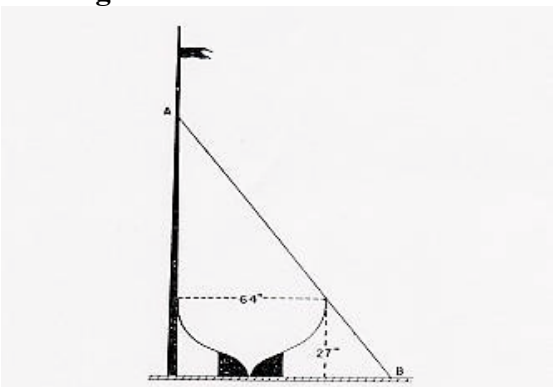
Wheatley, G. H., & Abshire, G. (2002). *Developing Mathematical Fluency*. Tallahassee, FL: Mathematics Learning.

LOOKING BACK

Rick Seaman

After a 25-year career teaching mathematics in grades eight to twelve, it is interesting to look back at the pedagogic moments that led to my return to university to gain a deeper understanding about teaching mathematics: That is, beyond what I had learned from reading, attending professional development seminars, preparing lessons, teaching, reflecting on my teaching.

As a high school senior



46. The lifeboat of a sailing vessel has a 64-inch beam and a 27-inch draught. When not in use, it sits at the foot of a mast. What is the shortest guy-wire, AB, which will just clear the lifeboat?

I can remember thinking on the first day of twelfth-grade algebra class that if I could solve problem 46 on page 333 near the end of the textbook (Petrie, Baker, Levitt & MacLean, 1946), there would be no reason for me to take the class. Successfully solving this problem would mean that I *must* know everything in the text leading up to the problem! Otherwise, how else would I be successful? I did not really believe this hypothesis but it was still fun trying to solve the problem. Little did I realize at the time that it was an optimization problem that required some knowledge of calculus and there was a lot to learn before I could successfully solve the problem.

While sitting in my desk I used to wonder about who these 'interns' were that both visited our class and taught us mathematics. Little did I know that I was to become one of these 'interns' teaching mathematics.

Internship, mathematics education classes, and graduation

"In all, I am very pleased with Rick and confident that Rick will make an excellent teacher and has a positive contribution to make to education." Those were the words of my cooperating teacher after my three-month internship. With my Master of Arts specializing in mathematics, and a Bachelor of Education with a major in mathematics and a minor in physics completed, I was looking for a teaching position.

I recall while attending one of my mathematics education classes there was a book we were asked to purchase that we never got around to reading or discussing in class. This book was George Pólya's *How to Solve It* (1957) a book about problem solving and the strategies used by mathematicians to solve problems. At the time this book sold for $2.25, but as the commercial on television would say today, I now consider it priceless. Mathematician and mathematics educator Alan Schoenfeld also recognized the book's value when on page xi in the preface of his book *Mathematical Problem Solving* (1985) he asked, "Why wasn't I given the book when I was a freshman, to save me the trouble of discovering the strategies on my own?" So what did I do with Pólya's book when I graduated? I tucked it away in my personal collection of math books [Math students rarely throw away their math books] and promptly forgot about it!

I got the job, the first ten years

I began to teach in the 1970s and I remember teaching what might be characterized as knowledge based skills such as solving equations, factoring polynomials, and the like. If there was any problem solving or thinking, it had to wait until the end of the chapter where those sections were labeled with surface features such as work, mixture, or uniform-motion problems. I confess that I rarely taught strategies to help students with their problem solving.

Roughly ten years later I reacquainted myself with *How to Solve It*, but this time I actually opened it up and started to read it. I wished that I would have read it earlier in my teaching career but in hindsight I am not convinced that I would have pedagogically appreciated the book enough. After studying the book I decided to help students with their problem solving by discussing Pólya's "How to Solve It" list/problem-solving model with them. However, I still sensed something was still missing.

The second ten years

A few years later, I was asked to introduce an internationally recognized mathematics curriculum in the high school where I was teaching. This program included a provincial high school mathematics curriculum, which was supplemented by many first and second year university topics in mathematics. As you can imagine, time became a major factor in covering this curriculum. To offset this problem, I applied Saxon's (1982) idea of incrementally developing concepts. This allowed for a concept to be presented and practiced for more than one homework set before the next facet of the concept was introduced. In this way, students worked on concepts over a longer period of time to help them retain their understanding of these concepts. In grade twelve the calculus portion of this curriculum became a partial review of the Saskatchewan high school mathematics curriculum [Which is another story]. These students still needed to pass their regular mathematics classes to graduate provincially but wrote separate comprehensive exams to graduate internationally.

Closer examination of Saxon's (1984) *Algebra II* text revealed that each lesson and corresponding assignment could contain both knowledge-based questions and problems to solve. I decided that this would be a good way to avoid the artificiality of problems being placed at the end of the chapter while also improving the students' problem-solving skills. I decided I would try to incrementally develop concepts and assignments as Saxon had done and teach problem solving and a supporting knowledge base every class.

Around this time I read an article written by Mayer (1985) that described a cognitive strategy similar to Pólya's that was made up of two phases: problem representation and problem solution. After reading the writings of Mayer (1985), Pólya (1957), Saxon (1982, 1984), and Schoenfeld (1985) I decided to teach and assign knowledge base skills and problem solving daily under the comprehensive umbrella of a cognitive strategy (see Figure 1).

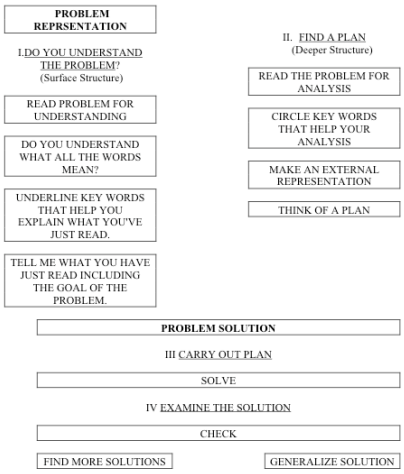


Figure 1. Cognitive Strategy

The last five years

Then as suggested in an article I read by Montague (1992) at the beginning of the term I had the grade nine students I taught memorize a cognitive strategy synthesized from reading Mayer (1985) and Pólya (1957). This provided the students with "hooks" for the problem solving process as they attempted to solve mathematical problems. Whenever a student was having trouble solving a problem, cognitive strategy gave me an entry point to answering their question. I would initially ask them to "tell me what they had just read including the goal of the problem" before I would respond. I would then make suggestions and use the cognitive strategy to guide the student through the remainder of the problem solving process. Because of the self-questioning strategies inherent in the cognitive strategy, students gained control over their thinking (Schoenfeld, 1985), utilized more representational strategies (Montague & Appelgate, 1993a, 1993b) and also slowed down their thinking process by reading for understanding before reading for analysis. Yes this was frustrating for some of the students!

To gain further insights into the students' understanding of the problem solving process I would give them an open-ended activity where they were asked the following question, "You have just been chosen to write an article in a national supermarket tabloid. Your headline for the article is: 'What my cognitive strategy for problem solving has taught me and what it means to me personally?'" A typical response was:

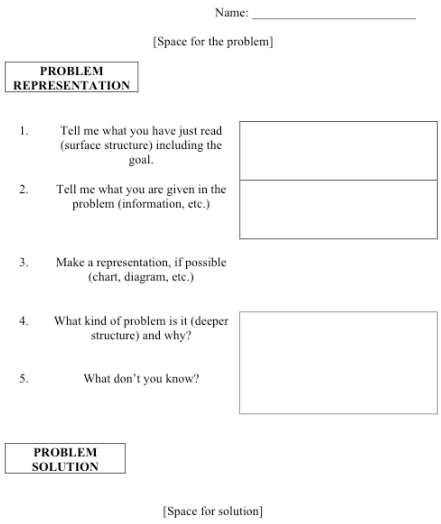
> Problem solving is very important in life. Without it, we wouldn't be where we are today. It has taught me to slow down my thinking. Before I hardly read the problem before I started calculating numbers, which never came out to be the right answer. Now, I've slowed down and underline, circle, and find the surface structure (what the problem is all about) before I even begin to analyze. Now I actually get the correct answers without having to ask someone. I think this theory will really help me in life. Not only in mathematics, but everyday life, other subjects and other things I usually wouldn't have the answers to. Maybe someday I'll be able to solve one of the world's great mysteries! [Student's response to the tabloid question]

I found that the more students were asked to classify a problem the more it became evident that they didn't classify a problem according to what was most influential in helping them solve the problem (deeper structure). Their categorizations contained superficial features such as question form, contextual details and quantity measured (Gliner 1989; Silver, 1977, 1979). These students, who made decisions based on surface features, were then instructed to perceive problems on the basis of their deeper structure (Schoenfeld & Hermann, 1982). In order to reinforce their choice of deeper structure, they had to justify their decision (Hutchinson, 1986, 1993), while acknowledging that deeper structures are

not unique for each problem but depend on how one 'chunks' their knowledge for problem solving success. As a result, when assessing students' work I assessed each student's choice and justification of deeper structure, and when taking up these quizzes I reinforced that there could be more than one way to represent and solve a problem.

> Well, problem solving has taught me many things. That problems must be represented before they are solved, and that categorizing problems according to deeper structure makes solving problems quite easier because you have formed a plan. The nice thing is that it makes problem solving understandable, and it helps out my grade nine year. [Student's response to the tabloid question]

Assessing students' problem solving quizzes was facilitated by using structured worksheets (see Figure 2) that contained room for one word problem at the top and spaces for each aspect of the cognitive strategy (Hutchinson, 1986, 1993).


Name: _____

[Space for the problem]

PROBLEM REPRESENTATION

1. Tell me what you have just read (surface structure) including the goal.
2. Tell me what you are given in the problem (information, etc.)
3. Make a representation, if possible (chart, diagram, etc.)
4. What kind of problem is it (deeper structure) and why?
5. What don't you know?

PROBLEM SOLUTION

[Space for solution]

Figure 2. Structured worksheet

The structured worksheets were scored analytically (Hutchinson 1986), with each component of problem representation and solution rated 2, 1, or 0, depending on the degree of understanding demonstrated by the student.

Lesson plans evolved to support the objectives of developing students' mathematical thinking and teaching the supporting knowledge base. The questions in their assignments were then divided into two categories: knowledge base and problem solving. In order to minimize any redundancy in the questions assigned, they were chosen according to their deeper structure. This allowed for questions with similar methods of solution to be incrementally assigned.

In the meantime

In the eighties and early nineties I was also a sessional lecturer at the University of Regina teaching Mathematics 101, a class that is required for a Bachelor of Education degree in elementary education and satisfies a degree requirement in the Faculty of Arts. I soon learned that most of the students in the class were math anxious and typically left taking this class until the end of their respective programs. I taught the class applying the ideas previously described. Students began to demonstrate expert-like retrieval strategies for solving a problem by stating, for example: "it was just a proportional reasoning problem". Others indicated they had transferred the application of the cognitive strategy to their other classes with success.

> Well, problem solving, to me has taught me to look at things (problems, especially) from a different point of view. I used to just give up on a problem after just reading it and not knowing how to attack it. Now, I can say that I don't give [up] so easily and I know of some ways to attack a problem. Personally I didn't feel that problem solving had anything to do with reality since the only time I used it was in math. I figured that we would never need this, but it seems as though I was wrong because it not only

> helps your thinking in math, but it affects the way you also handle things in life. [Student's response to the tabloid question]

At the end of one class one student in a 'thank you' card said, "I surprised myself by actually enjoying a math class!" I found that students I taught began to like mathematics and solving problems a lot more.

Reflection

Looking back on my years of research, thinking about and teaching mathematics suggested that instruction occur under a problem-solving umbrella supported by a cognitive strategy: a cognitive strategy that has students among other things classifying with justification problems according to deeper structure, while acknowledging the use of multiple representations when solving problems. Lessons and assignments would be supported by the incremental development of knowledge base and problem solving with students' assignments in problem solving scored analytically. I found teaching mathematics to students in this manner lead to students' improvement in their attitudes and beliefs toward mathematics and problem solving.

Return to university

It was time to read Pólya's book again, this time to prepare myself to return to graduate school and to research how I approached teaching mathematics. A year after my graduation, I was awarded a National Doctoral Thesis Award (Seaman 1995) for "...the evaluation and dissemination of new ideas in education...and assist in improving the quality of education." Today, as an associate professor of mathematics education at the University of Regina, this research has become the basis of what I employ to expand preservice mathematics teachers' ways of thinking about and teaching mathematics. My hope is that they will begin their teaching career with the pedagogical experience I have gathered in my teaching career.

> Hey Rick,
>
> Just wanted to "drop you a line" and let you know that I am loving this job. Every day just gets better and more exciting than the last. I'm sure that sounds naive and idealistic, but I've really had that good of a start to my semester. I'm super busy with lots of extra curricular stuff (SRC, volleyball, Outdoor Ed Club), but it has really helped me get to know the staff and the kids.
>
> I'm always trying to incorporate some of your EMTH ideas into my lessons... I often go back to my notes and cartoons we got from you in those classes. The longer I'm here, the more I understand what you were saying when I jotted down those notes in class.
>
> Anyway, I had a few minutes this morning so I thought I'd let you know that I really couldn't have made a better choice professionally!
>
> Thanks for all the advice. [Graduate e-mail]

> Hi Rick,
>
> Just wanted to send a little "thank you" note for the training you gave us! I just had a meeting with one of my vice-principals and she has been so happy with how I'm teaching "thinking" skills that she's asked a director to come and watch me teach. I can't believe how much I'm learning the more I'm in the classroom. I love this job! Hope things are going well. Keep educating your flock... we end up really appreciating it. Thanks. [Graduate e-mail]

> Hi Rick,
>
> … I was just thinking of you and your fantastic class a couple of days ago. I was answering some questions on a state job application and one of them was about classes or experience that had helped

develop my problem solving skills. I had to talk about your class, and how it changed the way I look at problems. No matter what the question, I still look for the deeper structure. Thank you for sharing your wisdom. [Graduate e-mail]

References

Gliner, G. S. (1989). College students' organization of mathematics word problems in relation to succcess in problem solving. *School Science and Mathematics, 8* (5), 392–404.

Hutchinson, N.L. (1986). Instruction of representation and solution in algebraic problem solving with learning disabled adolescents. *Unpublished doctoral dissertation*, Simon Fraser University, Burnaby, BC.

Hutchinson, N.L. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. *Learning Disability Quarterly, 16*(1). 34–63.

Mayer, R.E. (1985). Mathematical ability. In R.J. Sternberg (Ed.), *Human abilities: An information-processing approach* (pp. 127–150). New York, NY: Freeman.

Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. *Journal of Learning Disabilities, 25*(4), 230–248.

Montague, M., & Applegate, B. (1993a). Middle school students' mathematical problem solving: An analysis of think-aloud protocols. *Learning Disability Quarterly, 16*(1), 19–32.

Montague, M., & Applegate, B. (1993b). Mathematical problem-solving characteristics of middle school students with learning disabilities. *The Journal of Special Education, 27*(2), 175–201.

Petrie, P.A., Baker, V.E., Levitt, J.R., & MacLean, W.B. (1946). *Algebra: A senior course*. Toronto, ON: The Copp Clark Publishing Co. Limited.

Pólya, G. (1957). *How to Solve It (2nd ed.)*. Princeton, NJ: Princeton University Press.

Saxon, J. (1982). Incremental development: A breakthrough in mathematics. *Phi Delta Kappan, 63*(7), 482–484.

Saxon, J. (1984). *Algebra II: An Incremental Development*. Norman, OK: Grassdale Publishers, Inc.

Schoenfeld, A. H., & Hermann, D. H. (1982). Problem perception and knowledge structure in expert and novice mathematical problem solvers. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 8*(5), 484 - 494.

Schoenfeld, A.H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Seaman, C.R. (1995). Effects of understanding and heuristics on problem solving in mathematics. *Unpublished doctoral dissertation.* University of Regina, Regina, SK.

Silver, E. A. (1977). Student classification of word problems: Bases for classification and relationship to problem solving and other abilities. *Paper presented at meeting of American Educational Research Association*, April 1977, New York, NY.

Silver, E. A. (1979). Student perceptions of relatedness among mathematical verbal problems. *Journal for Research in Mathematics Education, 10*(3), 195–221.

ETHNOMATHEMATICS

Sam Lamontagne

The possibility of a student dropping out is a reality that all teachers face. The drop-out rate from Native communities in the past has been concerning, and there is a residual negative effect residential schools have placed upon the Native people. Educational institutions were once thought to be no more than colonial tools used by white oppressors. "Children as young as four were removed from their families...Children were forbidden to speak their language and unable to follow their traditional customs. As a result, they often became ashamed of their language, culture and family" (Our words, Our Ways, 2005, p. 3). The Native perspective has changed over the years concerning our school systems however the battle to keep Aboriginal students in schools remains. "In spite of 'an upward retention trend' more than half of all Native children (57 percent) drop out of school before graduation (as compared to 15 percent for non-Natives)" (Binda & Calliou, 2001, p.9). The impact of not receiving a high school diploma is detrimental "Those without the graduation diploma are more likely to encounter difficulty in entering and progressing in the work force, and tend to end up seasonal work, unskilled work, or be underemployed, unemployed and dependant on welfare" (Battiste & Barman, 1995, p. 157). In the past the failure to succeed in educational systems was blamed mainly on the child's learning inabilities. Today the blame has shifted towards the educational institutions and the curriculum they are providing. "More recent research has begun to focus on school culture or climate, posing the very pertinent question: How much do schools contribute to the problem" (Battiste & Barman, 1995, p. 161)? Some of the barriers concerning the ability to succeed in the school setting for Aboriginal students are; Language deficiency, Cultural dissonance and Inappropriate instruction.

An appealing approach towards tackling the three barriers listed previously would be the use of Ethnomathematics. Essentially, Ethnomathematics is the study of the relationship between mathematics and culture. "It refers to a broad cluster of ideas ranging from distinct numerical and mathematical systems to multicultural mathematics education" (Powell & Frankenstein, 1997, p.7). The ultimate goal of ethnomathematics is to contribute both to the understanding of culture and the understanding of mathematics, by using principles of knowledge and expanding them to address mathematical situations we are able to facilitate learning in a more sufficient manner. "A study of mathematics within other cultures provides students with an opportunity to 'put faces' on mathematics instead of erroneously thinking that mathematics is a result of some mystical phenomenon" (Vandewalle & Folk, 2008, p. 100).

"There is a considerable agreement among Native and non-Native educators that indicate English language skills contribute to Native youths dropping out" (Batiste, 1995, p. 163). If Aboriginal people struggled to learn English as their second language, then they must surely have difficulty understanding the linguistic intricacies of Mathematics. Furthermore the standardized way of testing, places an inadequate reader with limited English language skills at a serious disadvantage. The inability to fully understand standardized math problems creates an uneasy learning environment. Frustration begins to grow creating a 'snow ball effect' and the quality of learning eventually declines to a point of helplessness and eventual withdrawal. "Any obstacle to students' mastery of the language and literacy skills required for acceptable school performance will tend to deflect them away

from success" (Battiste & Barman, 1995, p. 164). The intent is that the program should use the student's mastery of the native language to assist in acquiring mastery of the English language. This supposedly happens, for example through the use of both languages in introducing primitive concepts in mathematics. Davison (1990) found that the student's knowledge of mathematics terminology in the Crow language was very limited. The problem exacerbated further by the students seeing little or no use for the mathematics they learned in school. An emphasis in school classrooms on textbook-dominating teaching only made the problem worse. "I would assert, from extensive classroom observation, that in predominantly native classrooms, it is critical that students hear, speak, and write much more English language mathematics" (Davison, 1990, p. 144).

Aboriginal culture is in a state of healing, years of assimilation and colonial rule have disfigured the once proud Native Nations. How can you challenge your students to problem solving ideas when they are cannot solve their own history. Teaching these lost understandings will facilitate a deeper understanding of the history and the cultures of Aboriginal people and will in turn make the learning of all subjects easier. "Becoming more familiar with Aboriginal worldviews helps teachers build cultural continuity into both the content and instructional approaches of all subject areas" (Our words, Our ways, 2005, p. 2). If you want to reach the people in some respect you must think like the people. There is an emergence of Native pride throughout Saskatchewan, Aboriginal people are not feeling as oppressed these days. In this sense, an ethnomathematics approach to the curriculum will help draw on traditional culture, while drawing attention on the mathematics needed by these students in an integrated society. Whether the illustrations are traditional or modern, they must engage the students' attention if the students are to be helped in understanding the important mathematical ideas.

Barriers such as language and culture make the instruction process that much more difficult. Therefore the onus is placed upon choosing an appropriate method of instruction that will facilitate the greatest amount of learning possible. Non-conventional approaches of teaching may lead to a better sense of learning. "There is strong evidence to suggesting that minority learners, in particular, have a strong preference for a more tactile, visual approach to mathematics instruction" (Davison, 1990, p. 145). The use of visual aids and other cognitive instructional aids may provide the bridge concerning the understanding of Mathematics. Western education often separates learning into distinct subject areas, whereas an Aboriginal perspective uses an integrated approach. "The making of a star quilt would be seen as an art involving geometry (including symmetry and rotations), an opportunity to meet a quilt maker from the community, and a way to learn cultural teachings regarding star pattern and quilt" (Our words, Our ways, 2005, p. 15). Therefore quilt making is more than the fabrication of material it is a communal experience that allows the opportunity to explore and learn about the importance of establishing and maintaining relationships.

According to Vandewalle and Folk (2008), there are six critical strategies when supporting English language learners in the classroom, the strategies listed are among the most frequently used.

1. Write and state the content and language objectives

Establishing a lesson set is crucial, if the students are not clear on the objectives or rules of the lesson they will struggle to find the meaning of the exercise. Providing a road map of what will be covered helps give direction to the task, also enabling students to build upon knowledge they already pertain.

2. Build background

Connecting mathematical situations with cultural symbols is useful in giving a clearer meaning of the task. You could use the construction of a Tee-pee as a addition exercise. 22 (yards of material) + 100 (feet of clothesline) + 12 (poles) + 12 (pegs) = 148 (materials needed to construct a Tee-pee) Along with the addition problem, a focus on the symbolism and ritual of constructing a Tee-pee would not only give meaning to the task but also promote Native culture (Monster.guide, 2008).

3. Encourage use of native language

"Research shows that students' cognitive development proceeds more readily in their native language" (Vandewalle & Folk, 2008, p. 101). So it is beneficial for students to begin solving the question using their own native tongue, problem solving should begin with a multitude of ideas and difficulty with a secondary language can hinder exploration of possible solutions. "The first language of some students is an Aboriginal language. As they speak English, these students are constantly translating their thoughts. This process may be difficult as the meaning of the words and the patterns of thinking in their first language may be quite different from English" (Our words, Our ways, 2005, p. 34).

4. Comprehensible input

Comprehensible input means that the message you are trying to convey is understood by the students. By simplifying the vocabulary within the question, the task becomes clearer which allows more time allotted towards the solution of the problem.

5. Explicitly teach vocabulary

Vocabulary development within Mathematics is crucial to the overall understanding of the subject matter. A popular learning tool is the mathematics word wall; the word wall is constructed whenever essential mathematical vocabulary is encountered. "When a word is selected, students can create cards that include the word in English, translations of the word in the languages represented in your classroom, pictures, and a student-made description in English or in several languages" (Vandewalle & Folk, 2008, p. 101).

6. Use cooperative groups

If you don't use it you lose it, so it becomes essential for English language learners to continually practice speak, write, talk and listen. Creating a comfortable environment where students are not afraid to step out of their comforts zones is crucial. Therefore the grouping of students influences the ability to learn effectively. Grouping individuals with language difficulties allows students to identify and assist others who may be facing similar struggles. However it becomes beneficial to integrate Bi-lingual and English speaking students within the groups as well. The diversity of the students will help facilitate learning the language of mathematics while simultaneously promote cross-cultural

appreciation. Incorporating First Nation content ensures a sense of pride and inclusion towards Aboriginal students. One symbol that is common to many Aboriginal peoples is the concept of the Medicine Wheel. There are a variety of approaches to teaching when using the Medicine Wheel as the foundation of learning. Ideas include using the topics of seasons, plants, animals, earth and sky. By modifying the Medicine Wheel with Mathematical language we are able to stick to the curriculum and promote a First Nations philosophy of understanding. "It encourages the inquiry approach and will assist students in developing their own environment ethics" (Binda & Calliou, 2001, p. 149)

In order to effectively instruct Mathematics to Aboriginal students an ethnomathematics approach must be present. Understanding the target audience is essential, and building upon their previous knowledge enables individual growth. Previous ways of instruction might have led to the perception of gained knowledge when in actuality it trained the component of calculated regurgitation, meaning students would work towards finding the correct numerical answer without a true understanding of the problem. As Davison (1990) states:

> The focus of attention in the continuation of these studies is to determine how familiar situations and tactile and visual approaches, integrated with systematic language activities, can be used to help students with below average language skills to improve their level of language functioning as well as their performance in a wider range of mathematics objectives (p. 147).

So then the question of teaching comes into play. Are we teaching students for the purpose of high test scores, or are we instructing students with the aspirations that they will develop their own individualized logic of understanding mathematics? If we are instructing standardized mathematical curriculum are we then ignoring cultural deficiencies that may be attributed to the inability to understand mathematics. Ethnomathematics enables minority students to accept the need to learn the mathematics needed for survival in our society and to be motivated to work to accomplish that goal.

References

Powell, A. B., & Frankenstein, M. (1997). Ethnomathematical knowledge. In A. B. Powell & M. Frakenstein (Eds.), *Ethnomathematics: Challenging Eurocentrism in Mathematics Education* (pp. 5-13). Albany, NY: SUNY Press.

Davison, D. M. (1990). An ethnomathematics approach to teaching language minority students. In J. Reyhner (Ed.), *Effective Language Education Practices & Native Language Survival.* Choctaw, OK: Native American Language Issues (NALI).

How to make a Teepee (n.d.). Retrieved December 5, 2008 from http://www.monsterguide.net/how-to-make-a-teepee.shtml

Vandewalle, J. A., & Folk, S. (2008). *Elementary and Middle School Mathematics: Teaching Developmentally.* Toronto, ON: Pearson Canada Inc.

Binda, K. P., & Caillou, S. (2001). *Aboriginal Education In Canada: A study in decolonization.* Mississauga, Ont.: Canada Educators' Press.

Battiste, M. A., & Barman, J. (1995). *First Nations Education in Canada: The Circle Unfolds*. Vancouver: UBC Press.

Our Words, Our Ways: Teaching First Nations, Metis and Inuit Learners (2005). Alberta, Canada: Alberta Education's Aboriginal Services Branch and Learning and Teaching Resources Branch. Retrieved November 25, 2008 from http://education.alberta.ca/teachers/resources/cross/ourwordsourways.aspx

ASSESSMENT FOR LEARNING IN SASKATCHEWAN MATHEMATICS CLASSES

Murray Guest

As the phrase assessment for learning (AFL) is used more and more in Saskatchewan, some math teachers find themselves wondering how their practice conforms to its precepts, if they are behind the times and if their practice is to be found wanting. In some people's minds AFL seems to be better suited to the humanities or perhaps even the sciences, but doesn't fit well with mathematics. This article looks at what I believe to be the practice of many math teachers in the province and compares that practice to the precepts of AFL. I am confident that math teachers have many teaching practices that follow AFL recommendations and that AFL is suited to student improvement in mathematics. There are areas of change to be addressed, but math teachers are well positioned with regard to AFL.

Assessment for learning involves using assessment in the classroom to raise pupils' achievement. AFL is based on the idea that students will improve most if they understand the goals of their learning, where they are in relation to them and how these goals can be achieved. This approach to education is based on decades of research from many sources. The results of the research are not in dispute. Assessment for learning is a research based theory of learning and teaching which has many components, all of which have been shown to improve student learning. These components include providing clear learning targets, exemplars of student work and continuous, high quality feedback to students. It also includes opportunities for students to revise their work based on the feedback they receive, and the provision for alternative proof of knowledge. It is anticipated that teachers using the AFL model will use the information gathered from student assessment to alter their teaching to address needs identified from the assessment process.

The assessment part of assessment for learning refers to the gathering of information regarding student understanding and using it to support student and/or teacher practice rather than matching a number to that information. Evaluation comes after assessment and is best not based on all assessments taken during the school year. Assessment can be more formal, using tests, writing samples or student projects to collect information regarding student understanding or it can be less formal, coming from watching a student work at math or a conversation regarding the current assignment or a one question quiz at the end of class to check student understanding of the day's work.

Math teachers have used some of the precepts of AFL before AFL was named. Math assignments with an available answer key allow students to self assess their knowledge and to obtain one form of continuous feedback. When teachers of mathematics walk around checking work and talking to students, they are providing feedback to students as well as gathering information regarding student understanding of math concepts. Handing back a piece of student work with corrections and written ideas for improvement can also offer students high quality feedback. These types of assessment for learning have been done for years in math classes. It is important to do this with explicit intent and that is

sometimes missing in the minds of the students.

Students must be aware of the reasons for having this feedback and also be aware of how to use this feedback to help them. Those we would traditionally consider good students are able to do this on their own without much prompting by the teacher, but weaker students are often unable to translate teacher feedback into better performance. I have regular quizzes in my classes, as many of us do. The change I made in my practice is to make it explicit to my students that the point of the quizzes is to have the student check to see what they do and don't know about the area they will be tested in. To increase the likelihood that this is the only message students are given regarding quizzes, I offer no marks for the quiz. The students are practicing, and I don't give a mark for practice. The only changes I made to conform to AFL are that I removed marks from my quizzes and I discussed the reasons for quizzes with my students.

Similarly, homework is practice. The point of doing homework is to gain automaticity with the material and to identify problems with student understanding of material. It is still practice and should not be assigned a mark. I justify this by asking how often I was marked based on a practice drive as I learned to handle a car or if I was judged as I refined my skills on the volleyball court. Once students understand the reasons for assigned practice and see that it works, they do it without complaint. Those who don't will learn of its value through trial and error. I still check homework so I can identify problems with student understanding, but that checking re-enforces the message I want to send regarding homework. It is useful in our goal of understanding math better, rather than something used to gather marks.

I also have students go through their own work using a list of standards they need to meet for the unit, which I provide for them. They then are expected to write about what they do well and what they struggle with and why they believe that is the case. This process supports student understanding of where they are with respect to math standards as well as meeting some of the province's requirements regarding communicating about mathematics. The process is collaborative, student centered and by students' own admission, useful.

Math teachers show the responsiveness that is a hallmark of AFL and have for years. When we look at the results of a mid-unit quiz, or know, through teacher-student interaction that a large portion of the class doesn't understand a concept, we spend more time with it and re-teach concepts. We spend individual time with certain students who we see, through informal assessment, need extra help to understand a concept. Math teachers already explain a concept in many different ways using visual aids, manipulatives and real world examples. When a student asks a math teacher for help, that is a self assessment the student makes and our response to this problem is collaboration with the student to help them understand the math concept they struggled with.

An area where math teachers may want to adjust their practice is in the area of making explicit all of the learning targets they have for students and being consistent in keeping to that learning target. I've only recently explicitly written on the board what I hope the

students will understand by the end of the class. An example would be "Today you will be able to find the reference angle for any given angle and you will find the exact value of a trig function using reference angles." This allows student to know exactly what is expected of them from the beginning of the class. I was surprised that many of my students did not know what I wanted them to know at the end of a class. By being very explicit about what I want the students to know by the end of every class, students better knew the purpose of each activity.

A final area of change regards evidence of learning which can take many forms. The traditional form for math teachers is the unit test and the comprehensive final. They offer students a chance to show off what they know by working a set of problems in a set amount of time. Alternatives do exist. Students can write regarding their understanding of various mathematical techniques – explaining how and why specific techniques work with a discussion of their strengths and weaknesses. Students can also devise or strengthen existing questions, with an accompanying explanation of why the work done reflects an understanding of concepts. They can work through real world questions either alone or in a group, grappling with the messy nature of problems that are not 'cooked' for the classroom. Although it can be time consuming, student interviews can give a very good picture of student understanding. By opening the door to alternate ways of showing understanding, we as teachers can also have our own students devise acceptable ways of proving their knowledge that we may not have ever thought of on our own.

This article holds that math teachers have been leaders, in some cases without knowing it, in the use of assessment for learning. While we have some challenges to address, we have less to apologize for than some would suppose.

THE STORY OF MATH 10: TONY AND VANESSA'S TALE

Ryan Banow

Author's note

When I first approached this project I had intentions of writing a history of mathematics story. As I further explored the content I realized that that was beyond my means and would not be overly intriguing. Instead, I contemplated and decided upon telling the story of a Saskatchewan family and show how all of this mathematical content is relevant in their lives. One way I would describe this story is that it is a year-long word problem. I wrote it so that it does not directly ask questions; rather, questions arise from reading it that need math to answer. It also introduces many concepts in an everyday context.

The story covers Concepts A-E of the *Saskatchewan Learning Math 10 Curriculum Guide*. I left off Concept F because that is to be covered differently in all classes as Math 20 preparation. I focused the story on the foundational objectives because I felt that if I hit those it would encompass the underlying objectives. I wrote the story to be light and fun. I envision it as a fun thing that students can look forward to hearing when they come to math class. I tried to make it humorous, but yet still realistic. I included many references to Saskatchewan culture, i.e. the

Roughriders, Co-op, RadioShack, and Ford. I think with the right attitude from the teacher and the right group of students this could really make for a fun and productive learning community.

When using this tool, teachers should read the students the story up until a point where something can be explored mathematically. There are no clear indications of when this is, but I think with a comparison to the *Math 10 Curriculum Guide* teachers would easily see where this story leads to development of concepts and examples.

Chapter 1: Linear equations and inequalities

There was a young married family living in rural Saskatchewan. The husband's name was Tony and the wife's name was Vanessa. The two of them were living a quiet life that had him working at the local Co-op grocery store as a butcher and she was working part-time at the Ford dealership as a receptionist. Things were going along smoothly as they were able to make ends meet.

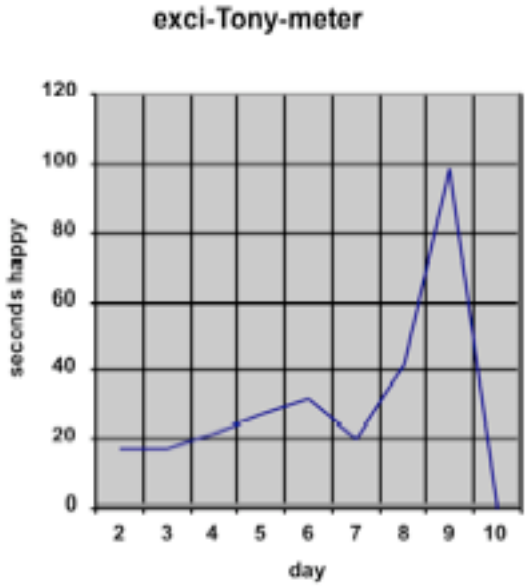
In October of 2001 Vanessa noticed some changes in herself. She went and saw the local doctor. The doctor informed her that she was seemingly pregnant and that she would be giving birth to a child in early July. Vanessa ran to the Co-op to tell Tony and they were both pretty ecstatic. They ran home to call their parents. Their parents were somewhat less excited because of the financial situation around having a baby. Tony had always been one to save money up just to go and spend it once he had enough for the new toy he wanted. In fact, just weeks earlier he bought a new *Ski Doo* MXZ 600.

It was already the end of October and they had until the start of July to save up money. At this time they only really had $400 saved. Their families and friends told them that they were going to need to spend about $3000 as soon as the baby was born. They had to figure out how much money they would need to save each month in order to reach that $3000 amount.

As couples do when they are preparing for the birth of a child, they started thinking about possibilities. Would the child be a boy or a girl? Tony scared himself when he thought, what if it is twins? If it were twins they would need to save more than $3000. He was comforted by the information from a friend of his whom recently had twins that he would not need to save any more than one and a half times the money of one baby. Tony and Vanessa took comfort in knowing that twins are not as expensive as two separate babies, but it was still more than they could fathom affording.

Chapter 2: Relations, linear functions, and graphs

Vanessa, being a sensitive woman, decided that throughout her pregnancy she should keep her mind focused on some different things…so she quit her job at the Ford dealership. She created something that she called the "exci-Tony-meter". Everyday when he came home from work, she would count how many seconds he would smile and be happy for. Tony is known as a serious guy, so if he was smiling you would know he was excited. She began to chart his progress on a graph. After the first ten days she had a graph that looked like so:



Vanessa was uneasy about what she was seeing on the graph, so if you can believe it, Vanessa stopped charting after this first graph because in her opinion it brought her down. She also seemed to think that she followed his patterns since she was so in tune to them. It is easier to block a person out when you are not studying them!

As the pregnancy progressed, Tony and Vanessa, *especially Tony,* noticed an increase in how much food Vanessa was eating. To him it appeared like it was increasing by the same amount each day. Just a little bit more on Monday, a tiny bit more on Tuesday, more again on Wednesday, and so on. Tony secretly got scared about how much food he was going to need to supply in the coming months, so he tried to track her eating to predict the future. From this he could create a graph representing the amount of food eaten over days. The data he collected was in grams because he bought a fancy new kitchen scale just for doing this; I said before that he likes to buy toys. He all-of-a-sudden turned into a gentleman and would prepare her plate for her to her specifications, then he would cause a diversion so he could find its mass in secret. Some of the data he had was on day 1 she ate 240 g, day 2: 248 g, day 5: 272 g, and day 7: 290 g. After making the graph he began thinking of how large of pieces of meat she was going to want by time the baby came along. He was very thankful that he was the butcher!

Chapter 3: Slope and functions

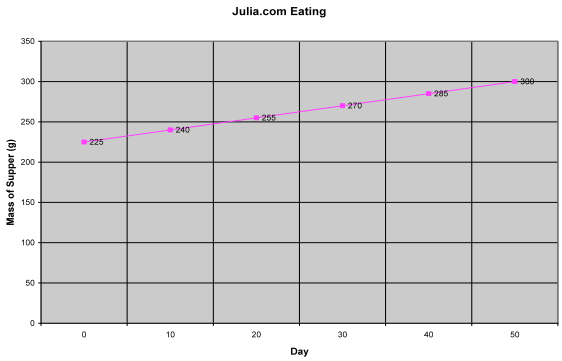
It was the winter and since Vanessa was still getting around quite well, they decided to take their last little vacation together before having a baby. Other years they would do a ski trip to the mountains because they both enjoyed it. Vanessa said that this year she wouldn't go skiing because she didn't want to risk anything. Consequently, Tony planned a trip to go skiing in the mountains!

This trip was quite boring for Vanessa, but she felt at least it was nice to get out of the house because she was a real people-watcher. She liked to stare out windows and see people's facial expressions as they talked kindly, argued, or gave the old silent treatment.

Tony's trip, on the other hand, was quite eventful. On the first day of skiing he started off again with just the bunny hills. He road up on the T-lift. This was simple enough because it wasn't too steep. Going down the hill was not as easy for him; he fell three times on the bunny hill. Tony took this as a need to get onto a hill more at his skill level. He road the chair lift most of the way up the mountain. The chair lift began at the bottom of the mountain and went up 2000m and over 3000m. This was a super-long ride, but Tony psyched himself up that he was ready for this black diamond run.

He got off the chair lift and found the run he wanted. It was called "Devil's Alley." He looked down the hill and it looked almost straight down to him, but keep in mind that he had just fallen three times on the bunny hill, so his head may not have been clear. Tony took a deep breathe and went down the hill. Somehow he forgot all about carving, breaking, steering, or even keeping his eyes open. He flew off the path and smacked right into a tree. 45 minutes later medics found him in among the trees. For the rest of the trip Tony and Vanessa spent time together. They people-watched: how romantic! It took six weeks before Tony's ribs began to feel better when he coughed.

As Vanessa's eating continued to rapidly increase, Tony began to wonder if every woman goes through this. He went on the internet and Googled "Woman eating way way way more during pregnancy." Astonishingly, he found a graph that charted another woman's eating habits during pregnancy. Oh the things that you can find on the internet!



From this Tony could come up with a value for how much this person's eating increased each day. He compared this to how much Vanessa was currently eating and wept.

Chapter 4: Direct and partial variation

As excitement increased and more questions were raised Tony and Vanessa started piling up huge phone bills. I mean HUGE phone bills. In March their bill came out well over $200. In that month Tony called his mother many times because he had a lot of questions to do with raising a child. Tony was always one to ask his mom instead of talking with Vanessa about it. Strangely enough, it never crossed their mind to get a long distance bundle, so instead they were paying a flat rate per minute. This rate was 15 cents/min.

After receiving this bill Tony looked into getting a little part-time job. When he was a boy he delivered newspapers, so he figured maybe this would be a job he could do again. He called around to the paper companies that had papers in their town and found that the best rate was $5 a day plus 10 cents for every paper delivered. When he sent in his information to take the job, they told him that they preferred to higher people under the age of 16. Another dream of Tony's over before it could begin.

Chapter 5: Sequence and series

As the time progressed into the spring, Vanessa and Tony began to be more excited and also more scared. You could really begin to tell that Vanessa was pregnant. Her weight increased in this fashion 120, 123, 126, 129, and 132 pounds each week as they went through April. On the other hand, Tony was beginning to get quite high strung and was eating a lot and quite sporadically to deal with his "issues." His weight increased over the same time period in this fashion 160, 161, 163, 167, and 175 pounds. The sequential increase was

good news for Vanessa because she was healthy, but it wasn't such good news for Tony.

Vanessa suggested to Tony that to get his mind to focus on different things he should either start buying baby supplies or take Tae-Bo. Tony opted to start buying baby supplies. He came up with what he thought was an ingenious pattern. Since he figured that he had about 10 weeks to go until they had the baby, he would buy one item in the first week, three in the second week, five in the third week, and so on. He never really imagined that that would add up to so many items!

Chapter 6: Consumer math

Since Tony had tied himself down to buying so many items, he decided to look for a new job. The first place he went to apply at was the Ford dealership that Vanessa had been working at. Tony went in and said that he was the salesperson of their dreams. Since he was coming with such high self-praise, they offered him a job where he would get paid exclusively by the sales he made. Tony had heard that the salesperson's usually got paid a wage and commission, but he shrugged that off because he knew that it would be easy to move these vehicles. He believed that they practically sold themselves. He would get 2% commission on each vehicle he sold. Since they had some huge new trucks on the lot in the range of $60 000 - $80 000, he figured that this was the job of his dreams.

After the first week, Tony sold zero huge trucks and even zero Ford Focuses. He thought of his hourly wage at the Co-op of $16.25 and realized how much money he could have made in this week if he would have just stayed working his 40 hours there. Tony thanked Ford for the week of free labour he gave them and he sought out another new job.

Tony went back to the Co-op, but they were already partway through training a new man for his job. He was out of luck. He needed to hit the pavement and find another new job. He looked in the newspaper classifieds, even though this was difficult for Tony to do because he was boycotting the newspaper after not letting him be a paperboy.

In the paper he found a listing for a business in the neighbouring town that needed someone to do landscape maintenance, also known as cutting grass. This was a lawn care business that was contracted to cut many lawns around the town. The ad stated that they would pay people employed by them $15 per regular sized yard cut. Tony eagerly called them about the position and did not state his age. Since the grass was beginning to grow very quickly, they eagerly hired him and he began being paid by the lawn.

Vanessa and Tony's income had gone down drastically since October. With Vanessa quitting her job and Tony switching to a lower paying job, the one thing they could take solace in was that they would not have to pay as much tax this year. Since they were having a child, their Income Tax Claim Code was going to change to allow them to keep more of their earnings. Tony also thought it was nice how he was going to be paying less tax overall too, but this was simply because he wasn't making as much.

With Tony's job he was being deducted on each pay cheque for EI, CPP, Federal Tax, Provincial Tax, and

the Health Plan. The business had a health plan to cover injury by lawn mower. After having the child, Tony's Federal and Provincial tax deductions would decrease. When Tony received his first pay cheque from cutting grass he realized that he was getting paid much less than $15 per lawn after deductions. His net pay, or the pay he was taking home, was much less than the predicted gross pay.

Throughout May, they were experiencing difficult financial times. Since they had always lived with enough money to get by, they never really worried about budgeting before, but now they needed to. Vanessa charted out the main categories of where the money was being spent. She compared this to the values suggested in *The Budget Book:* Savings = 5-10%, Food = 18-30%, Clothing = 8-15%, Transportation = 10-15%, Housing = 18-30%, Utilities = 5-9%, and Health and Other = 14-30%. She went over the numbers a few times and realized that they were probably paying too much for Other and Transportation. Because of their current situation she accepted the fact that they could not afford to have their savings to be in that range at this time either.

Vanessa needed to confront Tony about the budget. She was going to propose that he sell his new *Ski Doo* and to cut back on their satellite TV package. She figured that they could live without the sports package. When Tony was informed of these ideas he almost lost it. He suggested selling their car instead of the *Ski Doo* and he swore to fight to the death to keep the sports package, especially since they also had the food and home TV package. Needless to say, Tony ended up selling his *Ski Doo* for much less than he bought it for in the fall because it was no longer snowmobiling weather and he lost *TSN*, *Sportsnet*, and *The Score*. It was for the good of the family, but it was a very tough pill to swallow. Vanessa revisited their budget and was satisfied about what percentage they were paying into each category. They were now financially sound.

Chapter 7: Lines and angles

Did I ever mention that Tony was a visionary? He often thought well outside of the box. It was June and they needed to get a comfortable and functional room set up for the baby. They did have a spare bedroom that would have worked fine, but this was too regular for Tony's child. He figured, how would this child have any imaginative thought if it grew up in a square room? The walls met at perpendicular angles and opposite walls were parallel. That wasn't good enough for this Baby Einstein.

Tony proceeded to knock down a couple of walls and put up new ones. He left two of the room's walls up: these were the outside wall and the hallway wall. These two walls were across from each other and parallel. He then put up a new wall that was on an angle and he wanted to put up another wall at the other end of the room that was parallel to this new one. He had to Google a bunch of math websites to learn what angles had to match and what angle were supplementary. In the end he built this parallelogram room, and he just knew that his child was going to be brilliant!

Chapter 8: Polygons, triangles, and trigonometric ratios

In June, Tony bought his future child the *Get a Grip Sorter*. This was a toy with certain shaped blocks that the child

would fit into holes designed for them to fit into. Tony was pretty pumped about this toy because he had something similar when he was a child. This toy came with a circle, a square, a triangle, a star, an octagon, a pentagon, and a few more shapes. These shapes were very mathematical; therefore, Tony was ecstatic about the mental growth that his child would have with these devices.

The next week, in among all the items that Tony was buying for the baby, he bought a new TV. Somehow or someway this was a baby supply. I think maybe he thought that he was going to be watching a lot more TV in the coming months. Tony went to the local electronics store, then known as *RadioShack*, and compared a few models of TVs. Keeping in mind that this was 2002, the same year that the Roughriders acquired Nealon Greene, widescreen TVs were not overly popular yet. He first looked at regular 4:3 TV that had a 32" diagonal. Tony was eager to find out how wide and tall this screen was because the diagonal value wasn't that important to him.

The dealer then brought Tony's attention to a widescreen television. This fancy new device had a 16:9 aspect ratio, which meant no more black bars on the top and bottom of the screen! This TV screen if it were cut into two triangles by a diagonal had angles of 29.36° and 90° along the bottom of the screen. The first TV he looked at was 28" wide. He wanted to know how the store could call this TV a 32" screen even though it was much different in size than the 4:3 32" TV. The second widescreen TV he looked at was listed as a 37" screen and he wanted to figure out how tall and wide its screen was.

Tony ended up leaving the store with a confused look on his face and a feeling of accomplishment for saving his family forever from black bars because he purchased a 16:9 widescreen TV. Unfortunately for Tony, we now know that many movies are being filmed in a 2.35:1 aspect ratio. Even though he had good intentions to thwart black bars forever, it cannot be avoided.

Conclusion

Vanessa and Tony became the parents to a beautiful baby girl on July 6, 2002. Tony was soon offered the position at the *Co-op* because the new butcher's wife got pregnant and he ended up looking for a new job. Perhaps he could cut lawns! Tony accepted the position as the local butcher once again and their finances stabilized under the budget that Vanessa designed. Tony, Vanessa, and Vanony Jr. enjoyed a quiet life in rural Saskatchewan…without a *Ski Doo* and without the sports package. The End.

MATHEMAGIC I

Al Sarna

This article first appeared in the Spring 2001 issue of *Vector: Journal of the BC Association of Math Teachers*.

1. It's in the numbers

Consider the following 5 by 5 rectangular array of the first 25 positive integers:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Tell students that you will be able to tell them the sum of 5 numbers that they are about to select *in advance*. Write a number on a piece of paper, fold it, and give it to a student. The number in this case will be 65.

1. Have students circle any number in the first row.
2. Cross out the remaining numbers in the first row *and* all the remaining numbers in the selected column.
3. Repeat this process for the remaining rows.
4. Add the circled numbers.
5. Have the student read the number you wrote down.
6. Accept the fame!

A possible finished product may look like the following:

~~1~~	2	~~3~~	~~4~~	~~5~~
~~6~~	~~7~~	~~8~~	~~9~~	10
~~11~~	~~12~~	13	~~14~~	~~15~~
16	~~17~~	~~18~~	~~19~~	~~20~~
21	22	23	24	25

Solution 1

If you have students rewrite the array as follows they will come up with the reason why the sum will always be 65 (for a 5 by 5 array):

5(1)-4	5(1)-3	5(1)-2	5(1)-1	5(1)
5(2)-4	5(2)-3	5(2)-2	5(2)-1	5(2)
5(3)-4	5(3)-3	5(3)-2	5(3)-1	5(3)
5(4)-4	5(4)-3	5(4)-2	5(4)-1	5(4)
5(5)-4	5(5)-3	5(5)-2	5(5)-1	5(5)

No matter which column (or row) you pick, at the end you must have one each of 5(1), 5(2), 5(3), 5(4), and 5(5). You will also be subtracting 4, 3, 2, and 1. In other words, the sum will be:

$$5(1)+5(2)+5(3)+5(4)+5(5)-4-3-2-1=65$$

You may want to have younger students determine what the sum will be for a 6 by 6, a 7 by 7, and an 8 by 8 array. Older students who have a passing knowledge of arithmetic series should be encouraged to generalize the result for an n by n array. Incidentally, the sum for an n by n array is $\frac{n(n^2+1)}{2}$.

2. Your trigonometric number

1. Have students pick a number n such that $1 \le n \le 89$.
2. Subtract this number from 1980.
3. Take the *sine* (in degree mode) of the result.
4. Now take the inverse sine ($\sin^{-1}$) of the displayed result.
5. The answer is your original number.

Solution 2

Since $\sin(180-\theta)=\sin\theta$ and since $1980=11(180)$ all we are really doing is returning the reference angle, θ, that we subtracted from 1980. In other words $\sin(180-\theta)=\sin(1980-\theta)=\sin\theta$. Since n

(or θ) is such that $1 \le n \le 89$ we have $\sin^{-1}\sin\theta=\theta$. You may want to use 2340 instead of 1980. It works because 2340=13(180). Any odd multiple of 180 works.

3. Lightning addition

1. With your back to the blackboard have a student write any one or two digit number at the top of the board.
2. Have another student write any one or two digit below the first number chosen.
3. Have another student add these first two numbers and make this sum the third number.
4. Have students add the second and third numbers to arrive at the fourth number.
5. Repeat until they have ten numbers in the column. Then draw a line under the last number as they are about to be added.
6. The teacher turns around and immediately writes the answer down.

Solution 3

This is simply a *Fibonacci* sequence and the sum will equal 11 times the seventh number (the fourth from the bottom). Consider:

$$\begin{array}{c} a \\ b \\ a+b \\ a+2b \\ 2a+3b \\ 3a+5b \\ 5a+8b \\ 8a+13b \\ 13a+21b \\ \underline{21a+34b} \\ 55a+88b=11(5a+8b) \end{array}$$

With very little effort you'll find that multiplying by 11 is almost as easy as multiplying by 10.

4. Think of a number: A variation

1. Think of a one – digit number.
2. Multiply it by 5, then add 3 to the result.
3. Double the last answer.
4. Think of a *second* one – digit number and add it to the last result obtained.
5. When the student gives you the final result you can immediately tell them what two numbers they picked.

To find the numbers chosen you need to subtract 6 from the result. The first number is in the tens' position and the second number is in the units' position.

Solution 4

Let the first number be x and the second number be y. We will have $(5x+3)2$ before we add the second number. After adding the second number, y, and distributing we will have $10x+6+y$. When you subtract 6 the result drops out.

5. Psychic number prediction

1. Pick any three – digit number whose first and last digits differ by 2 or more.
2. Reverse the digits and subtract the smaller from the larger.
3. Reverse the digits in the above answer and *add* this value to the original difference.
4. Reveal that the final answer is 1089.

Solution 5

Let the original number be $100h+10t+u$. After we reverse the digits and try and subtract we have

$$\begin{array}{rl} 100h+10t+u & \\ -\underline{100u+10t+h} & (u<h) \end{array}$$

The problem here is that we have to borrow not only from the tens, but from the hundreds as well. The work proceeds as follows:

$$\begin{array}{rl} & 100h+10(t-1)+(10+u) \\ & \underline{-100u+\ \ 10t+\qquad h} \quad \text{(now need to borrow from the hundreds)} \\ \Rightarrow & 100(h-1)+10(t-1+10)+(10+u) \\ & -(\ 100u\ \ +\ \ 10t\ \ +\ \ h) \\ & \underline{100(h-1-u)+10(9)+(10+u-h)} \quad \text{(reverse and add)} \\ & +\underline{100(10+u-h)+10(9)+h-1-u)} \\ & =100(9)+10(18)+9 \\ & =100(9)+10(10+8)+9 \\ & =100(9+1)+80+9=1000+80+9=1089 \end{array}$$

The reason I ran through the steps this way is that the interested reader may want to generalize this to other bases. For instance, if x represents a positive number base then if we assume that $0\leq C<B<A<x$ the initial number can be represented by Ax^2+Bx+C. After running through the general algebraic manipulation you will arrive at the 'magic' number of $x^3+0x^2+(x-2)x+(x-1)$. The magic number for some bases is shown below.

$x^3+0x^2+(x-2)x+(x-1)_{ten}=1000+0+8(10)+9=1089$

$x^3+0x^2+(x-2)x+(x-1)_{six}=216+0+4(6)+5=245_{ten}$

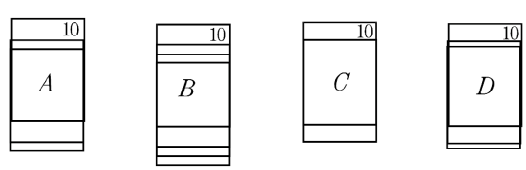
$x^3+0x^2+(x-2)x+(x-1)_{four}=64+0+2(4)+3=75_{ten}$

6. Those magic tens

These self-working card tricks are very popular and students are generally surprised at how easy they are to understand. As with any of the previous 'tricks' add your own presentation and flair to enhance the effect.

1. Take the four tens out of the deck and lay them down, face up, side by side.
2. Write down the name of a card (I'll tell you which one later) and ask a student to keep it hidden until you are finished.
3. With the remainder of the deck face down in your hand start dealing cards face up on the tens as you count backwards from 10. For example, as you lay the first card on top of the 10 you would say "nine", as you lay the second card on the pile you would say "eight", and so on.
4. If the card you are placing face up on the pile is the <u>same</u> as the number you are *saying* then Stop. That pile is finished. Repeat the process with the remaining tens.
5. If you get all the way to 1 without the card you are laying down being the same as the number you are saying then take *one more card* and lay it face *down* on the pile. That pile is dead.
6. When you have done this for all four tens you add the value of the face up cards. You can consider a face down card as having a value of "0". You count that number of cards, say 12, from the face down cards in your hand. The twelfth card will be the one you wrote down beforehand.

Solution 6



If we lay a 9 down and stop there are 2 cards in the pile. If we lay a card down and the next one is an 8 then we stop and

there are 3 cards in the pile. If we stop at a 5 then there are 6 cards in the pile. In general, if the values of the face up cards are *A, B, C,* and *D* where any (or all) could be zero (the case where the pile is dead that gives us a pile of 11 cards), then the number of cards in the four piles is:

$$(11-A)+(11-B)+(11-C)+(11-D)=$$

$$44-(A+B+C+D)$$

The number of cards left face down in your hand is:

$$52-[44-(A+B+C+D)]=$$

$$8+(A+B+C+D)$$

The last thing you did was eliminate $(A+B+C+D)$ cards from your hand. In fact, your magic prediction was the *ninth* card from the bottom. For example, if the sum of the face up cards is 15 then you are counting off 14 cards and the 15th card is the ninth card from the bottom. You remember this card when you are removing the tens from the deck. Some people remember the eighth card from the bottom and just count off the sum of the face up cards. It's entirely up to you - but the presentation seems more effective the other way.

Note: If all of *A*, *B*, *C*, and *D* are equal to zero then the last card played is the 'magic' card.

7. The sum on the bottom

1. Take a well-shuffled deck of cards and have a student pick any card and place it face down on the table.
2. Have them count *from* the value of the face down card as many cards as necessary until they get to 12. Face cards are considered to have a value of 10. For example, if the face down card is an 8 then they count "9, 10, 11, 12".
3. Select any of the remaining cards and repeat this process until they can't form a complete pile. If they can't form a complete pile then these cards stay in their hand.
4. Have them tell you the number of cards left over (in their hand) and you tell them the sum of the bottom cards.

Solution 7

Let the number at the bottom of each pile be $x_1, x_2, x_3, \ldots, x_n$. Therefore the number of cards in each pile is $(13-x_1)+(13-x_2)+\ldots+(13-x_n)$ where n is the number of piles. If R is the number of cards left over then we have:

$$\begin{aligned} & (13-x_1)+(13-x_2)+\cdots+(13-x_n)+R=52 \\ \Rightarrow\quad & 13n-(x_1+x_2+\cdots+x_n)+R=52 \\ \Rightarrow\quad & 13n-52+R=x_1+x_2+\cdots+x_n \\ \Rightarrow\quad & 13(n-4)+R=x_1+x_2+\cdots+x_n \end{aligned}$$

The right hand side is the sum of the numbers on the bottom cards. All the magician has to do is subtract 4 from the number of piles, multiply by 13, and add the remaining number of cards the student tells you. For example, if there are 7 piles and 3 cards remaining then the sum of the bottom cards is equal to 13(7–4)+3=42. Tip: Have the students lay the piles out in rows of 4 – this will make your work considerably easier!

8. I know what you like

This classic was done by David Copperfield on T.V. as an intro to his television special where he made the *Orient Express* disappear. His cards used the names of train cars such as Diner, Club, Shower, etc. Making up your own with student names gets a reaction. Since

you don't know my students let's try it this way.

Consider the following jobs you may want:

Movie star	Phys. Ed. Teacher	Board Official
Math teacher	French teacher	Politician

1. Start by showing *only* the following 4 jobs/cards: Phys. Ed Teacher, Politician, Administrator, and Math Teacher and have the students silently pick one. This works best with an overhead of them.
2. Explain that you are going to determine what they like best but don't want it to be too easy so you are going to add another 5 jobs (as shown above).
3. They are allowed to move up, down, right, or left on the grid but *not* diagonally.
4. Starting from *one* of the 4 **original** jobs make 4 moves (remember where you land).
5. Explain that they weren't going to be counselors so remove that card.
6. Make 5 moves from where you last landed. Then explain that we already have too many politicians so remove that card.
7. Make 2 moves from where you last landed. Then explain that administrators work so hard they probably don't want to be one so that card can be removed.
8. Make 3 more moves. Then explain that history teachers and board officials were such tough jobs to get so we may as well remove their cards.
9. Make 3 more moves. Explain that since many of them had already selected phys. ed but were now changing their mind that the card should be removed.
10. Make one more move and express your surprise that they all wanted to be math teachers!

Solution 8

First let's number the grid as follows.

1	2	3
4	5	6
7	8	9

Notice:

- a move to the left or right either subtracts 1 or adds 1
- a move up or down either subtracts 3 or adds 3-in fact, each move adds or subtracts an odd number
- we started on an even number (2, 4, 6, or 8)
- an even number plus or minus an odd number = an odd number
- an odd number plus or minus an odd number = an ever number

Now let's go back and look at the moves we made and remember that each step begins where the last step ended.

- When we first made 4 moves we went odd, even, odd, even. We could not have been on card 9 so we removed it. We are on an even card.
- When we made 5 moves we went odd, even, odd, even, odd. We could not have been on card 6 so we removed it. We are on an odd card.
- When we made 2 moves we went even, odd. We could not be on

card 8 so we removed it. We are on an odd card (1, 3, 5, or 7).

- When we made 3 moves we went even, odd, even. We could not be on cards 3 or 7 so we removed them. We are on an even card (2 or 4).
- When we made 3 more moves we went odd, even, odd. We could not be on card 2 so we removed it. The only cards left are 1, 4, and 5. Since we are on an odd card our last move forces us to card 4. Make sure you put the card you want forced in this position.

Clearly other variations are possible and the variations can get rather amusing depending on how far you want to go.

Finally

These are just a few of the many mathemagical 'tricks' you can use to spice up a math class or introduce a new lesson. Many more can be found in books listed in the bibliography. Relax, enjoy, and may the force be with you!

References

Fulves, K. (1983). *Self-working Number Magic.* New York: Dover Publications, 1983.

Fulves, K. (1984). *More Self-working Card Tricks.* New York: Dover Publications.

Gardner, M. (1956). *Mathematics Magic and Mystery.* New York: Dover Publications.

Gardner, M. (1992). David Copperfield's Orient Express Card Trick. *Mathematics Teacher*.

Mira, J. A. (1971). *Mathematical Teasers.* Barnes and Noble.

Simon, W. (1964). *Mathematical Magic.* New York: Charles Scribner's Sons.

***Innumeracy: Mathematical Illiteracy and its Consequences*: A Review**

Egan Chernoff

In April of 2008 I was fortunate enough to make my way down to Salt Lake City, Utah for the National Council of Teachers of Mathematics (NCTM) 2008 Annual Meeting and Exposition. While there were a number of highlights—the exhibitors' hall was worth the trip itself—the keynote address by Malcolm Gladwell (author of *Blink* and *The Tipping Point*) definitely stole the show. However, Gladwell was not the only major author at the conference. John Allen Paulos (author of *A Mathematician Reads the Newspaper*, *Mathematics and Humor: A Study of the Logic of Humor*, and *Beyond Numeracy: Ruminations of a Numbers Man*, and others) was also a plenary speaker. Given the conference theme—Becoming Certain About Uncertainty—coupled with my reading of Paulos' book *Innumeracy: Mathematical Illiteracy and its Consequences*, I was eager to hear him speak as well. For me, Paulos' books are to mathematics teachers what Asimov's books are to science fiction aficionados. In fact, my reading his books has not only spurred on personal areas of research, but has provided countless examples for my classrooms. In conversation with individuals at the conference, I found out that certain institutions adopt *Innumeracy* as a textbook for a wide variety of courses. While that initially caught me off guard, it got me thinking about *Innumeracy* in a new way: as a text, or supplemental material. In what follows I share my take on his book as a (supplemental) possible text with a focus on numeracy.

Innumeracy: Mathematical Illiteracy and its Consequences

That numerate people are literate does not imply that literate people are numerate. That statement either meant something to you or it did not! Whether it did or did not will depend on your numeracy level. Is it possible to change the statement in a way that will help you to understand? Let us use cars as an example: all convertibles are cars, yet not all cars are convertibles! At this point some may be offended by the approach I took to explain the concept. It may seem 'preachy' or condescending, it was not. In fact, neither is Paulos when he explains the consequences of innumeracy in everyday life. Paulos defines innumeracy as a person's inability to deal with number and chance in a comfortable manner. Innumeracy is a problem that inflicts even the most educated of people. If you are willing to face the fact that you may be a literate innumerate then this book is for you. In fact if you are educated and literate you may be asking, how is it that in the push for literacy, numeracy has been left behind? Is it due to the esoteric nature of mathematics or are we psychologically predisposed to being innumerate? The consequences of mathematical illiteracy are not so obvious yet in Paulos' book he attempts to show (via mathematical arguments) that lots of societal ills are due to innumeracy.

Paulos starts off the first chapter of his book by talking about a fundamental flaw of those deemed innumerate. The flaw he exposes is people's inability to deal with very common large numbers. He states that people have the inability to comprehend the size that is associated with very large numbers such as a million, billion or trillion. Paulos gives many diverse examples that deal with large numbers. Some examples include finding out how many seconds you have been alive for, how long it would take to flatten Mount Fuji via dump truck and how a man who is six feet tall when scaled to thirty feet tall would not be able to support his own body weight.

Another fundamental flaw of the innumerate (connected to large numbers) is the additivity of very small quantities. This point is where Paulos begins to make his connection between innumeracy and society's ills. People's inability to understand that their use of a single can of aerosol can lead to something horrific like the hole in the ozone layer is due to an inability by the innumerate to comprehend an infinite number of little units adding up to something significant. Paulos aptly titles this section of his book as practically infinite numbers. Here I tend to disagree that this concept is one that is not grasped exclusively by the innumerate. The concepts of actual infinity, potential infinity and (perhaps the hardest to understand) practically infinite numbers are topics that can stump even the most numerate of people. This implies that topics presented in the book can cause concern for the numerate and innumerate alike.

Paulos quickly shifts gears into the area of probability, which is the main mathematical focus for construction of arguments in the remainder of his book. The examples that he uses for probability are some of the more bizarre in the book and deal with the death of Julius Caesar and AIDS. These examples have been the source of some controversy. Paulos seems aloof in his dealing of such a tender subject such as AIDS. Yet, I feel he has done this in order to stress a major theme in his book

about the innumerate tendency to personalize. The Julius Caesar example in the book is the one that he wishes he could take back. In this example he makes a slight mistake, which for many turned out to be, an unacceptable error when writing a book on innumeracy. When writing a book with a tone that many feel is denigrating it is of the up most importance note to slip up.

This error opened the door for Peter L. Renz to write an article for *The American Mathematical Monthly* journal in October 1993. In it Renz proceeds to go over Paulos' calculations in excruciating detail. Renz goes over the Mt. Fuji example and the AIDS test and comes up with different answers and spends the remainder of the article providing an example where mathematical error had real life consequences. Directly after Renz's article the journal provided Paulos with an opportunity to reply. Paulos in response to Renz essentially says that Renz wrote an article that was nit picking and spending too much time on details. I find it interesting that Paulos would use that line of defense. After all, it is Paulos who says that one of the great pleasures of numeracy is finding internal inconsistencies (such as Renz did).

As you move into the second chapter of Paulos' book you start to see it for what it really is, a textbook. This is evidenced from topics such as scientific notation, expected values, probability, combinatorics, and statistics. Upon closer examination of the topics we see similarities to textbooks used to teach high school students with a focus on probability concepts. This is not a surprising development. The goal of any high school is to create numerate citizens and thus the topics that they are taught should relate to the topics found in a book titled *Innumeracy*. Butler states, "probability is the very guide of life." This quote nicely summarizes the common theme of his book.

Even though the topics in the book are similar to that of a text book, *Innumeracy* is nothing like a traditional text book. First, Paulos tries to use very elementary mathematics to explain difficult concepts. The examples that he uses are off beat and often involve humour to aid in understanding. The main difference in the book is in what is deemed an "application" of the mathematics. The traditional application uses math as a tool to help in real life situations. Instead, the math is applied to aide in an argument about real life situations encountered in society. A subtle difference in the use of applying mathematics; yet very effective in making his point. Paulos does talk about the misuse of the mathematics, but focuses more on the ill effect innumeracy has on society. The mathematics in the book is essentially used for debunking purposes in the assertions that he makes. These declarations although rooted in mathematical fact seem to deal more in the realm of psychology. The majority of chapter two is spent using mathematics to prove psychological claims of innumerate people.

Paulos states that people who are innumerate underestimate the frequency of coincidences and tend to personalize due to the impersonal nature of mathematics. Another psychological phenomenon discussed is filtering (which is also the source of many errors of the innumerate). Filtering is what determines our personality and we have to be aware of our tendency toward

innumeracy as it may go as far as to bias our judgments.

Mathematically Paulos explains that for a large collection the average value is the same as it is for a small collection. However, when speaking of extreme values of a large collection they will be more extreme than that of a small collection. The problem lies then in the fact that people tend to focus on the extreme value more than the average. If we are filtering out the common place then the rarity will become common at some point, our innate desire for meaning and pattern will lead us astray and our mind will start to try and make connections where there are none. This is evidenced in the media when rarity leads to publicity. This filtering effect can lead one to overestimate events and how they are connected. Soon a mind frame can develop that no one can live up to and may lead to a state of depression. I wonder if the drug companies are onto this and are trying to develop a pill that will cure innumeracy (and thus depression).

Many examples are used to drill home the point that people are psychologically predisposed to be innumerate. Examples such as the birthday problem, stock market scams, games involving dice, chance encounters with people and many others show that some mathematical problems are far from intuitive.

Perhaps people are psychologically predisposed to be innumerate, or at most, to be prone to a momentary lapse of innumeracy? How else can you account for all those numerate people buying lottery tickets? Paulos describes that the innumerate have a tendency to personalize events. For example, if I do not buy that lottery ticket what if my numbers come up? Paulos also explains a very interesting point that I wish he would have elaborated more on. He asks why such a small percent of the people that play the lottery do not play consecutive numbers. This question fascinates me and will continue to do so until I find an answer that suits my needs.

Some readers may have a tough time swallowing the claim that people are psychologically predisposed to innumeracy. If that applies to you then I suggest that you skip the third chapter entirely. The goal of Paulos' third chapter is to show that the tendency to confuse factual statements with sloppy logical formulations (another type of innumeracy) leads to the pseudosciences such as parapsychology, ESP, astrology, numerology, predictive dreams and belief in Aliens.

As I stated, if you were hesitant about the assertions of chapter two then chapter three may not be for you. However, it is an interesting read. Paulos states the esoteric nature of mathematics leads to misapplications of verities and leads to arguments that have escape clauses to account for anything. In fact, he alleges that mathematics is the easiest way to make impressive declarations that are devoid of any factual content, especially to an innumerate audience. Paulos asserts that the faulty logic (innumeracy) of not being able to refute a claim that something exists is often mistaken for evidence that the claim is true. This furthers his earlier psychological claims and how it may go as far as to develop a belief in the paranormal.

So if we are innumerate as a society, who is to blame? Paulos spends the next chapter (chapter four) focusing on

mathematics education. At the beginning of the chapter, he describes a situation with a former math teacher of his that seems to have soured him on teachers on an early age. Even though I think that he has made an intellectual error by using a personal experience as an introduction to a chapter, I agree with all of his statements made about mathematics education.

I do not feel that Paulos proved we are predisposed to innumeracy in his book. Yet his ability to mathematically back up his declarations makes me believe that most of the blame is squarely on the shoulders of mathematics educators. He states that important topics are lacking at the elementary level such as: estimation, inductive reasoning, phenomena, informal logic and puzzles. While at the secondary level combinatorics, graph theory, game theory and probability should be the focus. What is the reason for this? Paulos states the reason is that teachers are afraid that ten year old students will understand the topics and perhaps show them up. Whether this may be true or not is clouded by the fact that this is a generalization made from a personal experience by Paulos and thus does not allow the statement to get the credit that is deserves.

Paulos makes a number of valid assertions about mathematics education and teachers of mathematics and the connection to innumeracy. He declares that teachers are not capable and do not have enough interest in the subject. That blame falls squarely on teacher education. Other issues such as textbooks and lack of use of software are also to blame. He suggests many alternatives to get past these problems such as hiring specialists, salary bonuses, a swapping program with professors and teachers, millionaires sponsoring mathematicians and the use of humour (as seen in his book). Paulos stresses the importance of mathematics education and wishes to break down the esoteric nature of the subject that he feels is one of the causes of innumeracy.

Throughout the chapter Paulos focuses on other causes of innumeracy. All though the main focus is on education he also points out that psychological factors will still play an even greater role. The impersonal nature of the subject of mathematics provides an opportunity for the author to reinforce earlier comments on the tendency to personalize and filter which leads to bias in our judgments. He also discusses how framing of questions and anxiety add to the mix.

Paulos finishes off the chapter with discussion on a safety index that he has created and uses it as a segue into how the media can combat innumeracy. A lack of structure and flow starts to be seen in the end of chapter four and throughout chapter five. It is as if the author knew that he had some points to make and some to go back to and decided to just put them all in at the end of the book. This is seen by digressions and addendums made to earlier points in the book.

The last chapter of the book deals with statistics, tradeoffs and society. Many more complex mathematical concepts are explained including the voting paradox, prisoner's dilemma, type I and type II errors, the law of large numbers, correlation and causation, and regression analysis. The examples used in this chapter are graver societally than the ones shown in previous chapters. One wonders whether the author was leaving the last chapter to show that the

more difficult mathematics to understand leads to graver consequences in society. If that was his goal then his point is well made. Instead of talking about ice cream cones Paulos has focused his attention to elections, decisions made that better society as a whole, politician's decision making and lack of random sampling in the draft lottery of the United States (can you imagine!).

It starts to seem like the author is trying to make a direct correlation to the level of innumeracy and the ill impact on society. This may be true to an extent. The only problem is that he expresses that fractions and percents lead to as many statistical errors as does an error in regression analysis. This acts as a counter example to the correlation yet does not take away from the relative innumeracy that Paulos brings out in his book and the connection to types of societal pitfalls. Chapter five is full of mathematics that may even cause the most numerate of people to pause before they proceed.

In his closing the author focuses his thoughts on what his book is really about, probability. He is sympathetic that understanding probability takes some time to develop but that does not mean that all the related topics (ranging from conditional probability to distributions) should be more widely known in society. He is right in that innumeracy is not suited for such a technologically based society. The end of his closing is used to address the reason for the tone that is used throughout the book. What some may deem as preachy or condescending is actually anger that has stemmed from attrition. As a teacher of mathematics I feel that Paulos has slowly been encountering a personal slow destruction from a society that is entirely dependent on mathematics yet could care less about it, of which I have also felt. This is hard to deal with when you are someone who cares as passionately as he does about his subject. It is the numerate reader like Paulos who has the ability to make the connection to how innumeracy has infested society. I agree with Paulos that the esoteric nature of mathematics must be overcome so that all of society can benefit from its attributes. As he states, mathematics is too important to be left for the mathematicians. I applaud him for taking on the task that he has with this book. Paulos has been able to bring the seriousness of innumeracy that I have felt to the forefront and I will recommend his book to every math teacher I know in a hope to start a minor numeracy campaign that is sorely needed. He has been able to use elementary mathematics to debunk societal issues that may have other wise not been explained and has done so with a sense of humour to help people understand, even though this is no laughing matter.

References

Paulos, J. A. *Innumeracy.* Toronto, ON

Renz, P. L. (1993). Thoughts on innumeracy: Mathematics versus the world? *The American Mathematical Monthly, 100*(8), 732-742.

MATH OPEN REFERENCE (MOR): A REVIEW

Evan Cole

To the delight of many students and teachers, more and more classrooms are becoming equipped with newer technologies such as computers, data projectors, and SMART boards. But

these technologies need to do more than just offer a way of giving notes without the aid of chalk or markers: these technologies provide a new way of interacting with content when coupled with the right resources. This is where websites like Math Open Reference come into play.

Math Open Reference (MOR), located at http://mathopenref.com, is the brain child of John Page and is designed to be used by both teachers and students alike. The goals, as stated on the site, are:

1. To be a source of math information to students any time, any place.
2. To move beyond static, boring text towards engaging interactive content...
3. To provide instructors with the tools they need to move away from teaching, and towards learning facilitators.

Currently, MOR is focused on geometry, but Mr. Page has ambitions to grow his website to be a comprehensive resource for K-12 mathematics and to include content from engineering and the sciences. In developing this website John Page has kept in mind the National Council of Teachers of Mathematics' standards for content.

At present, the MOR website is divided into three main sections: "Plane Geometry", "Coordinate Geometry", and "Solid Geometry". Within each section are a number of related topics (e.g.: Plane Geometry contains topics such as angles and polygons) and each topic is broken down into its related subtopics. These pages contain one important improvement over textbooks and most websites – rather than just plain text and a few illustrations to reinforce a concept, MOR integrates fully interactive Java applets. Teachers can demonstrate concepts like naming angles by rotating the terminal arm to show where an angle is acute, right, obtuse, straight, or reflex using a data projector. Alternatively, teachers can link to the applets from a class website or blog and have students play with the applets on their own.

In addition to the geometry topics, there are also a few useful tools. One applet is a scientific calculator, which is useful to show multiple students at a time how to use scientific functions on a calculator. There are also applets for linear, quadratic, and cubic functions where the coefficient and constant terms can be adjusted with sliding controls and the effects are displayed on the graph. These applets allow students to engage with the functions and to better understand the effects of changes.

As the site grows beyond high school geometry, there are a few changes and additions that would make the site even more beneficial. While the site has a scientific calculator and applets for various functions, a graphing calculator would also be useful to determine values such as intercepts. It would also be helpful if there were a way (other than doing a screen capture) to export an

image as a .jpeg or to create a printout. This being said, Math Open Reference is a tremendous site that offers opportunities for teachers to enrich their lessons and for students to better engage with the ideas at hand.

Math Beyond School

Harley Weston

"When will I ever use this?" A question we have all heard. On Quandaries and Queries, the question and answer service on Math Central (MathCentral.uregina.ca) we have received this question many times since this service began in 1995. This question comes from both students, either as part of a homework assignment or from their own frustration or curiosity, and teachers who are looking to motivate their students. We have often responded with conventional applications of specific areas of mathematics but in recent years we have been able to supplement these responses with examples of our own which come from Quandaries and Queries.

The Quandaries and Queries service was developed to be an aid to students, teachers and parents who have questions concerning mathematics in school, and that remains the source of the majority of questions we receive, but we also receive questions from the general public. Questions from homeowners asking the amount of topsoil need to level an uneven garden, business people calculating percentage increase, golfers scheduling a tournament, artists designing a geometric object and many more. Some of these questions and their responses supply examples of ordinary people who need to use a specific topic in mathematics to answer a question that has arisen in their work or their daily lives. These questions and our responses are stored in a database. If you go to the Quandaries and Queries main page (MathCentral.uregina.ca/QandQ/) and use the Quick Search to search for the term **math beyond school** you will receive a list of approximately 300 such questions and responses, too many to browse effectively. The search can however be focused somewhat by adding a mathematical topic. For example searching for **math beyond school Pythagoras** returns a list that includes a question from a parent building an octagonal playhouse, a card player constructing a poker table and a parent building a garage; the list returned in a search for **math beyond school trigonometry** contains a question from an artist constructing a three-dimensional five pointed star, a farmer wanting to know the amount of fuel in a barrel lying on its side, and a biker constructing a ramp for a motorcycle trailer; and **math beyond school ratio** returns a question from a graphic designer on scaling a logo. In some cases the mathematics required to solve the problem is quite straightforward and in other cases the specifics are somewhat complex but the mathematical concepts might be exactly what your students are studying.

In the summer of 2007 we expanded our practical applications of mathematics by creating a collection of resources on Math Central called Math Beyond School (MathCentral.uregina.ca/beyond/). This section was developed by two people who were at that time students at the University of Regina, Natasha (Glydon) Olynick a teacher in Vanguard Saskatchewan and Stephen La Rocque who works for the Canadian

Mathematical Society in Ottawa. Three examples of resources on Math Beyond School are titled The Police, Navigation, and Medicine and Mathematics. These resources are stored in a database that can be searched by keyword or browsed by title, keyword and grade level. Natasha created the resources and Stephen developed the database.

I invite you and your students to sample our applications of mathematics either by searching the Quandaries and Queries database or visiting Math Beyond School.

I would like to expand the number of resources in Math Beyond School and in particular to include topics that teachers have found useful. If you have a favourite application of mathematics that you use with your students and are willing to share it with others I would very much like to add it to Math Beyond School, giving you recognition for the contribution. It might be a few sentences, a reference to a web site or even a whole lesson you have developed. If you and your students have found it useful then I am sure others will also. You can reach me at weston@MathCentral.ca.

MATH SUCKS

Jennifer Joachine

They teach us math
in middle-class North America
I have a test today, and
it will let me know I can do this,
and that

It will let me know I can add 2 + 2
(which equals 5)

haha, mockery

So after staying up till 2
and waking up at 6
I understand all 8 chapters

I took the test that they issue
in middle-class North America
and it lets me know I can do this,
and that

It lets me know 2 + 2
actually equals 4
(who knew?)

But it doesn't show me
how to add my heart to another

Or how to subtract racism, poverty and
sexism from the world

It doesn't show me how to multiply
a caring father
and a loving mother
with happiness
as the product

and it can't begin to explain
how to divide pollution out of our air

& that's just
the bare basics

vinculum

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STUDENT–CENTERED EDITION

The following excerpt is from the WNCP's Common Curriculum Framework (CCF) for K–12 Mathematics (http://www.wncp.ca/english/subjectarea/mathematics/ccf.aspx):

> ***BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING***
>
> *Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences. Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics. The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary (p. 2).*

Recognizing Saskatchewan's new mathematics curricula are based upon a student-centered approach to learning mathematics, the Journal of the Saskatchewan Mathematics Teachers' Society, *vinculum*, is seeking articles for a 'student-centered' edition. In other words, and with a very liberal sense of 'student-centered', we are seeking **Articles** and **Conversations** that focus on mathematics students. We also welcome submissions that fall outside of the October issue's theme.

Given the wide range of parties interested in the teaching and learning of mathematics, we invite submissions for consideration from any persons interested in the teaching and learning of mathematics, but, as always, we encourage Saskatchewan's teachers of mathematics as our main contributors. Contributions, student-centered or otherwise, must be submitted to egan.chernoff@usask.ca by September 1, 2009 to be considered for inclusion in the October issue.

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