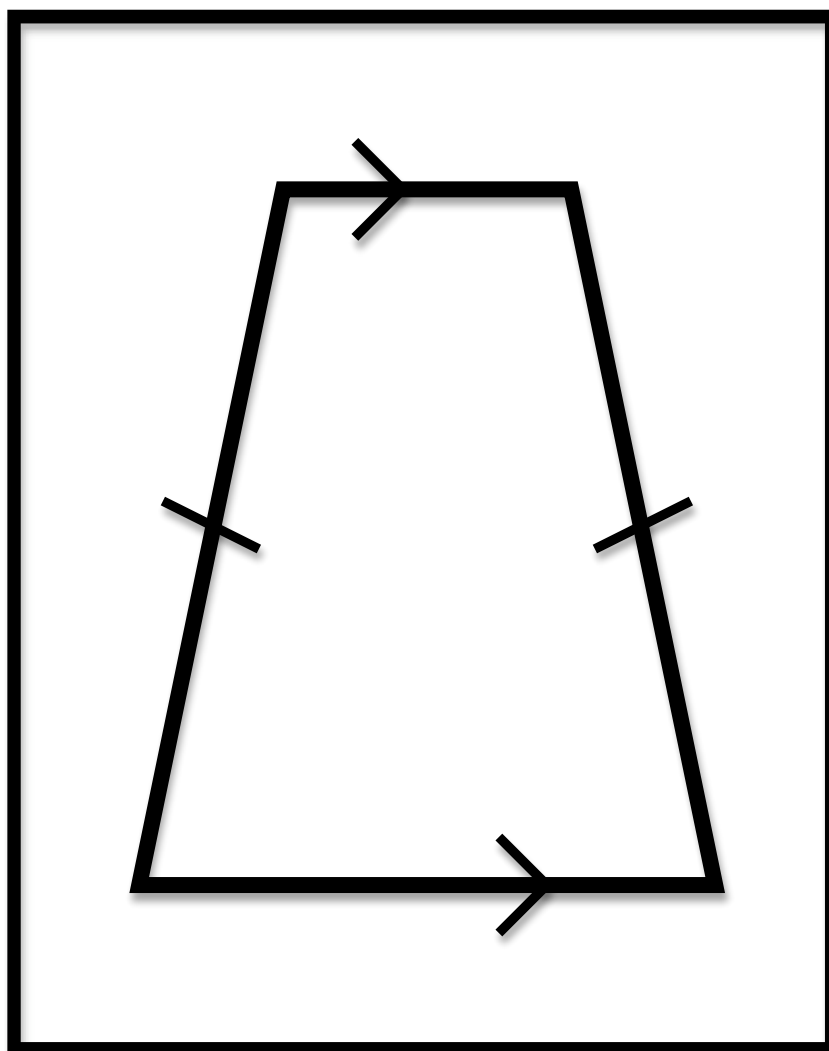


vinculum

Journal of the Saskatchewan Mathematics Teachers' Society

Volume 1, Number 2 (October 2009)

STUDENT-CENTERED EDITION



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SMTS objectives—as outlined in the January 1979 SMTS Newsletter—include:

1. To improve practice in mathematics by increasing members' knowledge and understanding.
2. To act as a clearinghouse for ideas and as a source of information of trends and new ideas.
3. To furnish recommendations and advice to the STF executive and to its committees on matters affecting mathematics.

vinculum's main objective is to provide a venue for SMTS objectives, as mentioned above, to be met. Given the wide range of parties interested in the teaching and learning of mathematics, we invite submissions for consideration from *any persons interested in the teaching and learning of mathematics*. However, and as always, we encourage Saskatchewan's teachers of mathematics as our main contributors. *vinculum*, which is published twice a year (in February and October) by the Saskatchewan Teachers' Federation, accepts both full-length **Articles** and (a wide range of) shorter **Conversations**. Contributions must be submitted to egan.chernoff@usask.ca by March 1 and September 1 for inclusion in the April and October issues, respectively.

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EDITORIAL: STUDENT-CENTERED EDITION

Egan Chernoff

Do you remember last winter? I certainly do. In fact, I will forever remember the winter of 2008/2009. Having moved from Vancouver in August of 2008 (stop laughing), our last winter was my first winter. (As the saying goes, you never forget your first time.) While walking over the university bridge in 50 below temperatures, I began to panic about a course I was offering in the summer. Given the conditions people had to put up with in the winter, surely no one was going to spend two weeks of their summer (vacation) stuck in my classroom. To my surprise, and delight, 24 individuals, interested in the teaching and learning of mathematics, and from all over the province, converged on the University of Saskatchewan campus to discuss trends and issues in mathematics education.

As our discussions continued, a theme began to surface: research findings, trends, and issues related to the teaching and learning of mathematics seldom spread beyond the mathematics education community. While ruminating over the theme that emerged, I revisited the SMTS objectives, which were outlined in the January 1979 SMTS Newsletter (and are found on the inside cover of each edition of *vinculum*). My attention was drawn to the first and second objectives of the SMTS, which are, respectively, “to improve practice in mathematics by increasing members’ knowledge and understanding” and “to act as a clearinghouse for ideas and as a source of information of trends and new ideas.” I also recalled, the main objective of *vinculum* is to provide a venue for SMTS objectives to be met. With the abovementioned objectives in mind, the main purpose of the course became the dissemination of the students’ newly gathered knowledge, through *vinculum*, to those individuals involved with the teaching and learning of mathematics in Saskatchewan (i.e., the members of the SMTS).

(Enough with the back-story.) In this edition you will find 24 articles, focusing on trends and issues in mathematics education, written by the 24 students who converged on the University of Saskatchewan campus in the summer of 2009. In other words, welcome to the student-centered edition of *vinculum*.

To be clear, and as you are about to read, the views expressed in each article of the student-centered edition of *vinculum* are those of its author, and not, necessarily, the views of the SMTS, the SMTS executive, or the SMTS Editorial Board. However, the abovementioned declaration was not made for hedging purposes. In fact, the declaration was made to document the fact that the student-centered edition of *vinculum* embodies a central tenet of our new mathematics curricula:

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds...Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics (WNCP’s CCF, p. 2)

One last thing, the SMTS executive welcomes two new members to the Editorial Advisory Board here at *vinculum*: Gale Russell (former editor) and Murray Guest. As most of you are aware, Gale and Murray are heavily involved in the teaching and learning of mathematics in our province. Their knowledge and experience will play a key role in shaping the purpose and direction of your journal.

PRESIDENT’S POINT

Stephen Vincent

Doing what has always been done happens automatically. I guess that’s why it is always done. If I do not plan and think about best practice in mathematics education I will, by default, end up teaching how I was taught or how I have seen others teach. Sometimes it is difficult to take a step back from our busy workload and think about what we do. By the time the end of June

hits, we feel like we need a break from thinking about school. We usually have some intentions of working ahead in August, but with most teachers I talk to, this is rarely actualized. Some things disrupt this annual cycle and cause us to take a closer look at instruction, assessment and student learning in our mathematics classroom.

Taking three-and-a-half classes as a grad student in July this year was the catalyst for me. I found myself asking questions such as: Is there a better way to teach this topic? How can I know that students have a full understanding of this area in mathematics? How can I make this relevant to my class? Am I being effective in the way I teach? What is the appropriate role of technology in my classroom? These are some of the questions that I was forced to think through in several of my courses. I didn't find definitive answers to all these questions, but I did read some insightful articles and research studies as well as engage in some meaningful discussions with fellow mathematics teachers and teachers of different grades and subjects. It is important to consider research findings and other vantage points in mathematics education.

That is the focus and rationale of this special edition of our journal. Not every aspect will be applicable for your classroom nor will you agree with everything written. However, it will make you think about what you do as a teacher and why you do it. My hope is that you read these articles with an open mind, investigate further on topics that are particularly interesting to you, and engage in discussion with colleagues the implications of various viewpoints. As the Saskatchewan Mathematics Teachers' Society we want teachers to think critically about their own classroom pedagogy in order to promote best practice in mathematics education throughout Saskatchewan. So: Do something different this fall and use this journal as an opportunity to take a step back and think about your practice as a mathematics educator, unless of course, that is what you have already been doing.

STUDENT ENGAGEMENT IN MATHEMATICS

Randi-lee Loshack

Engagement in mathematics classrooms is an integral part of students' academic success (Newmann, 1992). Engaged students make a "psychological investment in and effort directed toward learning, understanding, or mastering the knowledge, skills, or crafts that academic work is intended to promote" (p. 12). Engagement can be broken down further into three categories: behavioural, emotional, and cognitive engagement (Fredricks, Blumenfeld, & Paris, 2004). This article is going to focus on cognitive engagement in the mathematics classroom.

It is important to note that students may still perform well without being deeply engaged in knowledge, skills, or crafts. This is because they have memorized a series of steps or rules that enable them to get the right answer without developing the conceptual understanding behind them. This can be problematic, because once forgotten, rules are not easily retrievable without the concepts to support them (Hiebert & Lefevre, 1986, as cited in Ball, 1990).

In order for a teacher to cognitively engage students in mathematical concepts they must reflect on their own content knowledge. Shulman (1986) categorizes three areas of teacher content knowledge: (1) subject matter content knowledge, (2) pedagogical content knowledge and (3) curricular knowledge. Division of fractions is "often considered the most mechanical and least understood topic in elementary school" (Tirosh, 2000, p. 6). Using Shulman's first two categories as a framework, this article will revisit our knowledge with respect to the division of fractions and will examine what we can do to engage students into developing a deeper understanding of what it means to divide.

In order to engage students in mathematics, teachers themselves must have a thorough understanding of the content. With respect to subject matter content, it "is

more than just knowing that something is so, it is fully understanding why it is so, on what grounds its warrant can be asserted and under what circumstances our belief in its justification can be weakened or denied” (Ball, Hoover, & Phelps, 2008, p. 391). When dividing fractions, the most common calculation used to achieve the answer is to “invert and multiply.” It is important to have a clear understanding of the reasons for the invert and multiply algorithm.

Here is an important, but by no means exhaustive list of questions teachers need to ask themselves with respect to their own knowledge of fractions: (1) What does division mean? (2) Does that definition work for all real numbers or just natural numbers? (3) How can we visualize division of fractions? (4) Does it work for all kinds of questions? (5) Can we come up with a story to represent that expression? (6) Can we do all of the above when the divisor is larger than the dividend? and (7) Why do we “invert and multiply”? Looking at these kinds of questions with an open mind encourages educators to rethink what division of fractions is and how they can best portray it to students in a way that they will engage in the meaning of division of fractions. It also gives educators the knowledge base to be able to answer the question of “why?” It is important to begin to reflect on this and other topics in mathematics (e.g. why a negative times a negative is a positive) to exchange a deeper understanding for memorized rules.

Once teachers have a thorough understanding of the mathematical content, they can begin to look for powerful ways to engage students in mathematics. Herrington and Oliver (2000) state, “emphasis in school...has been on extracting essential principles, concepts, and facts, and teaching them in an abstract and decontextualized form” (p. 23). When students view mathematics as a series of steps and rules to memorize and follow, they begin to view mathematics as a “bunch of arbitrary, illogical rules” (Devlin, 2008, ¶ 1). Thus “when an algorithm is viewed as a

meaningless series of steps, students may forget some of the steps or change them in ways that lead to errors” (Tirosh, 2000, p. 7). Rather than just teaching the rules and letting the students practice, teachers must use a teaching strategy in which students can be part of the creation of their own knowledge. In this way, students can take a seemingly abstract rule and make it more tangible for themselves.

In order to do this, the educator must reflect and grow with respect to their pedagogical knowledge. Pedagogical content knowledge refers to the knowledge of various teaching strategies, how to use them, and the ability to be able to foresee, understand, and know how to react to any difficulties the learners may have (Shulman, 1986). This is an integral part of engagement. If a teacher is able to create an authentic task through which to deliver content, the students are more likely to be engaged and pull meaning from the activity.

With respect to dividing fractions, there needs to be a focus on looking past the memorization of the “invert and multiply” algorithm. This is an excellent opportunity to have students answer division questions using their previous knowledge and then ask, “Why does this work every time?” Students can investigate using manipulatives, working in groups, or any way they need to find the answer to “why” they “invert and multiply.” It is important during this time to keep questioning them with the same questions that teachers needed to first ask themselves, with respect to content knowledge. By having students construct their own knowledge it gives them ownership in their learning (Savery & Duffy, 2001). Hopefully, students will then be able to remember the algorithm because they understood it, not just because they memorized it.

In order to engage students more deeply in our mathematics classrooms, educators need to look at their own knowledge of the curriculum and how they deliver it to the students. Ball (1993) emphasizes that

“teaching and learning would be improved...if classrooms were organized to engage students in authentic tasks, guided by teachers with deep disciplinary understandings” (p. 274). As seen by using the example of division of fractions, once educators take a deeper look at their own understanding of division and how it pertains to fractions, they are able to engage students and help them construct their own meaning. In this way, they not only have students who are more eager to learn mathematics, see more connections throughout their studies, and are more successful, but the teachers themselves become more fluent in their own subject and pedagogical content knowledge.

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CONVERGING TRENDS: IS IT FATE?

Jacqueline Johnson

Ethnomathematics, new curriculum, and First Nations self-determination – are these recent ideas and realities all part of the same force? What do they have in common? How are they related and how do they affect a classroom teacher of mathematics? This article will attempt to illustrate the commonalties of these trends, as well as give teachers a reason to engage themselves in working towards finding practices that will support the realities of the present and create a bright future for Saskatchewan.

The definition of the term *ethnomathematics* has been evolving since it was first used by D'Ambrosio in 1985 (Bishop, 1988). It has evolved enough to make it useful for teachers in a mathematics classroom. The value for education in accepting the notion of ethnomathematics is realizing that all students have certain understandings about the way the world works, including the mathematics they have encountered in their world and have come to terms with in their lives (Boaler, 1993, p. 9). However, the way students have been taught

in regular North American classrooms does not acknowledge this pre-existing knowledge. The reasoning behind this method of teaching may have been efficiency. If a teacher can assume that all students have the same knowledge about a certain topic, the teacher can begin at the beginning and move at a certain rate through the material. The class did not need to take time to explore, debate or reflect because, it was thought, if they listen and practice they will learn. Many students did not. The mathematics lessons seemed to be unrelated to anything in the students' lives and therefore was considered *school math* that was done at school, but not good for anything practical (Boaler, 1993). Efficiency does not promote engagement and interest in the subject nor does it induce positive values towards mathematics, which is the aim of the new curriculum: "The aim of the mathematics program is to prepare individuals who value mathematics and appreciate its role in society" (Saskatchewan Learning, 2007, p 3). We can use what the students know when they come to school as a springboard for creating interest in mathematics and learning the outcomes in the curricula.

In the past few years we have seen governments and churches apologize to First Nations peoples for the way the students were treated in residential schools. There is no denying that at their worst the schools were abusive and at their best they destroyed family ties and security. Many aboriginal students still carry self-doubt and discount the knowledge they have attained through their personal journeys and many people see education as the key to change (Alberta Education, 2005). Teachers of any culture working in any setting can teach in ways that parallel the desires of aboriginal people. This means giving the students a voice in a classroom based on mutual respect.

Arguably, all students regardless of race, gender, or ability have a right to have their opinions heard and valued. "In a mathematical environment, students feel comfortable trying out ideas, sharing

insights, challenging others, seeking advice from other students and the teacher, explaining their thinking and taking risks" (Van de Walle, 2005, p. 39).

The Saskatchewan Curriculum states that if students are a part of a classroom rich in dialogue they will be exposed to more perspectives from which to make connections (Saskatchewan Learning, 2007). The mathematics classroom, in general, is starving to be richer than a place where students compete for high marks and have little ability to use the mathematics in any situation outside the classroom walls. "Traditionally, Aboriginal learning was often a multisensory small-group activity, beginning with observation and evolving into tactile, hands-on experiences. The classroom was the home and the village and, most significantly, the natural environment" (Alberta Education, 2005, p. 42). Traditional Aboriginal educational styles seem to be a good fit with our new curriculum and its commitment to focusing on the big ideas as compared to unconnected, isolated bits of information.

Ethnomathematics research would encourage teachers to find out what the students are bringing to class and then to develop meaningful problems to solve by building on the skills and knowledge the students possess. In this way all students' contributions are valued and all students value what they are doing and learning. Aboriginal people are asking teachers to take time and listen and get to know the students. Mutual respect is of the utmost importance for learning to take place. Our new mathematics curricula in Saskatchewan suggest that we teach for understanding through teachers and students sharing experiences and reflecting on the work and learning of all. These demands can all be integrated beautifully; indeed, the differences are practically indiscernible.

It will be a bit frightening to teach this way, until teachers get used to the new sights and sounds. It may feel like teachers have lost control at first because they will no

longer present themselves or be perceived as the one with the answers. Teachers will learn much more in a day than they did before. Teachers will need to make sure they are confident mathematicians themselves so they can recognize powerful discussion rich in mathematics content. They will have to practice steering conversations back on track, but it will be challenging, invigorating and meaningful for all. Today, teachers are at many different places on the continuum of teaching in an inquiry and co-operative manner, but as long as they jump in with both feet and begin to hone their skills in this new way they will move forward to the benefit and credibility of the profession, as well as the improved education of the students.

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ETHNOMATHEMATICS, CONSTRUCTIVISM, AND SASKATCHEWAN TEACHERS AND STUDENTS

Gale Russell

What is mathematics? The University of Cambridge defines mathematics as “the study of numbers and patterns in the most general sense” (Connecting Mathematics, 2004). In a list of related terms on the same website is ‘Mathematics in Culture’ which is defined as: “Mathematics that appears in everyday life or in history.” The notion that mathematics is somehow culturally bound is one that catches many educators, students, mathematicians, and the public at large off guard. Whether it is in movies, newspaper articles, or other works of fiction or non-fiction, mathematics is frequently presented as THE truth, THE language beyond question, even THE hope for mankind’s survival. Why then would such a concept of “mathematics in culture” be defined on a university website? One possible answer to this question lies in the discipline of ethnomathematics in which researchers argue that there is no universal language of mathematics, but rather that there are many and varied culturally relevant incidents of mathematical ways of knowing and being (Bishop, 1988; D’Ambrosio, 2006; Orey & Rosa, 2006).

If there is no universal mathematics, what is the mathematics that we teach in schools? Modern curricula are designed with the intention of providing students with the mathematics needed in the modern world. Nevertheless, like educators around the world, Saskatchewan teachers face the dichotomy that the mathematics of school curricula is not necessarily the culturally relevant mathematics (ethnomathematics) of their students.

Many researchers suggest that bridging between these seemingly two ends of a spectrum can be done through the use of cultural contexts in mathematics learning (D’Ambrosio 2006; D’Abreu 2000). Other researchers are concerned about such use of

cultural contexts. Roberts (1996) notes “Students may consider that links to school mathematics are an attempt to take over their cultural knowledge” (p. 45). Similarly, Meaney (2002) warns that “The use of indigenous activities must be done with respect and care or they become a tokenistic activity before ‘real’ mathematics is undertaken” (p. 170). How can Saskatchewan teachers find out what are relevant cultural contexts to use with their respective students, while at the same time also knowing how to treat the contexts respectfully? How can a teacher ensure that the real world learning activities that they bring into the mathematics classroom will not result in stripping away the cultural relevance of the contexts?

This dilemma increases when one considers how even in homogeneous settings (rarely seen in Saskatchewan), cultural values are not consistent between individuals (De Abreu, 2000). In addition, De Abreu notes that what someone from outside a culture deems to be of cultural importance may not in fact be important at all within the culture. Is the practice of making mathematics learning culturally connected and valuing asking too much of teachers?

Morris (2009) provides evidence that this challenge can be much more easily met through the use of “constructivist teaching and learning that allows learners to build on their knowledge, thinking, ways of knowing and doing, skills and mathematical language” (p. 6). Rather than the teacher attempting to find and make the culturally relevant connections in mathematics classes, instructional strategies should be used to help students bring their personal ethnomathematical understandings to the classroom.

Meaney, Fairhill, & Trinick’s research (2008) into the role of culturally-informed language in mathematics provides evidence that a constructivist approach to the teaching and learning of mathematics can effectively engage students in development of

mathematical understandings. The researchers demonstrate through class discussions how students identify expressions and words in their native language that represent a mathematical concept. For example, some te reo Māori students chose to use the word *whakawhānau*, meaning “making families,” for what in English is often called “collecting like terms.” Despite this being an example far removed from Saskatchewan, the notion of “making families” may have meaningful connections to some students whose First Nations and Métis’ cultures value the family.

In Saskatchewan, many of our First Nations and Métis students speak English as their first language; however, this does not mean that their culturally-based understanding of a word is the same as the meaning intended by the mathematics curriculum. An example of this type of dichotomy emerged through a discussion with Dr. Edward Doolittle of the First Nations University of Canada about grade three students’ understanding of the word “equal.” Researchers such as Faulkner, Levi, and Carpenter (2002) have shed light on how students believe that the equal sign implies an action must occur rather than representing the same amount of quantity. Dr. Doolittle’s reflections on the word “equal” brought forth a different dimension, namely that for many First Nations and Métis students, regardless of their first language, equal does not imply “same quantity,” but rather “fair” or “for the good of the community” (personal communication, January 1, 2009). This is not an error in the students’ use of the word equal, but what Atweh, Bleicher, & Cooper (1998) would refer to as the “dialect” of those students. Students who understand through such a dialect need opportunities to bring their personal understandings forward and to create additional meanings for the same word.

As Saskatchewan teachers embark upon the implementation of the new mathematics curricula, the incorporation of constructivist

teaching strategies with openness to ethnomathematical understandings will provide students with the opportunities to succeed mathematically. Moreover, this success will not only be in relation to the mathematics of the curricula, but also the ethnomathematics of the students. Students will then understand that the universality of mathematics lies in everyone's ability to construct, use, and share personally relevant mathematics.

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THE STRUGGLE OF MATHEMATICS CURRICULA IMPLEMENTATION

Tamara Schwab

Around the world, education is a dynamic and ever-developing field of research and practice. Throughout the last century there have been significant changes occurring within best practices due to the exponential growth in research, which has been conducted. Saskatchewan has been a part of these changes and most recently new mathematics curricula has been developed and implemented for kindergarten through grade eight, with the senior grades soon to follow. The development and implementation of a new curriculum is a time consuming and difficult task because so many factors need to be taken into account. The issues Saskatchewan is currently experiencing, as a result of the process of curricula implementation, need to be discussed with respect to public relations and teacher education if they are to be resolved.

There are many factors involved in implementing new curricula. Our province needs to take into account the lessons learned by other educational bodies that have already implemented new mathematics curricula. The most widely known example is the California 'Math Wars' which have been occurring over the last fifty years. When California introduced their new

curriculum in the 1960s, it was in response to public outcry after “the Soviet Union caught the United States off guard with its successful launch of the satellite Sputnik” (Schoenfeld, 2004, p. 257). American society felt that the programs being offered to their children were inadequate. It is clear that educators were not driving the curricular reform. Rather, the general public and politicians were the impetus behind proposal changes, which can be very dangerous. The new mathematics curriculum was poorly implemented and soon the state was forced by public demand to return to their previous curriculum, largely due to the public’s response to the new curriculum, which they had demanded (Schoenfeld, 2004). The reasons for this movement back to a traditional curriculum warrant a closer inspection, so that we can avoid creating such a reaction to the new mathematics curricula in Saskatchewan.

When the new mathematics curriculum was released in California, teachers were uncomfortable due to lack of preparation. As a result, many educators did not teach the new curricula or taught it in a traditional manner. This discomfort carried over to students, who became very confused in their mathematics classes. Parents felt lost and frustrated because they were unable to understand this new approach and did not know how to help their struggling children. Due to the parents’ lack of understanding regarding the value of the new curriculum they demanded change (Schoenfeld, 2004).

In Saskatchewan, both the ministry and school boards need to communicate with all teachers and the public regarding any new changes occurring in the educational system, especially curricular changes. “Teachers who [have] themselves been taught in traditional ways, [are] now being asked to teach in new ways and not given much support in doing it” (Schoenfeld, 2004, p. 272-273).

How can teachers be better prepared to consistently teach a new mathematics curriculum to their students with a full

understanding of the new curriculum and its instructional approach? How can better preparing teachers result in proactive communication with the public on the new curriculum being implemented? Both teachers and the public need an understanding of why changes are being made to curricula. Without this basis we will likely see the pendulum swinging back and forth between curricular approaches in much the same way as California has experienced.

Saskatchewan’s new mathematics curricula are based on the now widely accepted constructivist theory of learning. It is rooted in cognitive psychology and is based on the tenet that “children are creators of their own knowledge” (Van de Walle & Folk, 2004, p. 28). In other words, students build understanding through connecting new ideas to their prior or pre-existing knowledge. Children must be actively involved in their learning for connections to be made and learning to occur. According to Van de Walle and Folk (2004), when used properly, this approach to teaching and learning: is intrinsically rewarding; enhances memory; reduces the material that needs to be remembered; aids in learning new concepts and procedures; improves problem solving abilities; increases the potential for invention; and improves attitudes and beliefs. These are all firmly held goals of our society and developers of curricula need to share this foundation for our new mathematics curricula with both teachers and parents.

Sharing the foundations of our new curricula with teachers needs to be done through the constructivist theory of learning, which we are asking teachers to use in their classrooms. Many teachers will be more comfortable with the constructivist approach to learning and teaching if they have experienced it themselves. Teachers need this experience to appreciate constructivism and truly understand how it can be applied, rather than having someone simply explain it to them. When the teachers reach a level of comfort with this approach, the curriculum will be taught more consistently

for the students. The public's understanding of the reasons for implementing new curricula will naturally improve through interactions with teachers, the educational system's front line in dealing with the public. "If parents feel disenfranchised because they do not feel competent to help their children and they do not recognize what is in the curriculum as being of significant value... they will ultimately demand change" (Schoenfeld, 2004, p. 257). Many parents, and even teachers, in Saskatchewan are currently experiencing these feelings with our new mathematics curricula. With a proactive approach to informing the public and training educators, the feelings of helplessness and frustration brought forth by the implementation of new mathematics curricula will be greatly reduced.

We are fortunate in Saskatchewan to already have a foundation in place for collaboratively developing curriculum. "The province of Saskatchewan has a reputation for relatively amicable relationships between the various stakeholders in public education. Departmental officials, university personnel, teachers, trustees, and administrators" (Lyons, 1997, p.1) work together to conduct research and develop new curricula. We now need to expand the partnerships already in place for curricula development to aid us in implementation of new curricula, as cooperation can be seen as the foundation for successful implementation.

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THE SEARCH FOR DEEP UNDERSTANDING IN MATHEMATICS

Lisa Eberharter

In the movie *Pretty Woman* there is a man on the street saying, "what's your dream?" People may make different connections when they think of that phrase. It could remind them of where they were when they watched the movie, how much they like Julia Roberts as an actress, or it may even remind them of the trip they took to Hollywood. What dream do teachers have for their students? Teachers of mathematics (may) agree, one goal worth striving for is a deeper student understanding of mathematics.

In Saskatchewan, a renewed math curriculum is being implemented. The renewed curriculum brings with it new ideas for teaching mathematics. A constructivist approach to teaching is a central philosophy of research regarding the psychology of mathematics education (Ernest, 1993). There has been so much research, in fact, that the result has been a multitude of different facets that make "constructivism" (Simon, 1995). The focus of this paper is not on what constructivism is, but on what change the theory can bring to student learning and understanding. Constructivism can provide experts with a way to analyze the mathematical learning that goes on in a classroom, but it does not provide a particular model to follow in terms of teaching mathematics (Simon, 1995).

Simon (1995) states, "the question of whether teaching is "constructivist" is not a useful one and diverts attention from the more important question of how effective it is" (p. 117). Constructivism may not have its own set of ready to use instructional strategies; however, it does imply new goals for teachers and students (Simon & Schifter 1991). A question for educators faced with implementing a renewed curriculum is when, how, and why should I change my teaching style? Questioning the effectiveness of various methods of

instruction enables teachers to make a more informed decision about the instructional strategies they choose, and the impact on student learning.

What could a change in teaching style look like? Classroom tasks traditionally involve students working on routine problems by using an algorithm that was probably given as the sole representation by the teacher. However, student tasks can also be more complex, open-ended problems that encourage students to reflect and justify their personal understanding. Tasks can be based on real-life contexts and have multiple solution strategies (Stein, Grover & Henningsen, 1996). The selection of tasks used to enhance student engagement in a cognitive process is difficult, especially when the training of teachers is not adequate in this evolving “constructivist” theory.

Tasks that are grounded in real world experiences and mathematical models that are familiar to students is one way to build on existing cognitive structures (Simon & Schifter, 1991). Simon (1995) suggests the idea that, “students construct their understandings, they do not absorb the understanding of their teachers” (p. 122). A task should be presented by the teacher after some reflection on their own mathematical understanding and what the teacher perceives the students’ previous understanding to be (Simon, 1995). Not only is the posing of a real world problem important, but also allows students to solve the problem themselves.

Improving students’ problem solving abilities is not a new focus of teachers, but one that has been present for years. Just like people make different connections to the phrase, “what’s your dream,” teachers and students make different connections when solving problems. Therefore, finding a task that challenges students on a variety of different levels is a very difficult undertaking. The task could constantly evolve as the viewpoint of the students’ previous knowledge is discovered throughout the class discussion.

Once a rich problem is chosen, the direction the lesson will take in the classroom setting poses another challenge. Simon (1995) writes, “the only thing that is predictable in teaching is that classroom activities will not go as predicted” (p. 133). The success of any task posed by a teacher has variables that influence the effective implementation. A teacher takes a certain amount of risk by posing a problem without any real idea where the students will go with the task. If the students perceive the task to be difficult they may push the teacher to reduce its complexity by specifying procedural knowledge, or the teacher may get caught up giving too much direction (Stein, Grover, & Henningsen, 1996). However, if students take ownership of not only the problem, but also the solution they are developing to share with the class, the task will have personal meaning resulting in students becoming invested in the problem’s outcome.

The success of a task can be evaluated through a clear message to students that justification and explanation of their answers is just as important as the correct answer (Stein, Grover & Henningsen, 1996). The questioning that the teacher demonstrates throughout a lesson can also increase the level of understanding that the students achieve. Prompts, such as focused questioning, can be used by the teacher to help students clarify the strategy they used, take ownership of what they are learning, listen to others, and revise and modify their own strategies (Fleming Amos, 2007).

The expectation that all teachers will immediately be able to successfully change their teaching style, pose relevant tasks, guide students with appropriate questions, and reflect with students on the learning taking place is unrealistic. Teachers have not, in many cases, experienced this idea for themselves, and have not had the necessary preparation. If success is not experienced on the first attempt, the initial efforts are often abandoned (Simon & Schifter, 1991). Instead of abandoning ideas after the initial attempt, reflection on relevant questions,

level of difficulty, and time given for students to work on, a solution should be discussed and changes could be made to improve. Developing rich tasks for the renewed curriculum could make teaching more effective, learning a greater possibility, and deeper understanding of students a reality.

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PARENTS AS PARTNERS: CREATING A TEACHER/PARENT RELATIONSHIP

Claire McTavish

At present, Saskatchewan is in the process of implementing a new math curriculum based on the constructivist theory of learning. In essence, the curriculum changes strive to move away from procedural learning and toward the understanding of mathematics and the ability to solve problems. As with any educational change, a major component for success is parental involvement. For the purposes of this article we will discuss the

importance of a partnership between parents and teachers as they work together to improve student learning.

Creating a partnership. In past attempts at math education reform, parents and teachers have been labeled as enemies. Historically, particularly in California in the 1960s and in 1989, parents have been so influential that they dissolved attempts for math reform (Schoenfeld, 2004). In previous math reform literature written for educators, e.g., curriculum documents and National Council of Teachers of Mathematics (NCTM) articles, Peressini (1998) found that parents were generally portrayed as impediments to reform, positioned at the outskirts of the change, and were not recognized as significant contributors to the mathematics education of their children. It can be argued that this portrayal of parents in teacher education documents made it impossible for a partnership to be formed. This was surely a contributing factor to the decline of previous attempts at math reform.

Interestingly, the idea that parents were a barrier to change may have been a fallacy. Research has shown that parents (even those who vocally oppose reform in name) are supportive of fundamental teaching practices that are embedded in reform. Lehrer and Shumow (1997) studied parents' beliefs about mathematics reform in a community of supposed naysayers. "They [The parents] expressed reservations about the amount of talk during mathematics, the lack of focus on teaching algorithms...and the continued availability of resources, such as unifix cubes, to assist [in] problem solving" (p.47). After having parents observe the classroom practices associated with constructivist learning, the researchers were surprised to find that parents "generally believed that practices such as sharing solution strategies, inventing algorithms, and making mathematical conjectures were useful ingredients to mathematical learning" (p.54). Parents need a chance to experience the changes in mathematics in order to develop their own conclusions about what they believe to be valuable education for their

children. Being a part of their child's learning will break down the supposed barriers that impede change.

Teachers and parents are on a journey of learning together. Both groups play significant but not identical roles (Lehrer & Shumow, 1997) in students' education. It is important to realize that teachers and parents often come from the same background, as they have both been taught mathematics in a so called "traditional" way. Having students learn in a constructivist manner goes against most of the previous educational experiences of both teachers and parents. What teachers and parents will experience throughout this change will arguably be more difficult than what the students will experience. Civil, Berbuer & Quintos (2003) define "parents as intellectual resources and as such, we learn as much from them as they may be learning from us. Thus our intention is to engage in an egalitarian exchange rather than a teaching by transmission model" (p.9). As a basis for this discussion, parents should be invited into classrooms to observe, to participate as learners, or to help facilitate learning for students (Civil et al., 2003). It is not enough to tell parents about curriculum changes; rather it is vital that parents and teachers should be in constant discourse about their experiences.

On the home front. Not only can a partnership between teachers and parents create an atmosphere for curriculum changes to thrive, but "a match between adult-child interaction patterns at home and school appears to be advantageous for children" (Lehrer & Shumow, 1997, p. 55). Homework should match teaching practices in the classroom. In some cases, "home practices could place children in an uncomfortable position because they are in the middle of two different teaching 'cultures'" (Civil, Díez-Palomar, Menéndez-Gómez & Acosta-Iriqui, 2008, p. 12). In the same way that teachers are being challenged to change their practices in the classroom, parents are challenged to change the way they help their children with their homework. The nature of the homework

given to students will be very different from what traditionally has been sent home. Students will mostly be given "unique problems and tasks that help [them] to consolidate new learnings with prior knowledge, explore possible solutions, and apply learnings to new situations" (Saskatchewan Ministry of Education, 2008, p.18).

When it comes to homework, it is effective to question and listen (National Council of Teachers of Mathematics, 2006). "Active listening requires that we believe in children's ideas" (Van de Walle & Folk, 2004, p.40). To promote reflective thinking as students are working on unique homework problems, it is important for parents to be active listeners. Prompts such as "Tell me more" or "Why do you think that?" are non-evaluative ways that will give children the opportunity to expand their thinking. Waiting for a response requires patience, but also sends the invaluable lesson to the child that their understanding is what matters most (Van de Walle & Folk, 2004). Practical ideas that illustrate how parents can be effective facilitators for their child's mathematical education can be found at www.nctm.org.

It is clear that to fully undertake a curriculum change, an authentic partnership is required between parents and teachers. Although change can be uncertain, in this case, it is certain that parents and teachers must journey together for success to occur. With this solid parent/teacher relationship developed, improved student learning will ultimately be realized.

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HAVING CONVERSATIONS: THE CHANGING ROLE OF THE TEACHER IN PROBLEM SOLVING

Darcy Todos

“Understand the problem” by circling key words; “Make a plan” by using the current strategy being studied; “Carry out” the problem and write a concluding sentence, and “Look back” before moving onto the next word problem. George Polya’s (1957) approach to problem solving, adopted incorrectly by math textbooks as a linear mechanistic progression, has been an ill-fated attempt by teachers to redescribe the problem solving experience (Schoenfeld, 1989). In Saskatchewan, with the introduction of new math curricula that encourage a constructivist approach to problem solving (Western and Northern Canadian Protocol, 2008), many teachers are left questioning how their role will change

as problem solving is extended from the traditional approach of an application section within a unit of study to a strategy for teaching mathematics across the K – 12 spectrum. In using problem solving as a teaching strategy, the role of the teacher rests on the ability and actions needed to prepare, initiate, maintain and reflect on mathematical conversations designed to encourage students to actively develop their own mathematical understandings.

To prepare students for a problem solving lesson, a teacher’s role could be characterized by a lecture style of teaching that would focus the lesson on a “review-and-practice approach” (Simon, 1995, p.139). Teachers would review the problem solving “process,” to be modeled by their students, and any specific content or procedures needed so that students could practice solving a batch of similar problems. To prepare for a conversation in a mathematical area like measurement, a teacher’s job might first involve entering into an oral pre-assessment to determine what levels of thought (van Hiele, 1985) and language are present regarding measurement before the main conversations begin. For van Hiele, this pre-dialogue helps avoid the subject misunderstandings that arise when a teacher, operating at a different level of thought, imposes their own rigid problem solving framework onto their students.

Another important difference in the preparation for mathematical conversations requires the teacher to reflect upon which task should be chosen and anticipate what directions students might follow as a result. From the literature, support for a constructivist view of problem solving encourages the teacher to seek out and choose a “rich task” that would have a low “floor” and a high “ceiling.” The task should begin at a low enough level to include all students and should continue to evolve into areas of enrichment. In making the selection, the teacher then would place great consideration in choosing a task which will “engage all of the students in the class in making and testing mathematical

hypotheses" (Lampert, 1990, p. 12). This role of considering and selecting a task to extend student understandings through a conversation (Simon, 1995) is more difficult than selecting a list of problems to reinforce a procedure in a lesson. The teacher continues to reflect on selecting successive tasks, which are based on combining the connections already made by the class with a conceptual concept that the teacher anticipates, which might further understandings (Simon, 1995). The teacher is a decision maker. Although there is importance in the task that is chosen, the real focus of the lesson will not be the task, but the discussions students will have as a result of being engaged (Schifter, 1996).

In comparison to traditional methods, when a teacher initiates a problematic conversation, she will have to accept the role of being a patient facilitator. As the dialogue begins, teachers need to give students much longer wait times to begin the thought process (Schifter, 1996). The teacher's role in a direct problem solving activity begins without delay by demonstrating a procedure. Traditionally, Schifter (1996) points out that it would not be considered a fair task if the students' problem solving experience was not preempted with a demonstration, but from a constructivist view a teacher's role is to refrain from any form of instruction that would cause students to use one strategy over another.

Once a task is chosen, and the conversation initiated, the role of the teacher changes to accepting the responsibility of maintaining a conversation where the focus is not on obtaining a solution, but on student explanations (Lampert, 1990) and mathematical claims (Ball, Lewis & Thames, 2008). The role of the teacher is to enter into a *reflective practicum* (Schon, 1987) with their students. Teachers reflect on student conversations in order to frame questions that will further student reflections, helping them initiate and make mathematical claims. Asking questions such as, "Can you comment on your reasoning?" and "Will this always work and why?" is an

extension from the traditional question, "Did anyone else get the same answer?" Through ongoing reflections about what students are saying or anticipation of what they are thinking, the teacher's role becomes one of an active learner in the conversation (Simon, 1995).

One of the pedagogical hurdles teachers may overcome is the traditional role of telling students whether they are right or wrong. Teachers must encourage students to look to their peers to validate their claims (Lampert, 1990). Since validation rests with peers, the teacher must sit quietly and even allow the conversation to entertain a solution and explanation that may be mathematically incorrect. Even if students are at a stand still in the conversation, teachers must refrain from providing clues that would force the conversation towards a teacher determined outcome instead of an outcome driven by students understanding of the task (Schifter, 1996).

Although this process encourages the teacher to reduce their conversation, a teacher's input is still required to ensure that the math connections and claims made by students are correct and to ensure students safety in the case of a heated mathematical argument (Chazan & Ball, 1999). Chazan and Ball also advocate that it is still the role of the teacher to maintain the "direction and momentum" (p. 7) of the lesson, ensuring that the level of discussion is still well in the range of the majority of the students present. Viewing the teacher's role as a "coach" (Savery & Duffery, 2001; Schoenfeld, 1983) is an acceptable analogy, because as a coach uses a progression of skills to foster the active development of their athletes, teachers, through a scaffolding process of questioning, are actively building on student responses that would lead to developing the best understandings.

This strategy of teaching requires the teacher to be flexible with time. It is very difficult to predict how a problematic conversation will unfold. Will student comments reveal the understandings and

outcomes that the teacher wanted them to reach? Traditionally, where one period may have been adequate for the application of specific content, Simon (1995) would attest that in a problematic conversation, a lesson planned for one or two periods may evolve into several periods to ensure that the concept has been explored in numerous contexts (Schoenfeld, 1983).

Unlike a traditional problem solving lesson where teachers have grown quite comfortable with their role, the role of a teacher in conversations is not as defined, requiring teachers to access research and to interact with colleagues to share successes and failures (Schifter, 1996). There is no recipe; no scripted lesson that teachers can use to ensure that the lesson is following a “constructivist nature” (Simon, 1995; Schifter, 1996). It is a conversation, and as conversations are unique to the people that are having them, so too are they unique when they are used to teach mathematics through problem solving. As teachers become witnesses to the gains reached by students through conversations, they will come to value their changing role and see the importance of allowing the conversation to continue.

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SOMEWHERE TO START: ASKING OPEN-ENDED QUESTIONS IN YOUR MATHEMATICS CLASSROOM

Shirley Jones

Traditionally, factual, closed-ended questions that required little student understanding have been used in the mathematics classroom. Implementing the constructivist approach, using open-ended questions, may be a way for math teachers to become more comfortable with new math curricula and to better facilitate students' learning to meet outcomes. In order for students to experience long-term success in mathematical ability and understanding, they must be taught math in ways that ensure understanding rather than in ways that make it possible for them to memorize. "Questions that encourage students to do more than recall known facts have the potential to stimulate thinking and reasoning" (Sullivan & Lilburn, 2002, p. 1), develop long-term understanding, and open doors to future learning. This article will provide an overview of how open-ended questions can be used to strengthen mathematics programs in Kindergarten through grade nine classrooms.

Classroom Environment. Research says students learn best when using the constructivist approach which means working with a partner and in small groups (Van de Walle & Folk, 2004, p.28). Fello and Paquette (2009) state that a student's "understanding of overall math conceptualization improves with talking and writing in the math classroom" (p. 411), and that students must be coached to learn how to work in groups and how to respond to other classmates' answers in positive and respectful ways. This process, well facilitated, ensures that students continue to contribute and, therefore, do not risk stopping the learning process because they are anxious and uncomfortable about what their peers might think or say.

Initially, students may be reluctant to share ideas for fear of being wrong or expressing confusion. It is important to

emphasize the idea that "Confusion is something you go through, not a permanent state of being" (Carter, 2008, p. 135). Encourage the use of a "Thumbs Up, Thumbs Down, Thumbs Sideways" approach while the teacher circulates around the room so the students can share how they feel without being overly vocal. Showing thumbs up means: "We think we are on the right track." Thumbs down means: "We need help." While thumbs sideways means: "We are not sure and need more time to think." Another unique way to foster classroom acceptance of the questions and to lower anxiety is to allow each group of students to "phone a friend" (Fello & Paquette, 2009, p.412) when they come to a standstill in their thinking about the problem at hand. Each group asks another group about their findings, thus fostering positive student relations while keeping the lesson student-centered.

Writing Open-Ended Questions.

Teachers may choose to incorporate one of the following open-ended question writing methods just one day a week or when they feel it suits a particular lesson. Starting slowly and feeling comfortable with the approach is crucial to its success in a mathematics classroom.

METHOD #1 Working Backwards

- 1. Identify a topic** (e.g. topic is "averages").
- 2. Think of a closed question and write down the answer** (e.g. the children in the Smith family are aged 3, 8, 9, and 15. What is their average age? The answer is 9).
- 3. Create a question that includes (or addresses) the answer** (e.g. There are five children in the Smith family. The average age is 9 How old might the children be?).

METHOD #2 Adapting a Standard Question

- 1. Identify a topic** (e.g. topic is "space").
- 2. Think of a standard question** (e.g.. What is a square?).
- 3. Adapt it to create a good question** (e.g. How many things can you write about this square?).

(Sullivan & Lilburn, 2002, p.7-9)

Once students have been given the question to begin the lesson, the teacher circulates around the room and eavesdrops on students' discussions while facilitating student learning by asking questions that encourage deeper understanding.

When using open-ended questions, correct responses should not be given by the teacher because that gives students permission to stop thinking (Carter, 2008, p.137). Traditional lessons have an answer-driven focus. The constructivist approach is not driven by a single correct response. Students will be unfamiliar with this. If the teacher were to simply state, "Oops, I forgot my answer sheet at home" at the beginning of the first few classes, the students would learn that getting the answer is not the most important aspect of the lesson and they would soon realize that the teacher is not going to give them the answer.

Keeping a copy of Bloom's Taxonomy at hand while writing open-ended math questions helps one focus on asking higher-level questions. Mewborn & Huberty (1999) state that, "Teachers must learn not only to ask questions that provoke thoughtful responses but to follow initial questions with others that help students clarify and extend their thinking" (p. 226). Trying to predict possible areas of difficulty may help a teacher write follow up questions before each lesson, but for the most part, questions will have to be spontaneously generated while circulating around the room.

The teacher should treat incorrect responses as learning experiences. Students who have given an incorrect response often benefit from listening to others speak about their responses. If no definitive solution is reached at the end of class, the teacher should summarize main points, encourage students to sleep on it and return to it the next day. Teaching through open-ended questions is more time-consuming because some of the questions require more "think time" and oral responses take longer than traditional yes or no questions (Mewborn & Huberty, 1999, p.2).

As one's confidence using the constructivist approach increases, a teacher will realize its value, and the writing of open-ended questions and posing of further questions during class will become easier. Open-ended questions are an excellent way to reveal students' levels of understanding. Teachers and students will become accustomed to the teacher's relinquishing, to the students, some control over what happens in the classroom. Instead of lecturing their way through a lesson, teachers will actively listen, follow up with more questions, and encourage feedback.

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TRANSFORMING MATHEMATICS CLASSROOMS INTO LEARNING COMMUNITIES THROUGH MATHEMATICAL DISCOURSE

Carla Gradin

What is mathematical discourse?

Mathematical discourse is central to shape mathematical understanding and foster mathematical literacy among students (Knuth, 2001). Discourse in a classroom can be defined as "the ways of representing, thinking, talking, questioning, agreeing, and

disagreeing that is central to students' learning mathematics with understanding" (National Council of Teachers of Mathematics, 2006, p.489). Stein's (2007) adapted characteristics of the levels of discourse are found in the table below. Although various levels of discourse are found within classrooms, the intent is to focus on achieving a Level 3.

Levels	Characteristics of discourse
0	The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher's questions.
1	The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.
2	The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another's work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.
3	The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.

Curriculum connection. The renewal of Saskatchewan's mathematics curricula is based on a change in philosophy of mathematics education, such that "mathematics should be taught in a way that mirrors the nature of the discipline" (Stein, 2007, p. 285). The focus of mathematical discourse aligns itself with the Western and Northern Canadian Protocol (WNCP, 2006) goals for students to:

Use mathematics confidently to solve problems; communicate and reason mathematically; appreciate and value mathematics; make connections between

mathematics and its applications; commit themselves to lifelong learning; [and] become mathematically literate adults, using mathematics to contribute to society. (p. 4)

The teacher's role. The teacher plays a complex and central role in engaging students in meaningful discourse. "Left to their own devices, students will not necessarily engage in high-quality math-talk" (Bruce, 2007, p. 1). Teachers are responsible for establishing a risk-free classroom climate in which all students are comfortable sharing their ideas. Educators need to promote the value of mathematical understanding over simply stating the right answer. To avoid predictable conversations, such as teacher initiation → student reply → teacher evaluation, teachers must skillfully ask questions that aim to encourage and stimulate classroom discourse (Stein, 2001). Through the use of powerful questions that provoke thoughtful responses, teachers will be able to facilitate the direction of the students' learning and assist in the students' ability to clarify and extend their thinking. Mewborn & Huberty (1999) suggest a list of effective initial and follow-up questions to promote classroom discourse:

Does anyone have the same answer but a different way to explain it? Can you convince the rest of us that that makes sense? Why do you think that? Is that true for all cases? What assumptions are you making? How did you think about the problem? How does this relate to...? What ideas that we have learned before were useful in solving this problem? Do you see a pattern? (p. 244)

Limitations/difficulties for teachers. The nature of mathematical discourse is often dictated by "the reality of a teacher's classroom, which includes the competing demands of depth versus breadth in content coverage, the presence of students of dissimilar abilities and interests, and time constraints" (Knuth & Peressini, 2001, p. 325). In addition, mathematical discourse requires teachers to be comfortable with their own mathematical knowledge and teaching mathematics in a way that they perhaps did not experience as a student

(Bruce, 2007). The teacher must avoid simply giving the correct answer and allow students the opportunity to reflect and come to resolutions on their own, such as they would when encountering problems in the real world (Mewborn & Huberty, 1999).

Why venture forth? Research (e.g., Mewborn & Huberty, 1999; Manouchehri & St. John, 2006) suggests a number of reasons for engaging students in a Level 3 discourse, as described in the previous table. To list only a few reasons: language and articulation skills are enhanced, students become more active and engaged in their own learning and value the problem-solving experience, less repetition and practice is required, and a variety of concepts can be addressed in a single lesson when deeper connections within math and between math and other subjects are made. Mathematical discourse promotes a shift from a teacher centered environment to one in which students become active and engaged in their learning through their belief that they are responsible for understanding and sharing mathematics (Manouchehri & St. John, 2006).

Why value mathematical discourse? Although it is difficult to permit a class to end with unanswered questions, or venture away from predictable pedagogical strategies to disseminate curriculum content, rich mathematical discourse assists both teachers and students in a deeper learning of mathematics. Participating in a community of learners and collaborating among peers is a powerful way to “encourage students’ authentic engagement in the construction of mathematical knowledge” (Manouchehri & St. John, 2006, p. 551). Mathematical discourse is at the very heart of Saskatchewan’s goals of curriculum reform: to promote and achieve an inquiry-oriented learning environment in which students are actively constructing knowledge in ways, aligned with that of a mathematician.

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THE ROLE OF READING IN COMMUNITIES OF MATHEMATICAL INQUIRY

Heather Rowson

One of the most fascinating contemporary trends in the teaching and learning of mathematics is the growing emphasis on inquiry-based learning. Inquiry-based learning is rooted in constructivism; it is a process in which students “are involved in their learning, formulate questions, investigate widely and

then build new understandings, meanings and knowledge” (Alberta Learning, 2004, p. 1). Goos (2004) describes “*communities of mathematical inquiry*” as classrooms in which students “are expected to propose and defend mathematical ideas and conjectures and to respond thoughtfully to the mathematical arguments of their peers” (p. 259). In these classrooms, learning is a highly collaborative process.

Inquiry-based learning plays an important role in Saskatchewan’s new mathematics curriculum. According to the new mathematics curriculum for Grade 2, “inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience” (Ministry of Education, 2008, p. 21-22). Siegel, Borasi, and Fonzi (1998) argue that “language takes on new importance in classrooms in which knowledge is regarded as a social construction” (p. 379) and that “reading, writing, and talking are used to create opportunities for students to engage in mathematical inquiries” (p. 382). Indeed, the mathematics curriculum documents for grade two (and five) suggest integrating mathematics and English language arts to “move students’ inquiry towards deeper understanding” (Ministry of Education, 2008, p. 23) and both subject areas “share a common interest in students developing their abilities to reflect upon and communicate about their learnings through ... reading” (p. 37).

At the outset of their study, Siegel et al. (1998) posed the following question: “What functions can reading play in inquiry cycles developed in the context of secondary mathematics instruction” (p. 383)? Their study “confirmed that ... reading can serve multiple functions in mathematics inquiry cycles” (p. 385). Many of their findings are applicable at the elementary level as well. Space does not allow a full exploration of their “identification of 30 functions of reading that are specific to distinct elements of an inquiry cycle” (p. 387), but it may be

helpful to examine some of the ways in which reading supports mathematics inquiry as described in the abovementioned study and another conducted by Siegel and Fonzi (1995).

Siegel and Fonzi (1995) identified five groupings of reading practices; reading to: make public, comprehend, get an example, generate something new, and remember. It should be noted that in both studies the researchers used a rather broad definition with respect to “what counts as reading in a mathematics class” (Siegel et al., 1998, p. 409). Siegel and Fonzi (1995) admit they “had not anticipated the number, as well as the variety and complexity, of the reading practices in an inquiry-oriented classroom” (p. 643). According to the researchers, “neither textbooks nor rich mathematical texts, alone, were enough to support the development of broader conceptions of learning and mathematics” (p. 653). Students engaged with a wide variety of materials in order to explore how mathematics was “intimately connected to everyday life” (p. 654). Among other texts, the researchers suggest the students read: articles, pamphlets, posters, questionnaires, journal prompts, children’s literature, cartoons, posters, essays, textbooks, teacher-generated lists, student-generated questions, student-generated conceptual maps, and directions for making origami.

A major point of interest is that “of the 40 kinds of texts that were used in this class, 22 were generated by either students, the teacher, small groups, or the class as a whole” (Siegel & Fonzi, 1995, p. 653). This speaks to the importance of “community-generated texts” (p. 653) in the ongoing construction of understanding in the mathematics classroom. Siegel et al. (1998) claim that “students did not just read and discuss one another’s diagrams, theorems, or questions; often they also acted upon these texts, identifying patterns, suggesting revisions, and challenging interpretations” (p. 408). Through shared reading experiences, “students had opportunities to experience inquiry as a social practice that

involves the negotiation of meanings among members of a community” (p. 408).

Certainly the findings of these studies are interesting and offer cause for reflection. That reading is “bound up with doing inquiry” (Siegel and Fonzi, 1995, p. 661) may seem indisputable. The challenge facing educators in Saskatchewan as we work to implement the new curriculum is how to put this knowledge into practice. Many of the strategies detailed in the research studied (e.g., Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, & Perlwitz, 1991; Siegel et al., 1998; Siegel & Fonzi, 1995) are applicable at both the elementary and secondary levels and include: having students fill out a goals and beliefs questionnaire, using reader response journals, guiding students to engage with a variety of sources to identify mathematical questions for inquiry, facilitating shared reading experiences, having students create “thinking questions” and their solutions, and collaboratively constructing a “What did I learn?” list.

In working to build classroom communities of inquiry in mathematics, the focus should be on working together so students “become active participants in a collaborative search for meaning and understanding” (Ministry of Education, 2008, p. 21). We would do well, as teachers, to adopt that approach for our own professional learning as we continue to build our repertoire of inquiry-based practices.

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JOURNAL WRITING IN MATHEMATICS

Brian Crawley

Many educators associate communication in the classroom with subjects such as language arts or social studies; however, the reforms of the National Council of Teachers of Mathematics (2000) has made communication a curriculum priority for teachers of mathematics. The Western and Northern Canadian Protocol (2008) closely followed the NCTM standards and stated similar goals whereby students need to communicate and become mathematically literate. Typically, communication in mathematics has been verbal, but educators ought to consider writing about mathematical concepts as an excellent teaching tool, as well. Writing in mathematics enables learners to track their thinking and teachers are able to gain insights into the thought processes of their students.

Edwards (1992) described dialogue journals as "a process to help us see more clearly and develop new understandings. The writing acts as a scaffold or platform on which other ideas can be built" (p. 2). Edwards prompts students to record their thoughts, explanations, questions, and feelings about mathematical problems. Her students are able to examine and reflect on their own areas of strength and weakness as

learners. Written explanations of a student's problem-solving process allows the teacher to understand and assess the student's thinking and comprehension of mathematical concepts. One kind of prompt could be: *Your friend is sick today. Write a letter to your friend explaining how to solve this equation: $x^2 + 5x^2 - 36 = 0$.* Teachers can diagnose the journal entry and discuss areas of misunderstandings regarding procedural knowledge or computational mistakes.

Teachers often assume that students have understood the concepts and they are astonished when assessments show major misunderstandings. Chapman (1996) did a study using journal writing with her high school algebra class. When she instructed them to describe the graphs of $y = 2x + 1$ and $y = x^2 + 4x + 4$ the results were appalling. The main themes of the course had focused on the differences between the two functions; however, many students drew two straight-line graphs. Miller (1992) reported similar discrepancies. She asked students to explain why $0/5 = 0$ and $5/0$ is undefined. One student wrote, "because you can't take zero from anything" (p. 334). In an earlier test, students had written the correct answers for the above questions yet when probed for deeper understanding, Miller found that the mathematical comprehension was lacking. Her findings influenced her to: (1) reteach immediately, (2) delay an exam, (3) design a review, (4) initiate private discussions, and (5) initiate prompts and read them immediately to check for understanding.

Getting students to start thinking about their thinking or metacognition helps students to reflect about mathematical concepts, study habits, or general attitudes (Mason & McFeetor, 2002). Journal prompts can invite students to address the challenging aspects of learning mathematics. Karim, a tenth-grade student, described his study habits after the fourth test of the year when he wrote, "I closed my book and read the notes downstairs and I chatted with my friends online. That's why I only got 69%"

(p. 532). In an earlier entry after the second test, Karim had commented that he had never studied for a math test and it had not affected his marks. This process of having Karim reflect on his preparation may motivate him to change his study habits.

Writing allows students to see the steps used in problem solving and helps students make conclusions from the solution. Dougherty (1996) maintained that writing prompts offer students opportunities to reflect on particular solution strategies and to consider ways in which they learn. For example, students may respond to: "You know at least three ways to solve an equation. What is your favorite method? Why?" As students explain their choice of method they begin to understand their own problem-solving approaches and they become more aware of their strategies.

Journal writing can also give students who are often quiet or shy in class a medium through which they can express themselves without the risk of embarrassment. Baxter, Woodward, and Olson (2005) were surprised to discover that writing in journals provided low-achieving math students with an alternative strategy to communicate their mathematical ability. Students who had been afraid to speak out in class were able to express their knowledge in their journal writing. Some students preferred not to be praised in public and the journals permitted the teacher to encourage students privately. Affective journal prompts such as, "Which grade did you start having trouble in Math?" can give students the chance to express their attitudes and anxieties about mathematics (Mason & McFeetor, 2002). Some students can have strong emotions about mathematics due to past negative experiences. Journal writing provides an outlet for expressing these feelings of frustration or lack of confidence.

Many teachers do not use writing in their daily lesson plans because of the perceived extra time and effort involved in adding to their workload. Several studies have found the contrary. For example, Mason and

McFeetors (2002) claimed that writing requires only 5-10 minutes of time at the beginning or end of class. As well, journal writing can be a gut-wrenching experience for a teacher when they become aware of their students' lack of comprehension. Journal writing involves risk taking and often provokes reflective pedagogy. If an educator wants students to become actively engaged in the learning process, the introduction of journal writing is an excellent starting point. The benefits of having students become better at mathematics more than justifies the effort required to implement journal writing in the classroom.

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UNDERSTANDING VERSUS LEARNING IN MATHEMATICS

Lindsay Shaw

As teachers, we want our students to learn the mathematics content in the curriculum. However, this may be a misguided endeavour – there may be a lot of “learning” occurring in the math classroom, but not necessarily “understanding.” Whereas learning is an intake of knowledge, understanding is the deeper concept, which includes the application of the learned knowledge to other subjects and concepts. Therefore, the real focus must be understanding of the math content, such that the students have enough understanding for the next concept, the next grade, and for life. But how is the lofty goal of understanding accomplished? I propose *group mastery learning* as a method to increase student understanding of math content.

Mastery learning was first introduced by Bloom in the 1960's (Bloom, 1968). It is based on the idea that “all children can learn when provided the conditions that are appropriate for their learning” (Guskey & Gates, 1986, p. 73). The general concept is to create an environment that promotes understanding of mathematics. In a mastery learning classroom, this involves “organizing instruction, providing students with regular feedback on their learning progress, giving guidance and direction to help students correct their individual learning difficulties, and providing extra challenges for students who have mastered the material” (Guskey, 1985, p. xiii).

What Guskey does not include is an aspect of group mastery learning, where students first learn and understand the material through a group based problem solving approach. It is the melding of mastery learning and a constructivist teaching methodology that creates group

mastery learning and allows student an increased understanding of mathematics.

The group mastery learning theory is often difficult to visualize in the classroom. Below is one method, adapted from (Guskey, 1985), to implement the mastery approach into the mathematics classroom: *specify learning objectives → problem solving activities → formative test → corrective activities → enrichment activities → summative exam → classroom applications.*

Specifying learning objectives to sequence the curricular content makes learning and understanding more natural. This step should become less difficult with the implementation of the new Western and Northern Canadian Protocol (WNCP) curriculum. Next, the teacher needs to plan a variety of problem solving activities that apply the objectives. The reasoning for using a problem solving technique is that the research (e.g., Van Hiele, 1957, Van De Walle and Folk, 2007) has shown a number of benefits associated with applying this method. A problem solving approach to mathematics education is important because “(1) it helps students understand that mathematics develops through a sense-making process, (2) it deepens students’ understanding of underlying mathematical ideas and methods, and (3) it engages students’ interests” (National Council of Teachers of Mathematics, 2003, p. 20).

If the concept has not been mastered at this stage, then a series of corrective activities are required for the student to work on with the teacher, with a peer, in a group or individually. Once the objective is achieved then there are a series of enrichment activities (for all students) to deepen the students’ understanding, followed by a summative test. The above method is depicted in a sequential manner, but the idea of enrichment activities and applications can appear throughout the steps depending on the objectives and the class at hand.

There are a number of indicators of concept mastery: attaining 80% or higher on formative and summative tests, relating the mastered concept to other applications, or teaching the concept to others. The first indicator, namely attaining 80% or higher on tests, requires the student to rewrite a similar test after corrective instruction if this level of understanding is not obtained. The second indicator, relating the concept, requires application of a math concept through a variety of ways, such as in problem solving. The final indicator, teaching others, is a task, which is ongoing in the classroom as the students work through activities in their groups.

These are just three methods of assessing the mastery of a mathematics concept, but the overall benefit to the students through understanding is the best indicator of all. It has been shown that “mastery learning worked well in terms of promoting student learning ... [and] heightening their interest in and attitudes toward subject matter and their academic self-confidence” (Block, Eftim and Burns, 1989, p. 22). Not only did students’ learning and understanding improve with the mastery program, but also their overall attitude towards mathematics and school in general improved.

Further reading regarding mastery learning, constructivist teaching and teaching through problem solving will allow individuals to further individualize their own programs. Below is a small list, which may be useful as a start:

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WHAT DOES IT TAKE TO SUCCEED IN MATHEMATICS?

Christina Fonstad

“We sometimes think of being good at mathematics as an innate ability. You either have ‘it’ or you don’t” (Gladwell, 2008, p. 246). In other words, mathematical ability is typically connected to intelligence. However, Einstein suggested that there is more to learning than intelligence alone when he said: “It’s not that I’m so smart. It’s just that I stay with problems longer.” Schwartz (2006) stated that success in mathematics “is very much dependent on developing an *attitude*, one that includes perseverance, tenacity, and fearlessness” (p.50). Those who have this attitude or disposition work hard, experiment, take risks, make mistakes, and refuse to give up regardless of the obstacles encountered. But does having the *right* attitude really lead to increased success in math? Is it possible to “master mathematics if you are willing to try” (Gladwell, 2008, p. 246)?

Let us begin by taking a closer look at one of the groups of students who typically excel in mathematics. “For years, students from China, South Korea, and Japan – and the children of recent immigrants who are from those countries – have substantially outperformed their Western counterparts at mathematics, and the typical assumption is that it has something to do with a kind of innate Asian proclivity for math” (Gladwell, 2008, p. 230). In other words, Asian success in mathematics is often attributed to intelligence. However, Flynn (1991) found

that Chinese and Japanese Americans have mean IQs no higher than other Americans. If intelligence is not a major factor, is it an attitude of perseverance that makes particular students outperform in mathematics?

According to Gladwell (2008) “there is significant scientific literature measuring Asian persistence” (p.249). For example Binco (1992) conducted a study of first grade students in four elementary schools in both Japan and the United States. Japanese children persisted longer than American children with “the Japanese raw time-on-task mean of 13.93 minutes...and the American raw time-on-task mean of 9.47 minutes” (p. 413). Japanese children persisted roughly 40% longer on the task than the American children (Gladwell, 2008). While working with 290 Grade 9 to 12 mathematics students in upstate New York, Schoenfeld (1988) also noted a lack of perseverance among American students. When asked:

If you understand the material, how long should it take to answer a typical homework problem? [and] What is a reasonable amount of time to work on a problem before you know it's impossible? [the] “means for the two parts of the question were 2.2 minutes and 11.7 minutes respectively. (p. 160)

Schoenfeld stated that many of these students seem to believe “mathematics only applies to situations that can be solved in just a few minutes – and that if you can't solve a problem in a short amount of time – you should simply give up” (p. 14).

Boe, May, and Boruch (2002) noted the connection between student achievement and attitude among Asian students when studying the Trends in International Mathematics and Science Study (TIMSS), an international assessment of the mathematics and science knowledge of fourth-grade and eighth-grade students. The TIMSS consists of an assessment of mathematics and science content, as well as student, teacher, and school questionnaires

that seek information about students and instructional practices. Boe et al. (2002) compared test achievement to *Student Task Persistence (STP)* on the questionnaire. STP was determined by “the extent to which an individual student persists in providing answers to...[the] questionnaire, as measured by the percentage of questions answered out of all questions that were asked” (p. iii). It was concluded that STP and math and science achievement were indeed connected. “Countries whose students are willing to concentrate and sit still long enough and focus on answering every single question in an endless questionnaire are the same countries whose students do the best job of solving math problems” (Gladwell, 2008, p. 247-248). In other words, students from Singapore, South Korea, China (Taiwan), Hong Kong, and Japan are found at the top of both lists.

Based on these studies, it seems plausible to conclude that attitude plays an important role in the success of students in mathematics. The perplexing question now is how do we foster this attitude of perseverance, tenacity, and fearlessness in all learners? According to Lappan (1999), a past NCTM president, “students need dispositions that will enable them to persevere in more-challenging problems, to take some responsibility for their own learning, and to develop good work habits in mathematics.”

When it comes to Asian learners, certain individuals suggest their attitude of perseverance is cultural. For instance, Gladwell (2008) attributes Asian persistence to the labour-intensive cultural tradition of wet-rice agriculture. Similarly, Binco (1992) states “persistence permeates all of Japanese society” (p. 407). If cultural values of perseverance, tenacity, and fearlessness lead to success in math, teachers face the challenge of creating a *classroom culture* that also values these attitudes. What exactly would such a classroom look like? What teacher and student practices would be required, modeled, and embraced?

Answering these questions is the next step in finding out what it takes to succeed in math.

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SINGAPORE MATHEMATICS: CROSSING OVER TO CANADA

Diana Sproat

As the results of student achievement around the world become commonly known and reported to the public, accountability pressures mount to have our students perform well in international testing. With such remarkable results, other countries question what they can learn from looking at

Singapore, a tiny country in Asia, which has set itself apart as a world leader in mathematics achievement as measured by the well respected Trends in International Mathematics and Science Studies (TIMSS) (Ginsburg, Leinwand, Anstrom, & Pollock, 2005). What are the strengths, of what is referred to as ‘Singapore Math,’ and are any of these highly effective indicators seen in the curriculum and system of teaching mathematics in Saskatchewan?

Ginsburg et al. (2005) identified the strengths of Singapore’s system of teaching mathematics and found four key points that correlate with good test performance. Singapore students at an early age are streamed into a mathematics framework that best suits their ability, determined by a public exam. Students who perform lower are offered an alternative framework with the same topics introduced at a slower pace and reinforced with more practice.

A second indicator relates to the narrow focus of outcomes at each grade level. Textbooks reflect the degree of deep understanding in the amount of topics (usually fifteen per grade level), and the amount of pages devoted to each topic (twelve on average, as compared to two in the typical American textbook) (Leinwand & Ginsburg, 2007). With more time spent on lessons, mastery is expected and a spiraling approach, reviewing concepts each year, becomes unnecessary.

A third key point is the emphasis placed on teacher education. Teachers are carefully selected, are expected to demonstrate mathematical skills at a high level, and receive one hundred hours of professional development each year. Another correlation between student achievement and the system used in Singapore is the challenging tests that students are required to take and the high stakes placed on the results. Schools are ranked publicly and rewarded financially for high student progress over time.

These strengths give us some insight to the value placed on education in Singapore. “Education is seen as a passport to upward

mobility in Singapore” (Menon, 2000, p. 346). This is a culture in which “knowing mathematics is as important as knowing how to read well” (Ginsburg et al, 2005, p. 8). Billions of dollars are allocated by the government to ensure education is a top priority. Parents, who can afford it, pour hundreds of dollars into extra tutoring for their children each month, and those who cannot, have access to government run tutoring centers. Students are highly dedicated and hard working; typically spending considerable time on homework each day. These values are not easily adopted by a culture.

Menon (2005) offers questions to ponder in this regard. Do we want the competitiveness that comes out of ranking schools according to test scores? Should students be streamed at such a young age and their ability measured only by the test score achieved? How can we raise the level of prestige teachers and high achieving students receive in a culture that considers academics as “nerds” and glamorizes the “jocks” who will make the “megabucks”?

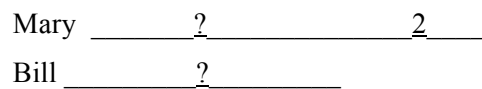
Only one of the identified strengths of the Singapore mathematics curriculum is clearly seen in the system of teaching mathematics in Saskatchewan. Changes to current mathematics curricula that emulate the approach taken by Singapore math is seen in the focus, or the narrowing of outcomes in the new documents. The move has been away from what has been referred to as curriculum that is “a mile wide and an inch deep” to one with fewer outcomes at each grade level, and an emphasis on learning with deeper understanding (Ministry of Education, 2008).

By decreasing the number of outcomes at each grade level, students are afforded more time to explore, investigate and make sense of mathematics. A curriculum centered on problem solving promotes deep understanding. Problem solving is the key component in the Singapore math program, and it is in the use of the locally developed model-drawing approach that Singapore

students excel in solving complex, multistep problems. This pictorial approach provides the bridge many students need as they move from the concrete to the abstract stage in understanding and the necessary link to later algebraic reasoning. It provides a symbolic representation that is a powerful tool in representing, understanding, and solving complex problems. Consider the strategies that might be used to solve this simple problem (Garellick, 2006):

Mary and Bill have \$10 between them. Mary has \$2 more than Bill. How much money does each person have?

Bar modeling offers a way to arrive at the solution that eliminates the less efficient method of trial and error or ‘guess and check’, which is often used by American (and Canadian) students (Garellick, 2006). The solution would be modeled with two bars representing each person, and Mary’s bar would be a little longer, representing the \$2 more she is known to have.



Removing the \$2 leaves two bars of equal length with a sum of \$8, as the \$2 was removed from the total \$10 Mary and Bill had. Students will then divide by two to see that Bill has \$4, and Mary, having \$2 more, equaling \$6.

Model drawing can be used to solve most word problems presented to students in grade three to eight Saskatchewan mathematics. Students are taught to use strategies that make sense to them in problem solving. The model-drawing approach, a part of the heuristic, ‘draw a diagram,’ can effectively be applied to problems that involve part-whole calculations, comparison, rate, proportion, and ratio. The diagram provides a picture of what information is given and what information is missing, making the solving of complex problems appear clear in the symbolic representation. This approach can become part of a large repertoire of

strategies students acquire as they develop conceptual understanding.

It is through looking at the success achieved by students in mathematics in Singapore, not only in test grades, but also in the processes they used to get there, that we are exposed to a powerful tool that can be used by students to enhance mathematical understanding and ability. The skills learned by reflecting and representing thoughts through construction of a model supports students in deepening their conceptual understanding and prepares them for higher level algebraic thinking.

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DESIGNING EFFECTIVE PROFESSIONAL DEVELOPMENT FOR TEACHER CHANGE: EXPLORING THE JAPANESE LESSON STUDY MODEL

Charlene Velonas

Mathematics education in the province of Saskatchewan is in the midst of a dramatic change. The shift towards adopting the Western and Northern Canadian Protocol's (WNCP) Common Curriculum Framework (CCF) will have a tremendous impact on the teaching and learning of mathematics. Professional development is critical to ensure effective implementation of the new curricula. In this article, the characteristics of effective professional development and how it relates to teacher change will be addressed. In addition, the Japanese Lesson Study Model will be explored as one of many alternative models of effective professional development.

A great deal of current professional development fails to meet the needs of teachers. Guskey (2002) presents two crucial factors that need to be considered in this regard. He suggests it is important to understand first, what motivates teachers to engage in professional development and second, the process by which change in teachers typically occurs.

The majority of teachers are attracted to professional development because they believe that it will "expand their knowledge and skills, contribute to their growth, and enhance their effectiveness with students" (Guskey, 2002, p. 382). Teachers hope to gain specific, practical, and concrete examples that can be incorporated into their daily classroom practice.

Professional development programs are usually created with the assumption that if you change teachers' beliefs and attitudes, this will lead to a change in classroom practice and behaviour, which in turn results in improved student learning. Guskey (2002) proposes a different model for the process of teacher change. He suggests that a significant change in teachers' attitudes and

beliefs will only occur after they see evidence of improvements in student learning. This suggests that the classroom behaviour and practices must change first. “[Teachers] believe that it works because they have seen it work, and that experience shapes their attitudes and beliefs” (p. 383).

Effective professional development, as defined by Loucks-Horsley, Love, Stiles, Mundry & Hewson (2003), has the following characteristics:

Is driven by a well-defined image of effective classroom learning and teaching; provides opportunities for teachers to build their content and pedagogical content knowledge and examine practice; is research-based and engages teachers as adult learners in the learning approaches they will use with their students; provides opportunities for teachers to collaborate with colleagues and other experts to improve their practice; supports teachers to serve in leadership roles; links with other parts of the education system; has a design based on student learning data and is continuously developed and improved. (p. 44)

The Japanese Lesson Study Model is one of many examples of an effective form of professional development. In Japan, it is widely known that participation in school-based professional development is considered to be a part of a teacher’s job. “*Kounaikenshu* is the word used to describe the continuous process of school-based professional development that Japanese teachers engage in once they begin their teaching career” (Stigler & Hiebert, 1999, p. 110). Even after the completion of a teacher-training program, educators in Japan are expected to continue to build pedagogical content knowledge (Shulman, 1986) throughout their entire career. One of the most common components of this model of continuous professional development is the concept of lesson study. “In lesson study, groups of teachers meet regularly over long periods of time (ranging from several months to a year) to work on the design, implementation, testing, and improvement of one or several ‘research lessons’” (p. 110). The concept of lesson study supports

Guskey’s (2002) proposition that to improve teaching, the most effective place to start is in the context of a classroom lesson.

Lesson study is essentially a collaborative problem-solving process. The first step in the process requires the study group to define a problem to examine. Typically this problem is one identified by teachers from their own practice, usually a concept that causes challenges for students. Once a problem has been identified, the study group collectively reviews research and plans a lesson. “The goal is not only to produce an effective lesson but also to understand why and how the lesson works to promote understanding among students” (Stigler & Hiebert, 1999, p. 113). The next phase in the process is the actual teaching of the lesson followed by a phase of evaluation and reflection. The lesson is then revised and taught again. Following this, a second evaluation and reflection phase occurs and then the results are shared with other colleagues.

The Japanese Lesson Study Model incorporates many of the Loucks-Horsley et al. (2003) characteristics of effective professional development. The lesson study model maintains a constant focus on student learning while promoting effective and reflective teaching. Through the use of research during the planning phase, teachers are building both content and pedagogical content knowledge. Teachers are engaged in the collaborative process; the professional development is relevant to them because it examines teaching in context. According to Stigler & Hiebert (1999) “teachers who participate in lesson study see themselves as contributing to the development of knowledge about teaching as well as to their own professional development” (p. 125).

The learning curve is steep for some mathematics educators in Saskatchewan, but professional development will help to make that task more manageable. Mathematics educators need to work together, establishing a strong community of collaborative and reflective professionals

who are committed to ongoing professional learning. Stigler & Hiebert (1997) summarize it well:

A true profession of teaching will emerge as teachers find ways and are given opportunities to improve teaching. By improving teaching, we mean a relentless process in which teachers do not just improve their own skills but also contribute to the development of Teaching with a capital T. Only when teachers are allowed to see themselves as members of a group, collectively and directly improving their professional practice by improving pedagogy and curricula and by improving students' opportunities to learn, will we be on the road to developing a true profession of teaching. (p. 21)

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INVESTING WISELY IN TEACHER TIME AND STUDENT LEARNING

Glenys Martin

As Saskatchewan moves forward with the new mathematics curriculum, new approaches to teaching and learning of math will occur in classrooms. Implementation of these changes will require school divisions to instruct and support teachers to

incorporate new pedagogy into their teaching. Professional development requires the investment of personal time and public money. Guskey (1985) defines professional development as the efforts that bring about change in the classroom practices of teachers and the learning outcome of students. If professional development money is spread too thin, significant change is unlikely. It is better to involve fewer teachers than to do a poor job with many (Ingvarson, Meiers, & Beavis, 2005; Morimoto, Gregory, & Butler, 1973). This article addresses features of good investments in teacher professional development.

The new mathematics curriculum requires a new approach to teaching and learning. Teachers require training to shift their teaching role from instructor to facilitator. Richardson (2003) recommends the inquiry model as a method for professional development. This model allows teachers to determine their individual goals and the staff's collective goals. It allows teachers to experiment with practices, and engage in open and trusting dialogue about teaching and learning. Incorporating mathematical problems within this type of professional development model is effective in developing important content understanding (Borko, 2004).

Studies have determined important characteristics for effective professional development. Richardson (2003) lists the nine key characteristics of research-based professional development. She states:

It should be school wide; be long-term with follow-up; encourage collegiality; foster agreement among participants on goals and visions; have a supportive administration; have access to adequate funds for materials, outside speakers, substitute teachers, and so on; develop buy-in among participants; acknowledge participants' existing beliefs and practices; and make use of an outside facilitator/staff developer. (p. 2)

Designers of professional development often ignore these characteristics. This is a concern because the traditional short-term transmission model of professional development is not particularly successful (Richardson, 2003).

The inclusion of Richardson's (2003) professional development characteristics in mathematics curriculum implementation will determine its effectiveness. Teachers' attitudes and beliefs about professional development are dependent on evidence of improvements in student learning (Guskey, 1986; Richardson, 2003). Providing evidence of student learning improvement early in professional development could encourage buy-in amongst participants. This evidence could also affect the goals and visions of teachers. Teachers often resist change that is mandated and engage in change that is self-initiated (Richardson & Placier, 2001). Making teachers aware of the potential for student learning improvement may initiate this desire for change. Adult learning without exploration and choice seldom results in a positive learning experience (Morimoto, Gregory & Butler 1973).

Professional development should provide teachers with delivery options. Just as K-12 students have preferred learning styles, adult learners also appreciate choice. Ingvarson et al. (2005) refer to a number of professional development models. At their schools, teachers could learn through action research projects, coaching, and mentoring. Others can choose formal learning from institutions. Online learning provides another professional learning opportunity. Teachers can build on their knowledge through participation or attendance at conferences and seminars.

Providing a variety of opportunities for teacher growth not only provides choice, it also provides multiple learning environments throughout the course of mathematics curriculum implementation. This supports Richardson's (2003) recommendation of long-term professional

development and allows the teachers opportunity to make use of an outside staff developer. Morimoto et al. (1973) found short-term professional development participated in over several hours or a few days had an implementation level of only 15 percent. Professional development focused over a longer period had greater success.

Professional development needs to provide math teachers opportunities to work together to make sense of their knowledge and skills in relation to their classroom practice (Battey & Franke, 2008). The first four of Richardson's (2003) characteristics are possible within a professional learning community. Any form of collaboration in a professional community allows members to struggle with other perspectives, contrary ideas and new insights (Murphy & Laferriere, 2003). Working with others is an effective way to reinforce learning and help demonstrate what is possible through the sharing of ideas and products (Brinkerhoff, 2006). In her study, Borko (2004) found that learning communities allow teachers to support each other in their shared goal of improving the learning and teaching of mathematics. Teachers came to view classrooms as learning environments for themselves and their students. Regular discussion with colleagues about teaching and learning helps teachers to develop a clear understanding of how children think and learn (Desimone, Porter, Garet, Yoon, & Birman, 2002). Research explains the benefit of learning communities in fostering teacher growth in mathematic concepts and instructional methods. If we are expecting teachers to create a community of learners among the students, professional development should encourage a parallel community for teachers (Borko, 2004).

Ingvarson et al. (2005) noted that well-designed professional development is not enough; school administrators' support is also required. Effective school administrators, one of Richardson's (2003) nine important characteristics, value teacher learning and support learning communities in their schools. People within the

educational institutions in Saskatchewan need to make wise investments in their professional development to improve student learning in mathematics.

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TEACHERS AS LEARNERS

Terry Johanson

At times in my mathematics classroom there was little active reflection on teaching philosophy, with hours being “too full” planning, marking, and organizing sport tournaments and practices. Admittedly, there were issues with the learning of *some* concepts by *some* students in my classroom, but solutions created to fix those situations included innovative (and dare I say brilliant) explanations designed to make them understand. Unfortunately, time was not taken to uncover students' foundational misconceptions or to try to understand how those misconceptions developed.

Saskatchewan schools are in the midst of mathematics teaching and learning reform. It is imperative that Saskatchewan teachers determine what the issues in mathematics education are and work to address those issues. To ensure actualization of curricula, school divisions and teachers need to be aware of barriers and investigate the types of teacher support that may reduce those barriers.

Schools are powerful communities that enculturate members into traditional ways of thinking (Putnam & Borko, 2000). Ball (1996) noted that school-based constraints may include: students unfamiliarity with approach resulting in resistance; parent resistance of movement away from traditional teaching; and administrator intolerance of chaotic classrooms and failure to provide resources.

These constraints can result in a disparity between teachers' beliefs and the teaching models they are able to implement (Ernest,

1988). Change in education is slow. Researchers ponder the same questions now that they have for decades. Labaree (1998) likened educational research with the Sisyphean task of rolling a huge rock up a steep hill, only to have it roll back down again, forcing researchers to begin again. Changes do not happen just because we decide to teach differently (Ball, 1996). Teaching for understanding requires a higher level of skills and a deep, relational mathematical understanding, with teachers having gone through a conceptual revolution themselves (Cobb, 1988). An additional challenge being faced is an incomplete knowledge of mathematics learning and teaching. Human understanding is complex; what teachers know about student knowledge and preconceptions will never be certain (Ball, 1996). There is no guarantee that a student engaging in a specific activity will correctly construct knowledge activities and even more frustrating is the realization that interventions *might* work with *some* children (Cobb, 1988).

Teacher development would reduce the barriers of school enculturation, pace of change, and teacher knowledge. A promising structure for teacher development is one which combines off-site learning experiences, which are unconstrained by classroom practice with ongoing support in classrooms (Putnam & Borko, 2000). The question that remains is what should happen during these learning experiences. Richardson (1992) recognizes the dilemma of agenda setting. Professional developers want to introduce teachers to specific content and see teachers' practice change in a particular direction. Professional developers also recognize that it is important to create an environment where teachers are empowered to own content and process. This is similar to a classroom teacher's dilemma between wanting to allow students freedom in developing their own learning and needing to lead student learning through curricular content. If balance between these goals is achieved, both teachers and professional developers will gain new

insights into teaching and learning (Putnam & Borko, 2000).

One conceptual framework for professional development is similar to traditional teaching by transmission of knowledge. In this model, teachers are taught how to use a new program or method. This method is derived from the idea that teachers are recalcitrant and do not change often by themselves, so someone from outside the classroom must decide what is good for students. Instruction focuses on the content and methods of teaching, and success is measured by the degree of implementation of new programs (Richardson, 1992). Teachers are viewed as conduits of information that follow directions given by experts. Teacher learners must use prescribed methods to fix instructional issues using ingenious materials and strategies designed to help learners acquire knowledge. Any breakdown in communication is regarded as a failure, which can be held up and contrasted to a success (Cobb, 1988), rather than recognizing that two people reasoning at different levels cannot understand each other. When classroom examples are reasoned and explained by a teacher at their own level of comprehension, students learn by rote how to manipulate with no understanding (van Hiele, 1985). When dialogue involves language and concepts not understood, adult and student learners face the same difficulty, resulting in fear, avoidance and anxiety (Van de Walle & Folk, 2005).

Richardson (1992) recommends a different conceptual framework for professional development, where teachers are professionals taking responsibility for their own construction of knowledge. An external consultant's role is to help teachers identify problems and develop solutions by exploring teacher beliefs and knowledge, reconstructing their philosophies, and altering practice. Ernest (1988) recognizes that reflection is key to changing instruction, where teachers may consider and struggle with: assumptions and viewpoints on the

nature of mathematics and mathematics learning; teaching alternatives; the school context affecting the choice of specific approaches or content; and how beliefs and experiences affect learning.

The dilemma of agenda setting can be addressed when a professional developer asks questions, guiding the construction of knowledge in a discourse community. A discourse community is a group of people that provides ideas, theories, and concepts for an individual to adopt to help make sense of their own experiences. Learning is not uni-directional, as the community changes ways of thinking when new members bring their own ideas to the discourse. For teachers to construct new roles, they need to communicate with teachers from different backgrounds in order to form new insights into teaching and learning (Putnam & Borko, 2000). As with a K-12 classroom environment, adult learners need to feel comfortable trying new ideas, sharing insight, challenging others, seeking advice, and taking risks (Van de Walle & Folk, 2005). In Richardson's (1992) framework for teacher learning, content or specific direction for change is not emphasized, and success is measured by whether change takes place in the eyes of teacher learners. Higher levels of thought allow teachers to reflect on the gap between beliefs and practice and narrow it (Ernest, 1988), and recognize the rich interconnectedness of new and existing ideas. As with student mathematical learning, large networks of information are more easily retrieved and transferrable to new ideas, creating an "I can do this" positive learning attitude with an increased potential for idea invention (Van de Walle & Folk, 2005).

The answer to the question of teacher professional development lies in the theories of learning that the new curriculum recognizes as valuable in mathematics classrooms. Van de Walle and Folk (2005) note that learners must wrestle with tasks individually or as a group, discussing solutions and strategies. Mathematics and mathematics teaching must be problematic

to encourage learners to wonder, search for solutions, and resolve incongruities rather than educators telling learners how to think or what habits to acquire. "We must believe in children's ideas. When we believe in children, they sense it and respond accordingly." (p. 40) Changing perspectives, we must believe in teachers' ideas, for when we believe in teachers, they sense it and respond accordingly.

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ABOLISHING MATH REFORM IN SASKATCHEWAN

Michelle Naidu

Math reform as a concept is not only obsolete; it is also incongruous with the epistemological framework it establishes itself within. We teach our teachers as if they are the proverbial empty-vessels that professional learning will fill with learning. Our teachers arrive with a plethora of previous experiences and ideas that shape their understanding of the world and new, “better” ideas can be “devastating; ...[can make] them profoundly uneasy” (Schon, 1987, p. 9). They have established theories regarding their students, their subject matter, and their roles and responsibilities (Clark, 1988). Math reform practices are often outdated by the time they reach schools, and they encourage teachers to see professional development as something done to them, rather than ongoing professional learning that is a natural part of teaching.

Teachers’ implicit theories are “robust, idiosyncratic, sensitive to the particular experiences of the holder, incomplete, familiar and sufficiently pragmatic to have gotten the teacher or student to where they are today” (Clark, 1988, p. 7). This complex web of theories that a teacher has constructed over time cannot be undone in a simple “lesson” about new ideas in mathematics education. Moreover, what we know about our students’ learning tells us we need to consider how the learner constructs learning and what situation that learning happens in.

According to the constructivist view of learning, learners must not only construct their own interpretation of experiences and ideas they encounter (Schon, 1987; Van de Walle & Folk, 2003), they also must link new and old knowledge to create an updated, coherent structure that is meaningful to them. This process happens, in different degrees, countless times an hour. However, in order for new information received to be valuable to a learner, it must first create disequilibrium with knowledge

they already believe as true. This will “force” learners into reflecting as to how the new information fits within their current theories, and allow learners to modify their knowledge. Secondly, the new information must in some way, strengthen the learners’ understanding (Sachs & Smith, 1988; Simon, 1995; von Glasersfeld, 1983). If both these conditions are not satisfied, the learner will not create a useful connection to the new idea. The process of challenging previously accepted pedagogy makes teachers feel incompetent rather than excited to be learning new things, making the process of math reform fraught with difficulties.

To further complicate this process, new theories of learning also state that “the situation in which a person learns, become[s] a fundamental part of what is learned (Putnam & Borko, 2000, p. 4). While professional development has traditionally focused on how learning is constructed, we now know professional learning needs to consider the circumstances learning occurs in, including: “(a) situated in particular physical and social contexts; (b) social in nature; (c) distributed across the individual, other persons, and tools” (Putnam & Borko, 2000, p. 4). All of these factors make changing the way we teach mathematics through a half-day session a virtual impossibility.

During professional development, teachers’ implicit theories should be challenged in the light of current research. There needs to be a reason for teachers to engage with the research, so challenges to their thinking need to be relevant and appropriate to the situation. In order for this to be respectful, professional learning needs to be ongoing, and should favor techniques like case study, creating and supporting discourse communities, and using technology because this will encourage continuous exposure to new research and sustained conversations about best practice with colleagues. These methods of professional learning establish teacher learning as an ongoing process rather than

an event where you learn to fix errors in how or what you teach.

Putnam and Borko (2000) state that when choosing how to situate teachers' learning, the goals of the professional development should be considered. It can be desirable to have teachers learn in the same setting that they teach as it is "intertwined with their ongoing practices, making it likely that what they learn will indeed influence and support their teaching practice in meaningful ways" (p. 6). However, if the learning is to encourage teachers to think about mathematics in new ways, Putnam and Borko also argue that "the pull of the existing classroom environment and culture is simply too strong. Teachers may need the opportunity to experience these and other content domains in a new and different context" (p. 6). Both approaches may be valuable depending on the goals, but mathematics reform, rather than situated, ongoing professional learning, is doomed to reduce the amount that the teacher can understand and apply. It also means the teacher is unlikely to make dramatic shifts in either pedagogy or practice.

Franke, Carpenter, Fennema, Ansell and Behrend (1998), Cobb and Steffe (1983), and Cooney (1994) contend that the most effective way to support self-sustained learning for teachers is to anchor their learning in understanding children's mathematical thinking. Franke et al. maintain, "helping teachers understand the development of children's mathematical thinking can provide the basis for fundamental change in teachers' beliefs and practices" (p. 79). Furthermore, self-sustained learning requires engagement at multiple levels:

...Struggling ourselves to understand how the teachers are thinking about the development of children's mathematical thinking not only allows us to better understand teacher development but also provides a forum for teachers and researchers to engage in multiple levels of practical inquiry. (p. 79)

In order to see true change in Saskatchewan's mathematics education, professional developers must distance themselves from a deficit model of change. Instead, those providing professional development must enter into their own struggle to understand teacher thinking. As a teacher's focus on student thinking of mathematics sustains itself, a focus on understanding teachers' thinking of mathematics education will sustain quality professional development. In this way, meaningful professional learning should be supported – not only to promote change in Saskatchewan, but more importantly to support teachers' natural desire to understand more about students' mathematical thinking. This makes professional learning a natural part of being a strong teacher, rather than an event designed to fix pedagogical errors.

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PROFESSIONAL DEVELOPMENT IN A RURAL SETTING: WHY IS IT SO IMPORTANT TO GET IT RIGHT

Mark Jensen

A large number of schools in Saskatchewan can be coined as rural, or non-urban. Being a mathematics teacher in these schools means that one may be the only teacher of mathematics teaching at the middle years and secondary levels. A teacher in this environment would not have the benefit of the number of colleagues that would be present in a collegiate atmosphere. These schools are typically located outside of urban centres where large collegiates or post-secondary institutions offer programming. As a result, teachers in these environments would need to travel some distance in order to receive professional development.

At a time when professional development budgets are both limited and strained, the cost to receive professional development for rural teachers is more expensive than those

who reside near urban settings. Bringing teachers together to a central location, within a school division still involves a large expense. Luebeck (2006) summed it up well when she stated, "Barriers of distance, time, and expense impede rural teachers from attending conferences, workshops, and college courses offered in more populated areas" (p. 35). As curriculum renewal unfolds, school divisions will need to grapple with the key question of "How do we train our teachers economically?" This article will identify some of the unique challenges that face rural teachers and it will identify characteristics of successful professional development along with some supports for both school division personnel and teachers.

Non-urban teachers do not have the access to peer-professionals that those in a collegiate atmosphere do. As a result, the option of sending one teacher for inservicing and then having that teacher inservice others is typically not available for the rural teacher. Luebeck (2006) states, "whereas new ideas and practices adopted by teachers in larger districts tend to 'trickle down' into the awareness of their colleagues through casual conversation or formal dissemination, there is no such potential for the lone rural teacher" (p. 35). The ability to share and network is limited to the occasional face to face meetings or by the use of technology. The use of technology is important in formal professional development settings, but it is no substitute for the informal sharing that spontaneously occurs when teachers of the same subject area share workplace space.

School divisions in rural settings often have to deal with the added pressure of training teachers who are not mathematics specialists to teach high levels of mathematics. In mathematics this can have a decided outcome on the experiences of the students as well as their teachers. Goldhaber and Brewer (1996) stated, "A teacher with a BA in mathematics, or an MA in mathematics, has a statistically significant positive impact on students' achievement

relative to teachers with no advanced degrees or degrees in non-mathematical subjects" (p. 206). Coupled with the challenge of upgrading a teacher's content knowledge alongside the geographical barriers, divisions will need to develop a concept for professional development that is both effective and economical if they aspire to keep their teachers current with the changes taking place in rural Saskatchewan.

Paek (2008) identified three important approaches when focussing in on a teacher's professional development experience. They are, "redefining mathematics teacher roles and responsibilities, making instruction public, and having new, customizable tools for teaching" (p. 12). Her research calls for teachers to be networking and discussing their roles as teachers. In a school division with geographic barriers, it is possible for teachers to be networked through technology. However, it is imperative that teachers be allowed some face to face time to discuss their roles and responsibilities. Quint (2006) pointed out that, "There is suggestive evidence that student achievement may be enhanced by professional development activities that involve teachers working together to align curricula with standards, review assignments for rigour, and discuss ways of making classroom activities more engaging" (p. 6). One must value the opportunity to network with colleagues. Too often the discussions have the tendency to go into directions that do not focus on the tasks at hand. Quint (2006) cautions us by stating, "If administrators want teachers' meetings to focus on instruction improvement, they must both provide guidance about how to do this and follow up to ensure that meeting time is used productively" (p. 7). The professional learning communities that many divisions have already in place could play a strong role with the improvement of teachers, provided it is structured, guided, and timely.

The second approach to making instruction public calls for teachers to collaborate and observe one another in the classroom. As mathematics teachers work

through the new curriculum and work with new instructional approaches, it will become necessary for them to feel comfortable with their networking community. This means they will need to be observed by colleagues, administrators, and divisional personnel, some of whom will have insight into how the instruction should take place. Paek (2008) states, "When instruction is public, teachers learn about the power of collaboration for improving their practice and lose the fear of observers in the classroom" (p. 12). Teachers are going to need to be students and instructors of new teaching methodology. The ability to network and showcase exemplars of instruction with one another will go a long way in breaking down existing rural barriers.

The last approach calls for teachers to access individual training to guide their growth. This is where divisional leaders along with school-based administrators will need to be certain that affordable professional development is offered. Allowing groups of teachers to attend seminars and conferences can save the division and teachers money. Also, utilizing organizations such as the Saskatchewan Mathematics Teachers' Society (SMTS), the National Council of Teachers of Mathematics (NCTM), and the National Council of Supervisors of Mathematics (NCSM) allows teachers to have access to online materials, journals, teaching strategies, and conferences. Teachers within their professional learning communities can study information from such organizations in a context that is useful for them.

One important issue that has not been mentioned yet deals with time. Luebeck (2006) states, "Teacher growth requires time, and effective professional development must be of sufficient duration, both in terms of total contact hours and the length of time spanning those hours" (p.37).

Teachers in rural Saskatchewan today feel pressures from many different places, both in the school and outside the school. It

is key that leaders ensure that teachers have the time that is necessary for a quality professional development experience and are committed to the offerings of professional development that exist in a variety of forms.

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THE EFFECTS OF OUT-OF-FIELD TEACHING IN SECONDARY MATHEMATICS ON STUDENT ACHIEVEMENT

Julie Helps

Teachers are often assigned to teach courses for which they have little or no subject specific training or formal education. Ingersoll (1999) refers to this as *out-of-field teaching*. While most parents assume their children are being taught by educators with a solid background in mathematics, this is often not the case. The study performed by Ingersoll found that over 33% of mathematics teachers in U.S. secondary schools were out-of-field. Several studies show positive correlations between in-field teaching and student achievement in

mathematics, yet the practice of assigning teachers to unfamiliar courses continues. Although many justifications are given for the abovementioned practice, the negative implications must be considered, and adequate support must be given to those who are expected to teach mathematics without the proper background.

Multiple research studies confirm the benefits of in-field teaching on mathematics achievement. In a study performed by Hawk, Coble, and Swanson (1985), the achievement of students in general mathematics and algebra courses was greater when certified mathematics teachers taught the courses. Although the discussion of how to certify a mathematics teacher is out of the scope of this paper, Hawk, Coble, and Swanson assume content mastery to be a prerequisite for teaching. The conclusions of this study suggest the reasons for improved student achievement are both content knowledge and more effective instructional presentation skills.

Similar studies have come to different conclusions, which at first sight seems to contradict the above statements, but upon further examination support the premise of this paper. Studies performed by Begle (1972), as well as a follow-up study by Eisenberg (1977), found no significant correlation between student achievement and teacher knowledge. It must be noted that in both studies the participants had experience with University math and were considered qualified mathematics instructors, which makes the studies inconclusive in terms of out-of-field teaching. However, this may lead to the theory that the main factor attributing to student achievement is not the grade point average of the instructor, but pedagogical content knowledge, which Shulman (1986) defined as “subject matter knowledge *for teaching*” (p. 9). Relating to these studies, it is not the amount of mathematics known (assuming at least some University training), but how well it can be applied in the mathematics classroom.

Much of the research into teacher effectiveness in the mathematics classroom assumes mathematical knowledge is fundamental to successful teaching, but not necessarily the only factor. Ferguson and Womack (1993) refer to a minor in mathematics as “the lower limits of adequate preparation” (p. 61). Ingersoll (1999) describes a mathematics minor as “a minimum prerequisite” (p. 27) of a mathematics educator. If these are the underlying assumptions, then why does out-of-field teaching occur? Ingersoll offers two different explanations for this situation. First, he suggests that teaching is regarded as a semi-profession, which allows society to view teaching as requiring less skill, and therefore specialization is deemed unnecessary. Second, the trend to move toward smaller schools for an increased sense of community and belonging makes scheduling difficult, and the disadvantages of out-of-field mathematics educators on student learning are being ignored.

Out-of-field teaching in the mathematics classroom has multiple implications for the learner. These teachers often rely on the textbook alone for instruction, limiting students’ critical thinking skills (Ingersoll, 1999). Ball and McDairmid (1990) caution “when teachers possess inaccurate information or conceive of knowledge in narrow ways, they may pass on these ideas to their students” (p.2). An effective teacher must be able to explain mathematical ideas in multiple ways and make the content accessible to all students (Hill, Sleep, Lewis, & Ball, 2007), which is difficult without the proper pedagogical content knowledge. Additionally, a teacher asked to teach an unfamiliar math course will most likely spend a disproportionate amount of time preparing for the new course, decreasing time allotted to other courses (Ingersoll, 1999). The teacher themselves may also be negatively affected, as Ingersoll found a correlation between out-of-field teaching and a decrease in teachers’ morale and commitment.

With the realization that a large percentage of mathematics teachers have no formal background in mathematics or mathematics education, how should one proceed? First it must be recognized that the context for which they know mathematics is their own high school education (Ball and McDairmid, 1990), which may not have included constructivist approaches. Ball and McDairmid also point out that teachers gain knowledge as educators over time, but that does not guarantee more knowledge about mathematics or mathematics education. It seems necessary to offer out-of-field teachers relevant professional development that meets their specific needs. Not only do they need to familiarize themselves with the curriculum, but the specific pedagogy that relates the mathematics to student learning. University courses specific to mathematics education are designed to help future educators link their mathematics understanding to student learning, but these courses have not been taken by out-of-field teachers. The concepts examined, focusing on pedagogical content knowledge, must be offered to new math teachers in some form. This extra assistance is likely to benefit the out-of-field teacher, as well as their students.

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ARE MANIPULATIVES FOR TEACHING OR UNDERSTANDING

Ronald Georget

Mathematical manipulatives are not the “be-all-and-end-all” of the classroom. Research has shown that a misuse of manipulatives can result in little or no improvement in student achievement (Van de Walle & Folk, 2005). However, there are numerous cases which demonstrate that the proper use of different forms of mathematical manipulatives can improve student achievement, academically and behaviorally. It is important to note that manipulatives are not tools for teachers to solely teach with, but a concrete representation for students to form connections for a more profound understanding of a concept.

It is important to realize that, “in order to ‘see’ in a model the concept that it represents, you must already have that concept—that relationship—in your mind” (Van de Walle & Folk, 2005, p. 36). This is why teachers see more meaning in using a model than students. If the student does not

know the relationship of the concept they cannot apply it to the model they are using. Furthermore, teachers must not impose their relations with the manipulatives, on their students because if, “the concept does not come from the model—and it does not—how does the model help the child get it” (p. 36). An example of this misuse of manipulatives can be seen when using manipulatives to do addition. A teacher can give a student blocks, but unless that student knows the concept of addition, he will only have physical objects in front of him with no mathematical relation (Van de Walle & Folk, 2005). This is a basic example, however, there are many in-depth examples, such in the work of Falkner, Levi, and Carpenter (2002). Their research centered on a kindergarten class that had trouble with this number sentence: $4 + 5 = \underline{\quad} + 6$ (p. 202-203). All of the students believed that 9 was the missing value. The teacher decided to model the statement using manipulatives, even though the students had a different understanding. Together they represented the statement with piles of blocks. The students were then able to vocalize that 3 blocks were needed to balance the two groups, but when the teacher returned to the statement, the students still believed that 9 was the missing value from the statement. Here the manipulatives were being used to teach the concept and as a result, the students did not benefit. This precisely affirms the idea that manipulatives are used to reinforce a concept and *not* to teach one. If the students had worked on the concept then been given a variety of manipulatives, their connections would have been more meaningful.

Students should have access to a variety of manipulatives in order to help them work through a problem or explain what they are thinking. This practice should be encouraged by teachers. Students should be able to select the manipulative they find most engaging because this is what will make sense to the student. Teachers have a well developed understanding of the concepts, which influences how they use a certain

model (Van de Walle & Folk, 2005). Students must create their own understanding of concepts and this will be done through different techniques than those teachers would choose. This is an opportunity for teachers to learn from the students through their explanations and representations to see how different manipulatives can reach the same conclusion.

There are many benefits, aside from academic achievement, to using mathematical manipulatives. Ernest (1994), Sowell (1989), and Rust (1999) concur that attitude, participation, interaction, and performance improve with consistent use of manipulatives in the teaching of mathematics. They found that students' attitude was positive, participation increased, and the students enjoyed math when they were able to use manipulatives. Only once students have a basic understanding of a concept, and are enjoying working with manipulatives, they can then create connections from real-life to mathematical theory through exploration (Van de Walle & Folk, 2005). The more connections a student has made, the better the understanding of the concept will be. This strong understanding will make it easier to create new connections to future concepts.

Sowell (1989) states that, "mathematical achievement is increased through the long-term use of concrete instructional materials" (p. 498). Manipulatives have too important of a role to only use occasionally. These tools help students reach a deeper understanding. Corneille (1995) found that, "children's conceptual understanding came from their explorations with manipulatives and from the decisions they made as they solved problems with those tools" (as cited in Rust, 1999, p. 6). These are just two of the many testimonials that researchers have stated in approval of mathematical manipulatives. Granted every individual group of students will react to the manipulatives in a different manner, but how can educators pass on this exciting

experience, which can increase the students' understanding of concepts?

From the research presented, it is clear that there have been, by far, more results to support the argument that when manipulatives are properly used, academic achievement improves. Encompassed in this achievement are other attributes such as attitude and participation, which improve as well. However, there are some reports that disagree with the present argument made and demonstrate that the use of manipulatives has no additional effect on student achievement. The question then arises, did these teachers use manipulatives to teach the concept or as a support for the students further development of understanding? Overall, it can be argued that the proper use of manipulatives will help their students connect to the math they are doing in class. These connections will internalize the math for students and make it easier for students to remember as they move on to more complex concepts.

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TO USE OR NOT TO USE: GRAPHING CALCULATORS IN THE SENIOR MATHEMATICS CLASSROOM

Kim Andersen

As technology becomes more powerful and more easily available, teachers need to look at how it is affecting teaching and learning in the classroom. Gone are the days when calculators could simply add, subtract, multiply and divide. There is very little that calculators and computer programs (e.g., www.wolframalpha.com) cannot do. As teachers, I contend, we need to embrace the new technology and use it to enhance student learning. Sure, students can simply plug information into a calculator and get an answer without ever understanding what they are really doing. But, there are also many ways to incorporate the calculator as a learning tool to help students understand the mathematics. As teachers, I contend, we need to look at our beliefs about mathematics education, and adapt and develop new and different activities and teaching methods to effectively use calculators to improve student learning and understanding.

When students are allowed to explore and discover patterns and generalizations, they are using previous knowledge as a base level to create new connections. As students are able to build these connections for themselves, they will not only have a better understanding of the concept, but will also retain the information and be able to use it (Van de Walle & Folk, 2004). The Western and Northern Canadian Protocol (WNCP) agrees that calculators and technology help to create a learning environment that increases student curiosity and should be used as one method to increase mathematical understanding (Western and Northern Canadian Protocol, 2008). With

the calculator, “less emphasis is placed on facility with paper and pencil calculations and more emphasis on mathematical concepts and their relationships” (Saskatchewan Education, 1996, p 19).

Tan and Forgasz (2006) did a study of calculator use in classrooms and found that 80% of teachers believed that calculators helped students understand the mathematics better. They found that graphing calculators were useful “in providing graphical representations, saving time from tedious calculations and sketching, allowing students to explore mathematical properties, motivating students and aiding investigations and explorations by students” (p 254). By using the calculators, students can focus on the concepts that they are trying to learn, without being bogged down by all the calculations.

To use calculators in this way, teachers not only need to change their style of teaching, but also their method of assessing. By asking questions such as ‘Sketch the graph of $y=2(x+1)^2-3$, students can simply put this into their calculator and copy the sketch into their books, without knowing what they are doing. However, if the student is given a graph and asked to find an equation for the graph and to provide their reasoning, they still need to understand the concepts, but can use their calculators to check their answers (Ruthven, 1990). Using a calculator to check their answer (it) can provide students with a variety of information. “A check which reveals that a formula is incorrect, particularly a graphic check, also provides further information which may help in revising the formula” (Ruthven, 1990, p 441).

Many students come into the classroom with math anxiety. One way to help combat this anxiety is by using a calculator. Ruthven (1990) observed that the “use of feedback from a graphic calculator can reduce uncertainty and thus diminish anxiety” (p 448). Loyd (1990) believes that there is also a positive impact related to attitudes towards mathematics. She also hypothesizes that

students who are allowed to use calculators achieve better results because students “are more relaxed, more confident, or have a more positive attitude toward the testing situation” (p 20). For some students, their calculator is like their security blanket. Even though they may not need it, without it they feel lost. Not only senior students benefit from using calculators; students at all grades and ability levels benefit from the use of calculators. They have a greater motivation to work together, more self-confidence in problem solving, and more positive attitudes and enthusiasm about mathematics. They show more persistence and a willingness to seek alternative solutions. Researchers also indicate that the learning of basic facts and skills is enhanced through the use of calculators (Saskatchewan Education, 1996).

If calculators can help students to succeed, are we holding them back? Studies have also shown that there are no significant differences in procedural skills for students who use calculators, compared to those who do not (Burrill, Allison, Breaux, Kastberg, Leatham & Sanchaz, 2002). When using the calculator, students still need to know what operations to use and to do this they still need to understand the mathematics. Calculators are only a tool (Saskatchewan Education, 1996), and only one tool that can help enhance teaching and learning in the classroom. “The use of technology should not replace mathematical understanding” (Western and Northern Canadian Protocol, 2008, p 9).

When talking about how to use calculators, we need to think about what our goals for mathematics education are. What do we want students to be able to do when they leave our classroom? “The basic skills we require in today’s world are not proficiency with operations, but rather those of reasoning, communicating, problem solving and applying knowledge to new situations” (Saskatchewan Education, 1996, p 20). With the help of calculators, we are able to build these skills and prepare our students with what they really need once they leave the mathematics classroom and

go out into the real world. “If we are to prepare our students of today for the world of tomorrow, we must develop their ability to use a calculator effectively and efficiently” (Saskatchewan Education, 1996, p 20).

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**GRAPHING CALCULATOR USE:
CHANGING THINKING ABOUT
MATHEMATICS AND INSTRUCTION**
Murray Guest

Studies show that using graphing calculators in math class will, in general, increase student understanding and achievement in mathematics from middle years to university (Barton, 2000; Dunham & Dick, 1994; Kastberg & Leatham, 2005). A common provision to these research findings is that graphing calculator use must be “intelligent” (Martin, 2008, p. 20) or “used appropriately” (Barton, 2000, p. 5) for them to have positive effects. There should also be a shift in thinking about mathematical relationships and a change in both the types of questions posed in class and a shift in the role of the teacher in a class in order to maximize the effectiveness of graphing calculators.

The appropriate use of graphing calculators can have many positive effects in mathematics education. Graphical and numerical reasoning can become more important to students in addition to the algebraic thinking and manipulations that are currently honoured in math classes (Goos, Galbraith, Renshaw, & Geiger, 2001). It is possible to have students explore connections between numerical, graphical, and algebraic representations of a problem, decide what counts as a solution to a problem, exercise autonomy from the teacher as sole arbiter of what is correct, engage in useful collaboration with peers, and present their solutions for critique by the class with graphing calculators (Goos et al., 2001).

These goals cannot be fully achieved when graphing calculators are used occasionally because students may see them as useful only when a teacher tells them they are useful, when a teacher says they are useful. Graphing calculators should be available to the entire class every day so their use becomes natural and students have the time to explore the strengths, weaknesses and uses of the graphing

calculator (Kastberg & Leatham, 2005; Martin, 2008). The calculator must also be used with thinking beyond looking at mathematics as a collection of algebraic algorithms and techniques which is too constraining to achieve best results with the graphing calculator (Kastberg & Leatham, 2005), although algebraic techniques can be strengthened with the use of graphing calculators (Forster, 2004). Additionally, teachers can best help students achieve the gains from the use of graphing calculators by moving away from the center of the class with the role of the one who knows “the” answer and moving into the role of a guide and resource for the student as she makes her own knowledge using the graphing calculator as one of several tools.

The power of the graphing calculator is illustrated by a shift in thinking about the sign of equality as a comparison between two functions rather than creating an equation to be algebraically manipulated to find a solution set (Yerushalmy, Leikin & Chazan, 2004). When using simultaneous equations, for instance, $y=3x+2$ and $y=-2x+7$, a math student may justifiably use thinking about relating two functions $3x+2=-2x+7$ to yield graphs which will show where the two functions share a common point of intersection $(1,5)$. This method of solving simultaneous equations is used in many parts of Saskatchewan, but consider how this mode of mathematical thought, the comparison of two functions, becomes useful when exploring an equation where no algebraic manipulation will yield answers, expressions like $x^2=\sin(x)$ [with points of intersection $(0,0)$, $(0.87672662,0.76864886)$] or $2^x=x^2$ [with points of intersection $(-0.7666647,0.58777476)$, $(2,4)$, $(4,16)$]. By comparing the graphs of two functions and using the intersect function, good numerical answers are available in addition to an appreciation of the relationship between the graphs of two functions. Because the graphing calculator allows different ways of conceiving of mathematics, students are able to approach more types of problems in ways

that are unattainable with algebraic thinking alone and are able to use these approaches in a way that can better illuminate mathematical ideas.

Beyond the shift in thinking which can occur with intelligent graphing calculator use is the opportunity for students to explore questions in many ways. Someone may look at graphs and intersections, or try to use algebraic manipulations, or use a table of values approach to make sense of the above equations. Students can then start making decisions about what methods are strongest, what level of precision is appropriate for the question, and what ways they think about functions and equations without having to use the teacher as the arbiter of what is right and wrong in mathematics. Students will also share their insights more easily with graphing calculators. They are likely to swap calculators and explain to each other how they arrived at their answers, both technically and mathematically (Goos et al., 2001).

Questions that are based in reality, are more open ended, and have avenues for mathematical exploration embedded in them are better suited to graphing calculator use. There are textbooks that support this view of mathematics (Cosenza et al., 2006; Smedley & Wiseman, 2004) so that teachers do not have to create materials entirely on their own. Without these open-ended questions, the use of graphing calculators to supplement algebraic manipulations will rightly be interpreted by students to be of secondary importance.

With a wealth of research to show the benefits of graphing calculator use, more materials designed to support the types of questioning, and thought that maximizes the power of graphing calculators, it becomes increasingly difficult to justify continuing to teach with a predominantly algebraic, technical view of mathematics, one that leaves the graphing calculator as an afterthought rather than a main part of mathematics learning. Instead, teachers can use open-ended questions, which place the

student at the center of answer construction, thinking about and discussing the merits and weaknesses of answers, aided by graphing calculators.

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DISPELLING THE MYTHS: ONLINE LEARNING AND MATHEMATICS

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What is it about math that causes us to groan when someone talks about teaching it as an online course? Is it the formulas, the links that need to be made, or maybe all the symbols? It seems to be ingrained in people's minds that in order to learn math you must have a teacher looking over your shoulder. Some people are skeptical about math being taught effectively online. Teaching online can mean either asynchronously (at a different time, i.e., lessons are posted and students read or watch them when they choose to) or synchronously (at the same time, i.e., students are watching a live feed of the teacher and able to talk to the teacher).

There are two important myths related to the teaching and learning of mathematics online: children cannot learn mathematics online, and constructivist teaching methods are not possible in online mathematics. These myths will be dispelled and some ideas on why online learning is important will be presented.

Myth #1: Children cannot learn mathematics online. Bernard, Abrami, Lou, Borokhovski, Wade, Wozney, et al. (2004) conducted a meta-analysis of 232 studies and Cavanaugh, Gillan, Kromey, Hess, Blomeyer (2004) conducted a meta-analysis of 14 web delivered schools. Both analyses concluded that there is no significant difference in learning effectiveness between online learning and face-to-face learning. In fact, so many studies have come to this conclusion that researchers coined the term *the no significant difference phenomenon* to describe the comparison of online versus face-to-face learning.

Hughes, McLeod, Brown, Maeda and Choi's (2007) *Assessment for Algebraic Understanding* test was used to compare student's achievement in similar mathematics curricula in three virtual schools and three traditional schools throughout three different states. Students' demographic characteristics showed few differences. Students were scored using an exam based on four subscales: patterns and relations, using algebraic symbols, mathematical models, and analyze change. Analysis of the results concluded that online students outperformed face-to-face students in this study.

These results tell us that students can learn mathematics effectively online. We must remember though, that the quality of instruction is an important factor in determining if students succeed. A student will not do as well in an online class, or face-to-face class, if the instruction is done poorly.

Myth #2. Constructivist teaching methods are not possible in online mathematics. In Saskatchewan, the new mathematics curricula promote constructivism, the idea that students need to construct their own knowledge, which they do through problem-solving, group work and discussion. This sounds very challenging to implement in an online mathematics course, but there are many online tools to help students do this.

In a synchronous setting, students and teacher can talk to and see each other, using webcams, projectors, microphones, and online whiteboards. If there are two or more students at each receiving site those students can be grouped to discover mathematics together. On the other hand, if there is only one student at each site, the students can be grouped and each group 'given' a virtual classroom to meet in. If the students are using webcams they will be able to see what each person is doing and talk about it, or without webcams they will need to converse on the topic and use a shared whiteboard to diagram their models or thoughts. In an

asynchronous setting the same things can happen; students need to set up a common time when they will meet in a virtual classroom.

Another way for communication to happen is through discussion boards. Discussion boards allow students to think about a topic such as “what is zero?” critically before responding in the discussion. Meyers (2003) did a study of students who each enrolled in both a face-to-face class and an online class concurrently. In a comparison done by the students on the discussions, Meyers summarizes, “The threaded (online) discussions were often more ‘thoughtful,’ more reasoned...” (p. 61). This shows that higher-order thinking skills are used in online discussions, which is an important skill when students are constructing their own knowledge.

Technology offers many mathematics applets (interactive online activities) and virtual manipulatives for mathematics. If you Google math applets you will receive approximately 1.48 million results. There are hundreds of great math applets on the internet that can be very useful in an online class. For example, The National Library of Virtual Manipulatives (<http://nlvm.usu.edu/en/nav/vLibrary.html>) has pattern blocks, geoboards, algebra tiles and much more. These applets can all be used by students or groups of students in a virtual classroom. They are usually free and convenient to use, allowing students more tools to learn mathematics.

Why do we need to offer mathematics online? Online mathematics is a way to supply quality mathematics to schools that do not have a math teacher or to students who live in remote areas. It may also be used as a way for a group of schools to offer students, at the secondary level, all three pathways in the new math curricula. In a group of three schools, each math teacher would offer one of the three pathways to their own students and online to the other two schools. This allows all students to choose which pathway(s) they wish to take

and teachers to focus on one curriculum (or two in a doubled graded classroom) in a single class period.

Research has shown that children can learn mathematics online and constructivist teaching methods are possible in online mathematics. So: Individuals can quit being afraid of online math and embrace it. The implementation of mathematics online will not be seamless. There will be bumps in the road and technological issues to deal with, but it will be worth it. Online learners will have new tools and a new perspective to apply to mathematics. It is a positive step in education, which will benefit students, teachers and schools.

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CURRICULAR EDITION

As detailed in a recent Editorial:

At the present point in time, and with respect to the teaching and learning of mathematics in the province of Saskatchewan, we are in the midst of major change. Saskatchewan's recent adoption of WNCPC (Western and Northern Canadian Protocol for collaboration in education) Mathematics' Common Curriculum Framework has introduced new mathematics curricula to the province for grades K-9. Further, the province will adopt new mathematics curricula for grades 10, 11, and 12 in 2010, 2011, and 2012, respectively.

More specifically, the implementation dates were/are as follows:

Curricula	Implementation date
K, 1, 4, 7	Fall 2007
2, 5, 8	Fall 2008
3, 6, 9	Fall 2009
Workplace and Apprenticeship 10 Foundations of Mathematics 10 Pre-Calculus 10	Fall 2010
Workplace and Apprenticeship 20 Foundations of Mathematics 20 Pre-Calculus 20	Fall 2011
Workplace and Apprenticeship 30 Foundations of Mathematics 30 Pre-Calculus 30	Fall 2012
Calculus 30	January/February 2013

With the overwhelming change (detailed above) both ahead of us and upon us, the Journal of the Saskatchewan Mathematics Teachers' Society, *vinculum*, is seeking articles for a curricular themed edition. In other words, we are seeking **Articles** and **Conversations** that focus on school mathematics curricula. We also welcome submissions that fall outside of the April issue's theme. Given the wide range of parties interested in the teaching and learning of mathematics, we invite submissions for consideration from any persons interested in the teaching and learning of mathematics, but, as always, we encourage Saskatchewan's teachers of mathematics as our main contributors. Contributions, curricular themed or otherwise, must be submitted to egan.chernoff@usask.ca by **March 1, 2010** to be considered for inclusion in the April issue.



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