

The



# Variable

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January/February 2017

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## Notice to Contributors

*The Variable* welcomes a variety of submissions for consideration from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants, teacher educators, researchers, and students of all ages, although we encourage Saskatchewan teachers of mathematics as our main contributors. Submitted material is assessed for interest, presentation, and suitability for the target audience.

Please submit articles by email to [thevariable@smts.ca](mailto:thevariable@smts.ca) in Microsoft Word format. Articles should be limited to 3000 words or less; authors should also include a photo and a short biographical statement of 75 words or less. Accepted contributions are subject to revision. Editors reserve the right to edit manuscripts for clarity, brevity, grammar, and style.

The Saskatchewan Understands Math conference is for K-12 math educators and all levels of educational leadership who support curriculum, instruction, number sense, problem-solving, culturally responsive teaching and technology integration.

For more information or to register, visit [www.stf.sk.ca](http://www.stf.sk.ca).



Saskatchewan Mathematics Teachers' Society



# Leading Together

The SMTS, SELU and SPDU presents ...

# #SUM2017

Who: K-12 teachers, coaches, consultants, coordinators, superintendents and directors

Where: TCU Place, Saskatoon

When: October 23-24, 2017

Cost: \$315 (early registration) | \$375 (regular)

## Keynote Presenters

**Lisa Lunney Borden**, St. Francis Xavier University

**Steve Leinwand**, American Institutes for Research



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## Message from the President



Well, I'm not going to lie. I sure wish I would have written this President's message over the Christmas break as a kindness to myself, something to take off of my to-do list for January Michelle. Because after last week (heck, even the last few hours), it's impossible to pretend that it's business as usual in our teaching lives.

While this is a space that we normally share because of our love and interest in the teaching and learning of mathematics, there are very real concerns and threats infiltrating our own and our students' day-to-day lives in new and scary ways. As such, I encourage you to find ways to actively engage politically in whatever way that make sense for you. Maybe this means coffee with your neighbours to talk about your own educational reality, or writing to your elected officials about your concerns. Maybe it means volunteering. Maybe it's donating money. Or maybe it's just a daily affirmation to your students that your classroom is a safe space, and that they are loved and valued. As my dear friend Anne Schwartz says, "you do you." But there is a key element of *doing* here. At the same time, while your students and neighbours may need your support now more than ever, I also encourage and remind you to practice self-care.

Closer to our purpose of supporting the teaching and learning of mathematics, I'd also like to challenge you to think about those possibly slightly neglected statistics and probability outcomes of the curriculum. Your students need the ability to read, decode, and decipher data in their world. While we need to be creating safe spaces for them to process their own experiences regarding the changes in the world around them, we can simultaneously provide them with mathematical understandings about how data is and can be used (and misused). There is so much potential within these outcomes as a framework to notice and wonder about current and historical uses of data in the media, in politics, and elsewhere in our lives. And, of course, I encourage you to share with other teachers what you and your students get up to, or what you plan to try, by contributing to *The Variable*.

'Till the next edition, take care of yourselves.

Michelle Naidu



## New Year, New Schedule

*Editorial*

**H**appy New Year!

Do you have any New Year's resolutions? Perhaps you would like to incorporate number talks on a regular basis in your classroom, learn how to use the activity builder in Desmos, or even simply remember to take some time for yourself during the school year. If you're looking for some ideas, Saskatoon math teacher Amanda Culver shares her own resolutions in her article "2017 Teaching Resolutions" (p. 8).

We—that is, *The Variable* team—have a resolution, too: to bring you great, relevant content all year round. To ensure that we can continue to do so, we have decided to change our schedule from a monthly to a bi-monthly release. So from now on, you can look forward to a new issue of *The Variable* every two months, starting with the current issue: January/February, which will be followed by March/April, May/June, July/August, September/October, and November/December. As always, all issues of *The Variable* will continue to be free to access and download on our website at [smts.ca/the-variable/](http://smts.ca/the-variable/).

Whether you teach in Kindergarten or Grade 12, we hope that you continue find this periodical relevant and valuable for your teaching. If you have any questions or comments about what you read, don't hesitate to get in touch by emailing us at [thevariable@smts.ca](mailto:thevariable@smts.ca)—we would love to hear your feedback. And if you would like to share your perspectives and experiences with a wider audience—for example, in the form of a lesson, essay, or story—please consider contributing to *The Variable* (see p. 42 for more information). A New Year's resolution, perhaps...?

We look forward to hearing from you. In the meantime, happy reading!

*Ilona Vashchyshyn and (on behalf of) The Variable team*



## Problems to Ponder

Welcome to the January/February edition of *Problems to Ponder*! This collection of problems has been curated by Michael Pruner, president of the British Columbia Association of Mathematics Teachers (BCAMT). The tasks are released on a weekly basis through the [BCAMT listserv](#), and are also shared via Twitter ([@BCAMT](#)) and on the [BCAMT website](#).

Have an interesting solution? Send it to [thevariable@smts.ca](mailto:thevariable@smts.ca) for publication in a future issue of [The Variable](#).



British Columbia  
Association of  
Mathematics Teachers

I am calling these problems ‘competency tasks’ because they seem to fit quite nicely with the curricular competencies in the British Columbia revised curriculum. They are non-content based, so that all students should be able to get started and investigate by drawing pictures, making guesses, or asking questions. When possible, extensions will be provided so that you can keep your students in flow during the activity. Although they may not fit under a specific topic for your course, the richness of the mathematics comes out when students explain their thinking or show creativity in their solution strategies.

I think it would be fun and more valuable for everyone if we shared our experiences with the tasks. Take pictures of students’ work and share how the tasks worked with your class through the [BCAMT listserv](#) so that others may learn from your experiences.

I hope you and your class have fun with these tasks.

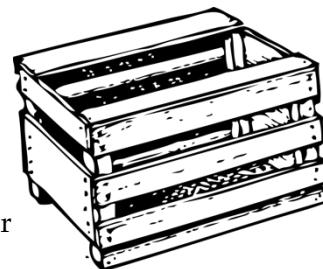
*Michael Pruner*

### Intermediate and Secondary Tasks (Grades 4-12)

#### Milk Crate

A certain milk crate can hold 36 bottles of milk. Can you arrange 14 bottles in the crate so that each row and column has an even number of bottles?

Extensions: What is the smallest array that can fit 14 bottles under this rule? What about 15 bottles?



Source: Mason, J., Burton, L., & Stacey, K. (1985). *Thinking mathematically*. Essex, England: Prentice Hall.



#### Mountain Bike Race

In a mountain bike race, there are 25 racers. The track can only fit five racers at any time. Devise a strategy to determine gold, silver, and bronze. How many races are necessary?



### Tethered Goat

A goat is tethered by a 6-metre rope to the outside corner of a shed measuring 4 m by 5 m in a grassy field. What area of grass can the goat graze?

Extensions: What if the rope was fastened to the middle of one wall? What if the rope was 20 m long? What if the shed was circular?

Source: Mason, J., Burton, L, & Stacey, K. (1985). *Thinking mathematically*. Essex, England: Prentice Hall.



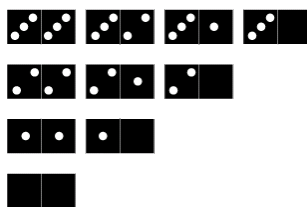
### Primary Tasks (Grades K-3)

#### Coloured Dice

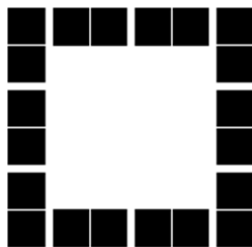
Roll 3 different coloured dice. What are all the possible ways to get a total of 5 points?

#### Dominoes

These are the “double-3 down” dominoes.



Use these dominoes to make the following square, such that each side has eight dots:



Source: Domino square. (n.d.). Retrieved from <http://nrich.maths.org/146>



*Michael Pruner is the current president of the British Columbia Association of Mathematics Teachers (BCAMT) and a full-time mathematics teacher at Windsor Secondary School in North Vancouver. He teaches using the Thinking Classroom model where students work collaboratively on tasks to develop both their mathematical competencies and their understanding of the course content.*

## Reflections

Reflections is a monthly column for teachers, by teachers on topics of interest to mathematics educators: reflections on classroom experiences, professional development opportunities, resource reviews, and more. If you are interested in sharing your own ideas with mathematics educators in the province (and beyond), consider contributing to this column! Contact us at [thevariable@smts.ca](mailto:thevariable@smts.ca).



### 2017 Teaching Resolutions

Amanda Culver

A new year means a new start. A time to reflect on the previous year and a time to set goals for the year ahead. In two weeks, I get to teach math again! As a French Immersion teacher, I love teaching my French courses, but I also miss working with math. In particular, knowing that this will be my second time teaching the Foundations of Mathematics 20 course, I am excited to come at the course from a different *angle*, after doing a lot of *reflection* at the end of last year (puns intended). And so, in the spirit of the season, I have developed a list of resolutions to guide me this year. The resolutions (or goals) that follow mostly apply to this course, but also extend to my extra-curricular mathematical involvements.

“A new year means a new start. A time to reflect on the previous year and a time to set goals for the year ahead.”

#### 1. Practice more math.

As I don't teach math frequently, I can often go quite a while without doing any “challenging” math. What I mean by “challenging” is math that goes beyond just the basic facts—the math where you write a little and think a lot. I have now gone about seven months without teaching math, and, as happens with any other language, I get rusty.

I do co-run the math club and math circle at our school (two different extra-curricular math groups), so I do engage in some mathematical thinking for about two hours a week. However, I am often working on other things during this time span, and really only sit down with a question if a student brings one forth.

So, one of my many goals this year is to actually *do more math*. As I write this, I am sitting with my math circle group, after school on a Friday. I have plopped myself down beside a student who has quite a few questions, and I've been working with him on solving multi-variable systems of equations. Baby steps! I definitely want to get back up to being able to answer calculus-related questions without having to fact-check as often as I do.

#### 2. Find and share applications with students for each topic/theme taught.

Arguably the most common question in a math classroom is “When will I ever use this?” Last year, I was proud of myself when I was able to engage students with trigonometry by making connections to blood spatter analysis. My goal this year is to

share more connections to careers where the math we are using in Foundations 20 is applied in practice. (If you have any suggestions for me, I'd be glad to hear them!)

### **3. Find picture books that share math concepts used in a secondary classroom.**

I grew up and went to school in Ontario, where education degrees are specialized based on a combination of two of four different grade levels. I went to school for K-6 (primary/junior) education, and later earned my certifications to teach K-12 (with specialties in French and mathematics).

One thing I particularly enjoyed about working in the elementary grades was the plethora of picture books I was able to use to engage students. Unfortunately, at the high school level, math topics become more and more abstract, and there are fewer and fewer "elementary-styled" resources I can bring into the classroom. *What's Your Angle, Pythagoras?* by J. Ellis (2004) is a nice tie-in when introducing Pythagorean Theorem, albeit in Grade 7. I create my own mnemonic devices and songs, but it would be nice to have some picture books to supplement the other resources available (and how awesome would it be to have this in French?!).

### **4. Partner with a math teacher. Learn from others.**

As a French Immersion teacher, I often wind up in my own little bubble. Last year, I had the pleasure of collaborating with the marvelous Sharon Harvey, and I absolutely loved the experience! She challenged me to rely less on the textbook and to find some fun activities for this group of students. Unfortunately, we are no longer teaching at the same school. This year, I am partnering with the Foundations 10 and 30 teachers so that we can develop a clearer pathway for students taking the Foundations courses, making sure that they are well prepared for the next class.

### **5. Try something new, and don't spend as much time with the textbook.**

Last year, I really wanted to try some flipped lessons (and students really wanted some videos in French, so that they could become more familiar with the vocabulary), but I couldn't fully commit, especially since it was the first time I was teaching Foundations 20. Now that this will be my second time teaching the course, I feel a lot more comfortable trying out new ideas and strategies. Rather than committing to the term "flipping," I would instead like to make at least one video to supplement my in-class lessons.

Also, now that I have taught each unit, I know where I need to spend more time and where I can incorporate more activities. I would particularly like to incorporate more up-out-of-your-seat activities, especially with our parabolic functions unit. I also have some new ideas for teaching linear inequalities.

I am looking forward to growing as a teacher this year in committing to these and other goals. As I have come to learn that life isn't all about work, I wanted to share some of my "non-math-related-but-still-teaching-related" resolutions:

1. Talk to more adults about non-school-related things.
2. Leave school by 6pm, and leave work at school.

3. Take more “me time.”
4. Be more forgiving, with myself and with students.
5. Read for pleasure.

Do you have any teaching- or math-related resolutions this year? We’d love to hear them!



*Amanda Culver has been a French and mathematics secondary teacher within the province of Saskatchewan for four years. She aims to make her classroom a safe and supportive space to be and to learn mathematics. Amanda’s closet is full of math t-shirts, and she got a “pi” tattoo on Ultimate Pi Day. Needless to say, she loves math!*

Do you have a lesson, reflection, review, or other thoughts or ideas that you’d like to share with the Saskatchewan mathematics education community – and beyond? Consider contributing to *The Variable*! See page 42 for more information.



## Spotlight on the Profession

### In conversation with Jennifer Brokofsky

*In this monthly column, we speak with a notable member of the mathematics education community about their work and their perspectives on the teaching and learning of mathematics. This month, we had the pleasure of speaking with Jennifer Brokofsky.*



*Jennifer Brokofsky is the K-12 Coordinator of Mathematics for Saskatoon Public Schools. She is passionate about mathematics education, and believes in empowering students and teachers to feel ownership of, and become deeply engaged in their own learning. Her Masters work in Educational Technology and Design strongly influences her practice and her belief in the importance of technology as a tool to enhance and extend learning opportunities for all.*



*Thank you for taking the time to have this conversation! To start off, could you tell our readers a bit about the work that you do as Coordinator of Mathematics for the Saskatoon Public School Division?*

As the Coordinator of Mathematics, my job is to support and advocate for mathematics within Saskatoon Public Schools. My work provides me with opportunities to be a researcher, learner, leader, designer, and collaborator. Some days, I facilitate professional learning opportunities for teachers or work in classrooms with students. Other days, I collaborate with school-based administrators and teachers on strategic plans for mathematics. Every day, I work closely with a fantastic Staff Development Team who have a wealth of expertise to share around mathematics, literacy, technology, First Nations and Métis education, English language learners, student supports, and leadership. I love that every day provides me with an opportunity to learn and to serve the many mathematicians within Saskatoon Public Schools.

One of my favorite parts of the job is working closely with the teachers who are a part of our Mathematics Leadership Communities. These communities of educators from Kindergarten to Grade 9 receive ongoing professional development in our division. Initially, the professional learning focused on strategies for instruction, assessment, and mathematics content. Over time, however, the communities have evolved to take on the roles of advisor, contributor, and co-leader in mathematics within our division. When I look back at the work we have collectively generated and shared with our division this past year around computational fluency, mathematics in Kindergarten, and mindset, I know that it would not be as widely received or relevant without the feedback, ideas, and leadership of these amazing teachers.

*As you completed your Master of Education at the University of Saskatchewan in Educational Technology and Design, you must be intimately acquainted with the advantages, challenges, and complexities of teaching and learning with (digital) technology.*

*Ever since the introduction of pocket calculators, there has been pushback against the use of technology in the math classroom. What is, in your view, the role of technology in the teaching and learning of mathematics? And although they seem to multiply every week, what are some of the online tools and resources that you would recommend?*

Technology in mathematics is often seen as either a calculator or an online game. The calculator can be a great substitute for paper and pencil calculations, especially when the purpose of the task is not calculating the correct answer but using the computation to solve a problem. Online games can be a great substitute for traditional concrete games and can provide the opportunity for students to play in an environment that is customized to meet

“Technology in mathematics is often seen as either a calculator or an online game. This view limits the potential of technology within our classrooms.”

their individual needs. In both cases, the technology is enhancing the mathematics through substitution and augmentation. However, this view limits the potential of technology within our classrooms.

Ruben Puentedura’s [SAMR Model for Technology Integration](#) encourages educators to think about using technology for purposes beyond substitution and augmentation. As mathematics teachers, we need to think about integrating technology in ways that not only

enhance, but actually *transform* mathematics learning. Transformation has two steps in the SAMR model: Modification (significantly redesign a task) and Redefinition (create new tasks that were previously inconceivable).

Within our division, we are exploring a few different types of technology that have the potential to transform mathematics learning. For instance, our secondary teachers have engaged in several professional learning opportunities around use of [Desmos](#), a free online graphing calculator and Activity Builder. Our elementary teachers are beginning to develop ways to [connect robotics programing and mathematical concepts](#), a project showing tremendous promise. We also continue to promote the use of video creation as a way for students and teachers to communicate their mathematical thinking to others (see the following blog post for an example: <https://jenniferbrokofsky.wordpress.com/category/three-act-math-videos/>). In these ways, the technology is providing unique opportunities for students to explore and engage in mathematics that were not possible in the past. As we continue to develop the use of these types of technology in our classroom, we can harness their potential to redefine learn experiences for our students.

*Outside of the classroom, you use technology, including [Twitter](#) and your blog, [jenniferbrokofsky.wordpress.com](http://jenniferbrokofsky.wordpress.com), to reflect on teaching and learning, to share ideas and resources, and to communicate with colleagues around the world. The online math teacher community is certainly very strong and growing. How has social media contributed to your own professional growth? In your view, why are math teachers in particular so eager to connect and share online?*

Social media has been a powerful tool for my own professional learning. Every time I connect with educators from around the world, I have an opportunity to expand my thinking, gather new ideas, and discover new research-based practices. Learning today is no longer limited by your geographical area; through social media, the world has gotten smaller, but the opportunities to learn from others have increased exponentially. For me,

technology has been the biggest “game changer” to my professional learning. Not only does social media provide me with a new way to access information but it also provides me with opportunities for ongoing conversations with educators from around the world.

Over the past five years, I have seen math teachers from around the world become very active on social media. I would like to attribute this to teachers modeling the very same qualities of mathematicians that they hope to foster in their students—after all, within our classrooms we impress upon students that mathematicians collaborate with others, clearly communicate their thinking, share ideas, and become members of a community. Online, I see mathematics teachers doing these very same things. Twitter hashtags such as [#MTBoS](#) and [#mathchat](#) provide opportunities for teachers to share, collaborate, and communicate with other teachers in a community that supports and enhances mathematics instruction.

“I have seen math teachers from around the world become very active on social media. I would like to attribute this to teachers modeling the same qualities of mathematicians that they hope to foster in their students.”

*Will (and should) blogs, Twitter, online conferences, and other social media ever replace in-person professional development?*

No. Whereas I see the huge potential of all forms of digital learning, I also still see the benefit of learning side-by-side with colleagues in face-to-face situations. It is not about one platform for learning replacing the other—it is more about acknowledging the value that each can have in professional learning and finding ways for the two to work together. It is also about educators identifying the best fit, moment-to-moment, for their own professional learning and then having the opportunity to tap into the platform that works for them. Ultimately, I think that face-to-face learning in partnership with online learning has the greatest potential to impact mathematics education.

*On your blog, in addition to sharing strategies to support children’s learning of mathematics in the classroom, you have also discussed ways to support their learning and appreciation for mathematics at home. What are some key tips that you share with parents who are looking to do so?*

This question is complex, as so much of it depends on the child and what their interests and needs are. There simply is no one-size-fits-all response. However, I do often tend to make three recommendations to parents:

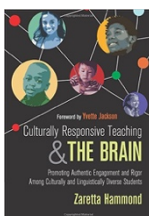
1. **Play**—Find games that you can play with your child that invite them to use math. Common games involve cards and dice that require computation, but be on the lookout for opportunities to use math in other ways, too. Games that require logical reasoning and problem solving are also mathematical, and can help your child see math as more than just computation. (See my article “Building on Mathematical Thinking Through Play” in the July 2016 edition of [The Variable](#) for a few of my favorites.)
2. **Talk**—Find ways to talk about mathematics as it exists in your world. Look for examples of math in grocery stores, as you are driving in the car, on television, and in your kitchen. If you find yourself using or seeing math, share that with your child and

start a conversation. These conversations can help them see that math is alive in our world and is useful.

3. **Value**—If you value mathematics, so will your child. Share with your child that math is a valuable and important subject for them to learn *and* that you will support them along their learning journey.

*In wrapping up this interview, perhaps you can give our readers some homework. As a coordinator of mathematics and a longtime classroom teacher, you must be familiar with the (overwhelming number of!) resources that are available for mathematics teachers at both the elementary and secondary level. Could you share a few of your favorite resources with our readers who are looking to grow in their practice? What's currently on your professional reading list?*

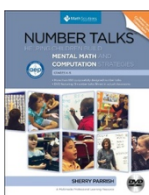
I am constantly reading and exploring ideas to support mathematics instruction. As a lifelong learner, it is a passion of mine to embrace new opportunities to learn from some of the leading thinkers in this area. My current favorite resources include the following:



[Culturally Responsive Teaching and the Brain: Promoting Authentic Engagement and Rigor Among Culturally and Linguistically Diverse Students](#)

by Zaretta Hammond

This book is probably the single most impactful book I have read in a long time. Hammond skillfully connects brain-based research with cultural understanding and ideas for engaging our students in culturally responsive ways. Although it is not specifically focused on mathematics, the opportunities to connect the research and ideas to mathematics are plentiful. I would recommend this book to *all* teachers.



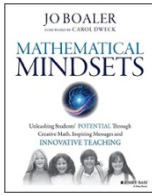
[Number Talks Common Core Edition, Grades K-5: Helping Children Build Mental Math and Computation Strategies](#)

by Sherry Parrish

I strongly believe in giving students a voice in our mathematics classrooms.

As members of our community of mathematicians, their thinking and ideas need to be shared with others in ways that can generate conversation. The ideas in this book provide starting places for such conversations for students in Grades K to 8. I am also looking forward to the new book focusing on fractions and decimals, which should be available shortly.





Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching

by Jo Boaler

Jo Boaler is my math hero! Her ongoing work on mathematical mindsets is inspirational. As educators, we need to accept the challenge of not only teaching our students the curriculum, but also to believe in themselves as mathematicians. All students can do mathematics—the challenge is to help them believe that they can, and then to support them in their learning. Like Zaretta Hammond's book, I would recommend this book to *all* teachers.

*Thank you, Jennifer, for taking the time to share your expertise and perspectives with our readers. We look forward to continuing the discussion in the future.*

*Ilona Vashchyshyn*



# Attending to Precision with Secret Messages<sup>1</sup>

Courtney Starling and Ian Whitacre

**M**athematics is a language that is characterized by words and symbols that have precise definitions (Devlin, 1998). Many opportunities exist for miscommunication in mathematics if the words and symbols are interpreted incorrectly or used in imprecise ways. In fact, we find that imprecision is a common source of mathematical disagreements and misunderstandings between students, as well as between students and teacher.

“Imprecision is a common source of mathematical disagreements and misunderstandings between students, as well as between students and teacher.”

To help students learn to communicate more precisely, we devised an activity called a Communication Game that involved relating trips along a number line to arithmetic equations. A key feature of this game is that it places students in situations that require precise communication with their classmates. Students find out for themselves how successfully they communicated, and they give feedback to their peers to help them communicate more precisely in the future. Communication Games, more broadly, can be incorporated in different mathematical domains.

## A Classroom Example

Our example of a Communication Game had its origins in an introductory unit on integer arithmetic. We used this unit with fifth graders who had no previous integer instruction. The unit involved reasoning about number line trips and relating such trips to arithmetic equations. A simple example of a number line trip would be to start at 5, go 3 spaces to the right, and end at 8. This trip would typically be expressed by the equation  $5 + 3 = 8$ . At the beginning of the unit, we restricted number line trips to the right side of zero so that we could establish foundational ideas that would be important when students began reasoning about negative integers. Although addition and subtraction of whole numbers were familiar topics for the students, relating such equations to number line trips required careful attention to the details of the correspondence. We found that students were imprecise in relating equations to trips. Therefore, we introduced the Communication Game as a way to improve precision.

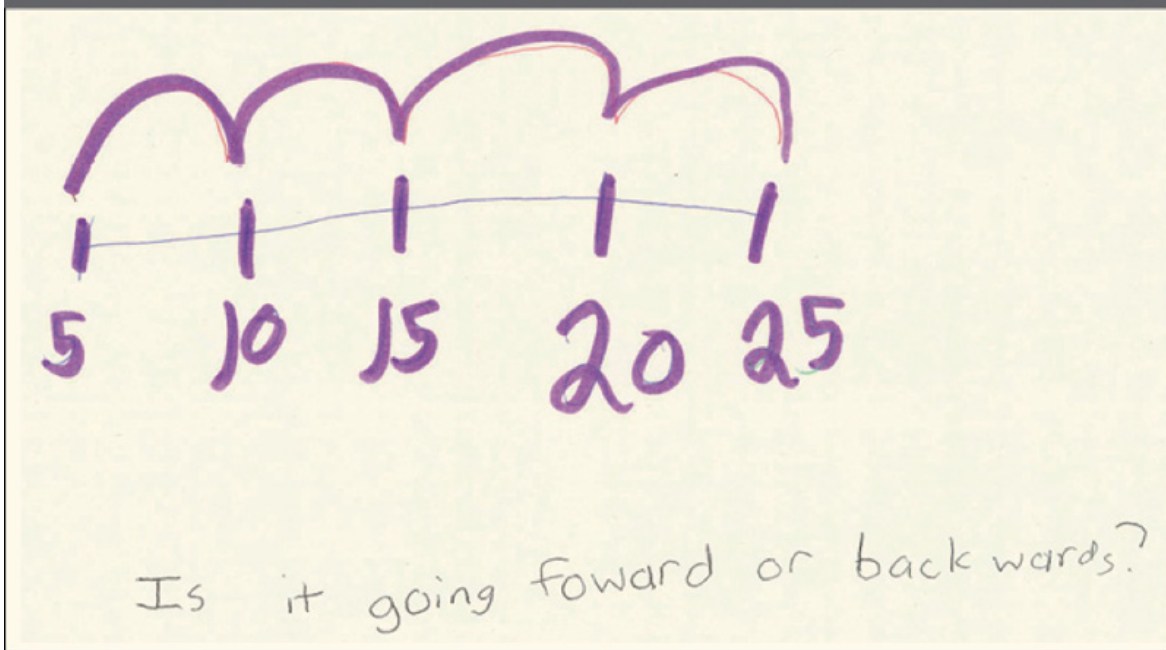
**Step 1.** Students were assigned to groups. Two “secret messages” (equations, in this case) were given to each group. Students had to communicate their secret messages to a partner group by “encoding” them as drawings of number line trips. This task required students to consider such issues as how to illustrate traversing distance on the number line and how the order of the addends would affect the drawings. In the first game, group A was given the messages  $20 + 5 = 25$  and  $5 + 20 = 25$ ; group B’s messages were  $9 + 4 = 13$  and  $13 - 4 = 9$ .

**Step 2.** Students worked in groups to encode their secret messages. Group A decided to distinguish its two equations by drawing the first trip from 20 to 25 and the second from 5 to 25. In this way, the students attended to one important issue in precisely relating

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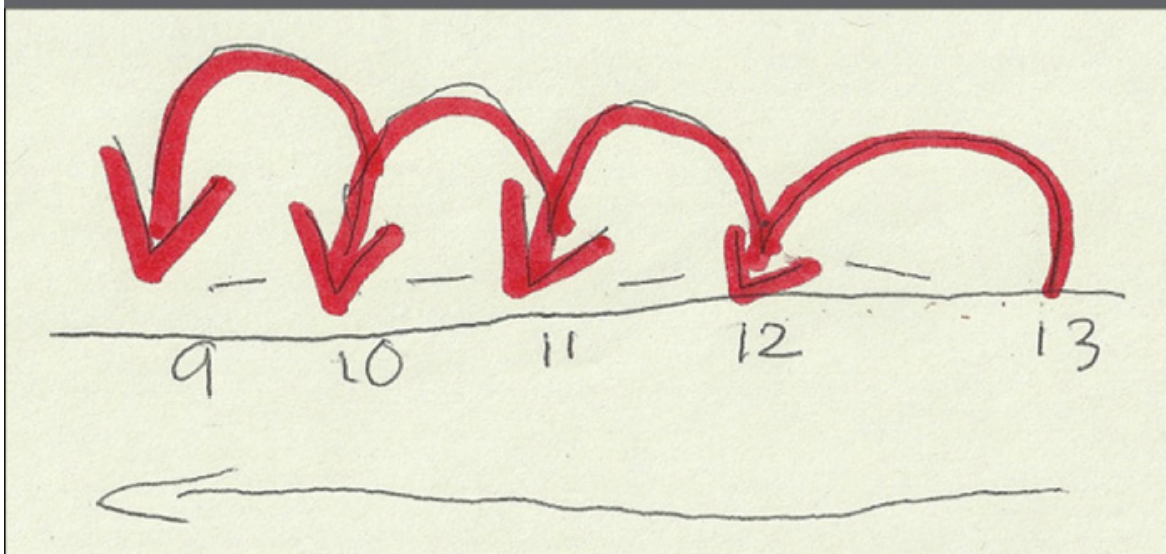
equations to number line trips. On the other hand, they did not attend to other matters of precision, such as indicating the direction of their trips. (See Fig. 1.)

Fig. 1 Group A's drawing for  $5 + 20 = 25$  is on the top; group B's question appears on the bottom.



Group B, by contrast, focused on the issue of direction and decided to use arrows to show whether a trip was going to the right (as in  $9 + 4 = 13$ ) or to the left (as in  $13 - 4 = 9$ ). (See Fig. 2.)

Fig. 2 Group B produced this drawing for  $13 - 4 = 9$ .



**Step 3.** Group A found it easy to interpret group B's messages because the direction of the trips was clear. Group B, on the other hand, was unsure about the intended direction of group A's trips. That group also questioned whether the trips were additive (as in  $5 + 20 = 25$ ) or multiplicative (as in  $5 \times 5 = 25$ ). The groups did their best to decode the messages that they had received. They also prepared feedback to improve precision in future communications.

**Step 4.** The groups then met and held a conference to share their interpretations and reveal their secret messages. Group A had interpreted group B's messages as intended, whereas group B was unsure about the intended meanings of group A's drawings.

**Step 5.** The teacher helped to facilitate the conference by asking group B, "Is there anything that you would suggest changing or adding to their drawing that would make it more clear?" Student Kiyana suggested, "Adding arrows going this way or that way." She further explained, "This way to add [pointing to the right] and this way to subtract [pointing to the left] 'cause you go lower or higher." Other students agreed with Kiyana's suggestion.

In subsequent activities, it became the norm for students in the class to use arrows to indicate the direction of number line trips. As students played these games, we saw improvements in their attention to precision.

The episode above highlighted improvement in one aspect of precision: indicating the direction of a number line trip. Other aspects were addressed in interactions and games that occurred later. For example, trips related to equations like  $-5 + 10 = ?$  were consistent with Kiyana's description of addition moving to the right. However, when students began exploring trips such as  $10 + -5 = ?$ , they had to reason through the idea that adding a negative number would have the opposite effect as adding a positive number. In fact, this new possibility led to a discussion of the important point that two different equations could be used to represent the same trip (e.g.,  $10 - 5 = 5$  looked the same as  $10 + -5 = 5$ ). In cases of adding or subtracting a negative integer, the class agreed that although there could be more than one correct equation, there were correct and incorrect correspondences between equations and number line trips. Thus, precision did not necessarily imply one unique answer. We used a Communication Game as a catalyst for these discussions. Whereas it was the students' responsibility to communicate effectively, we devised messages that would likely lead to miscommunication and give rise to important discussions.

In addition to the issue of direction, differences were found in how students illustrated how number line distances were traversed. Some students seemed to think in terms of representing the equation itself by showing the starting and ending points and then illustrating the distance traveled as one big jump. Other students used their drawings to represent the strategy that they had used to traverse the distance. For example, the drawing in Figure 1 showed the distance from 5 to 25 being traversed in jumps of 5 units, giving it a repeated-addition structure, as opposed to one big jump from 5 to 25. Thus, it became necessary to clarify whether the drawings were intended to simply match the original equation or to illustrate students' strategies. This Communication Game created opportunities for such matters to be negotiated and discussions to take place.

### **Student Ownership of the Need for Precision**

To appreciate the value of using a Communication Game, consider an alternative approach. Rather than giving students the opportunity to attempt to communicate secret messages to one another, the teacher could instead have emphasized the importance of precision and

instructed students to always make arrows when drawing number line trips. Such an approach may have resulted in students drawing arrows; however, students would only have experienced the need to attend to precision as a rule imposed by the teacher. By

“When our students participated in Communication Games, they gained an appreciation for the need to be precise after receiving practical feedback from other students.”

contrast, when our students participated in Communication Games, they gained an appreciation for the need to be precise after receiving practical feedback from other students when they were not entirely successful in their communication. We believe that they experienced an intellectual need for precise communication (Harel, 2013). When participating in Communication Games, students see the natural consequences of the precision of their encoded messages on their ability to communicate. We found that students were motivated to communicate successfully, so they worked to make their drawings more precise.

### What is a Communication Game?

A Communication Game is an engaging and productive activity that we have used to improve students’ mathematical understandings while encouraging attention to precision. The goal is for each group of students to communicate one or more secret messages effectively to their partner group. We suggest groups of two to four students; however, group sizes may vary depending on the nature of the messages and the intended discussion. A Communication Game creates a need for precision in student-to-student communication, rather than the teacher having to play an evaluative role. The process of facilitating these games involves several of the Mathematics Teaching Practices described in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). Students’ participation in these games also creates opportunities for them to engage in mathematical practices and to develop their mathematical communication skills.

“A Communication Game creates a need for precision in student-to-student communication, rather than the teacher having to play an evaluative role.”

We devised a Communication Game to consist of five steps. We describe these steps with connections to the Mathematics Teaching Practices and the Common Core’s Standards for Mathematical Practice (SMP) (CCSSI, 2010):

1. *A teacher hands out previously devised secret messages to student groups.* A secret message may be presented as a solution, equation, definition, expression, or other representation. Each group receives different secret messages that the teacher deliberately chooses, designing the messages to orient students to reason about particular issues (Mathematics Teaching Practice 2: Implement tasks that promote reasoning and problem solving).

2. *In groups, students encode their secret messages through illustrations or other written descriptions.* A Communication Game involves small groups of students using and connecting two distinct types of representations: the message and its encoded form. Students encode each message into a different form, such as a drawing, graph, or story, which their partner group will attempt to decode. The teacher typically specifies the type of representations to be used for encoding but can also allow for students to choose their type of representation. By focusing students’ attention explicitly on relationships between representations, a Communication Game creates opportunities for students to clarify their understanding of the nuances of those relationships (Mathematics Teaching Practice 3: Use

and connect mathematical representations). A Communication Game puts students' thinking on display in ways that highlight salient features, specifically those aspects of representational relationships about which students might disagree. While students are working to encode a message, the teacher has opportunities to ask questions to clarify how students are thinking (Mathematics Teaching Practice 8: Elicit and use evidence of student thinking).

“A Communication Game puts students' thinking on display in ways that highlight salient features, specifically those aspects of representational relationships about which students might disagree.”

3. *Partner groups exchange and decode messages.* Partner groups trade papers and attempt to decode the messages that they have received. The task of decoding messages should not be trivial or procedural. Like the task of encoding, it should promote mathematical reasoning and problem solving (Mathematics Teaching Practice 2). The task of decoding is also intended to create situations in which students struggle to make important distinctions and grapple with how to interpret other students' representations (Mathematics Teaching Practice 7: Support productive struggle in learning mathematics). This phase provides opportunities for students to share and discuss their interpretations with their partners and to justify their interpretations based on the available evidence (SMP 3: Construct viable arguments and critique the reasoning of others).

4. *Partner groups meet to share their interpretations and find out how successfully they have communicated.* In conferences between partner groups, students share their interpretations of the encoded messages that they received. Then the secret messages are revealed, and students find out how successfully they communicated. It is important for the teacher to model appropriate, respectful ways of interacting during these conferences. In particular, the emphasis is not on right or wrong, but on clarity of communication. The teacher celebrates details of successful communication and orients students to focus on improving communication. Both within groups and in conferences between groups, Communication Games encourage students to attend precisely to meaning to communicate effectively (Mathematics Teaching Practice 4: Facilitate meaningful mathematical discourse).

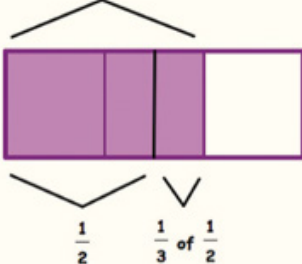
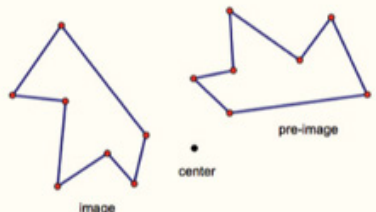
5. *Partner groups offer feedback to facilitate improvement in future communications.* In the conference between partner groups, students have the opportunity to provide feedback to improve future communications. This step is crucial because it provides the opportunity for students to take ownership of precise communication. The more precise that students are in using words and symbols in correct or conventional ways, the more successful their communication becomes. A Communication Game can position students to respond to one another's ideas and provide constructive feedback (Mathematics Teaching Practice 4).

The broad purpose of Communication Games is to encourage attention to precision (SMP 6). This purpose is especially emphasized in steps 2 and 5. In step 2, students make decisions related to precision when they encode their secret messages. In step 5, they give and receive feedback to improve precision.

### **Creativity in Communication Games**

Communication Games give students opportunities to learn and understand the language of mathematics. Ideally, they also allow opportunities for creativity. At first glance, it may appear that precision and creativity would be incompatible; however, we have found the capacity for creativity to be advantageous in Communication Games. In the classroom

episode above, our students drew number line trips, but a variety of representational forms may be used. Story problems are a great example. A wide range of contexts is possible, allowing many different story problems to be written to correspond to a single equation. However, with the help of precision, a story problem may be decoded as corresponding to the intended equation or to an equivalent equation. (See Table 1.) In this way, Communication Games touch on Mathematics Teaching Practice 2 from *Principles to Actions* in that they “allow multiple entry points and varied solution strategies” (NCTM, 2014, p. 10).

Table 1 These examples show how Communication Games can occur in various domains.	
<p>Ratio and Proportion: Students encode proportion problems by contextualizing them as story problems (SMP 2: Reason abstractly and quantitatively). Students write story problems, thus giving students the opportunity to be creative and to personalize their mathematical work while requiring them to attend precisely to proportional relationships between quantities.</p>	<p>Secret message: <math>\frac{3}{5} = \frac{9}{x}</math></p> <p>One possible encoded message:</p> <p>3/5 of the players on a team are right handed. There are 9 right-handed players. How many players are on the team?</p>
<p>The Number System: Students encode expressions or equations involving rational numbers into story problems or drawings. Such activities afford a focus on meaningful connections between real-world situations and operations involving rational numbers. Students can develop a better understanding of operations involving rational numbers by creating their own representations.</p>	<p>Secret message: <math>\frac{2}{3} \div \frac{1}{2} = 1\frac{1}{3}</math></p> <p>One possible encoded message:</p> <div style="text-align: center;">  </div> <p>Our drawing shows that there are</p> <p style="text-align: center;"><math>1\frac{1}{3}</math> copies of <math>\frac{1}{2}</math> in <math>\frac{2}{3}</math>.</p>
<p>Functions: Students encode functions by evaluating them at values of their choosing and encoding them as coordinates of points. Students receiving the encoded message will then have to work backward to find the equation. Relating multiple representations can help students deepen their understanding of functions (Brenner et al. 1997).</p>	<p>Secret message: <math>y = 2x + 9</math></p> <p>One possible encoded message:</p> <p>Our message is the line that passes through the points <math>(-3, 3)</math> and <math>(1, 11)</math>.</p>
<p>Geometry: Students encode transformations into drawings. The secret message may take the form of the description of a transformation, which students encode by drawing a figure and its preimage. This activity requires precision and also affords students the opportunity to see a transformation as both an object (here, a message to be communicated) and a process (rotate the figure 120 degrees) (Sfard 1991).</p>	<p>Secret message: 120 degree rotation</p> <p>One possible encoded message:</p> <div style="text-align: center;">  </div>

## Versatility in Communication Games

Communication Games are versatile enough to be adapted to all domains of mathematics, as well as to other content areas, to promote understanding and precision in multiple topics. We regard Communication Games as a regular classroom activity that can be used to help students relate concepts and representations. (See Table 1 for a few examples.)

## Interpretations and Feedback

Communication Games provide authentic opportunities for students to be precise and creative in sharing their interpretations of mathematical representations and in providing feedback to one another. It is important that teachers pay attention to students' interpretations and ideas to support their learning (Jacobs, Lamb, & Philipp, 2010). We believe that the better a teacher and fellow classmates understand students' thinking, the more conducive the classroom environment will be to student communication and learning (Horn, 2008). The opportunities created by Communication Games allow for students to actively participate in mathematical practices and to experience ownership of the mathematics that they are learning.

“Participation in the games helps both students and teacher better understand one another and can help students learn to speak the language of mathematics.”

We find that students enjoy Communication Games and benefit from them. Participation in the games gives a class access to the details of students' thinking, which helps both students and teacher better understand one another and can help students learn to speak the language of mathematics. As with learning any language, this process is gradual and can be aided by multiple experiences using the language for practical communication. We hope that this article gives readers a new tool that can be used in classrooms. If you use Communication Games with your students, please share your experiences with us.

## Acknowledgment

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## Is Cancel the Right Word?

### How Students Understand “Cancelling” When Reducing Rational Expressions

Daniel Woelders

Throughout their academic journey, students accumulate mathematical ideas, attitudes, strategies, and vocabulary. As we introduce new curriculum, much of our time is spent trying to understand what students know. This semester was no different, as a group of Grade 10's funneled into my classroom and we began a series of mathematical conversations that would continue for the remainder of the course. Amidst these conversations was the notion of “cancelling”. This word was carelessly thrown into explanations, often to describe the process of *getting rid of* a term. For example,  $2x - 2x$  would be described as “*cancelling 2x*”; thus, one might speculate that cancelling results in zero. However, this term was also used when describing  $2x/2$ , where the 2 again gets “cancelled.” This term is so frequently used in algebra that it often creates confusion with students, and this confusion became very evident in our classroom when students were required to work with linear equations. During one particular lesson, students were asked to rearrange the equation  $2y = 4x + 2$  into slope-intercept form ( $y = mx + b$ ). Final answers from students took the form of  $y = 2x + 2$ ,  $y = 4x + 1$ ,  $y = 2x$ , and  $y = 2x + 1$ . When asked how the students came to these conclusions, the term “cancel” came up over and over. It seemed that the way students understood the process of “cancelling” in rational expressions played a big role in the errors that they produced in reducing algebraic expressions. This experience fueled a desire to shed light on student beliefs and provided

“Cancelling’: This word was carelessly thrown into explanations, often to describe the process of *getting rid of* a term.”

an excellent opportunity to investigate how students understand “cancellation” (or “reduction”) of rational expressions.

There were 50 Grade 10 students involved in this investigation, half of which had completed a unit on factoring (a skill that plays a significant role in reducing rational expressions). Given the diversity of perspectives on this topic, it seemed appropriate to gather data from

every student in the form of a survey (see Appendix). This survey addressed not only what students understood about “cancellation” and the rationale for it, it also targeted common misconceptions in reducing rational expressions when working with variables, radicals, and integers. Additionally, the survey was intended to be a self-discovery tool for students, a way to test their methods and verify reductions.

As the surveys were compiled and analyzed, there appeared to emerge a pattern in how students perceived “cancellation,” leading to three general lines of thought. These three general groups include: a) *cancelling when the numerator and the denominator have a common factor*, b) *cancelling when a term in the numerator and denominator have a common factor*, and c) *cancelling when each/all numbers in the numerator and the denominator have a common factor*.

#### I. Cancellation occurs when the numerator and the denominator have a common factor

This method, of course, lends itself well to reducing rational expressions. In fact, it describes the reason some rational expressions are reducible. Ideally, this would be how all of our students understand reducing. Unfortunately, even among those students who have been

exposed to factoring, this method is seldom employed by students, and made up only 20% of those surveyed. Students who used this method were identified as explicitly factoring before the division (“cancellation”) occurred (Fig. 1).

Figure 1: Q4A and Q4E shows students factoring the numerator prior to reducing the rational expression. The explicit written factored form was consistent with those who held this strategy for reducing.

Students who understood reduction in this way often used language such as “factor,” “take out,” or “remove” to explain why the cancellation is allowed. In other words, it is essential that a common factor first be identified before the reduction occurs. Understood in this way, it becomes irrelevant whether the expression included variables or radicals, as long as a common factor could be found. In general, students using this technique demonstrated a stronger vocabulary, were better able to justify why the reduction occurred, and were able to verify that the reduction was correct.

The problem with this strategy, of course, is that it requires that students be able to factor various types of polynomial expressions. Given students’ limited exposure to polynomials besides linear expressions, it is not surprising that only a select number of students were able to factor when given a question outside the context of the factoring unit. So how did students who were unable to factor understand reduction? The second group of students reflects a common notion held by Grade 10 students.

## II. Cancellation occurs when a number in the numerator and the denominator have a common factor

Students who understand cancellation in this way place the emphasis on “a number”; often, this is understood as “a term in the numerator is divisible by the denominator.” Students who reduced expressions in this way were under the impression that only a single term need be divided by the denominator.

Of course, no error would be detected when the numerator had a single term, as was the case with Question 4c) (Q4C) (Fig. 2). This train of thought was likely established early for mathematics students with the reduction of fractions. Students are often encouraged to reduce fractions by considering the greatest common factor in the numerator and denominator and then “cancelling” it out.

Figure 2: Q4C shows the reduction of a monomial by monomial. Students using method II seldom have any problems when there is a monomial numerator

This strategy helps students reduce fractions, but seems to become a stumbling block when binomial expressions are introduced (Fig. 3). Students who understand reductions in this way often look for a single term in the numerator that can be reduced, the same way that a fraction can be reduced. Expressions

like  $\frac{6x+8}{2}$  are seen as  $6x + \frac{8}{2}$  and a simple fractional reduction is employed. These students often explained their reduction using words like “divide into,” “fraction,” and “like term.” One student in particular explained “I believe cancellation is allowed where there are equivalent values of the same multiple of the same term are in both the numerator and the denominator.” The interpretation, simply put, is that cancellation is allowed if you have common factors and they belong to like terms. This explains why the student chose to divide into the 8 instead of the  $6x$  in Q4A (Fig. 3). The student sees the  $6x$  as being an unlike term, and thus not divisible by 2.

Figure 3: Students who look to divide single terms by the denominator often struggle to reduce binomial by monomial expressions. Students frequently choose to reduce the term that is a “like term” with the denominator.

An additional element of these students’ understanding was the close association of “elimination” with the word “cancellation.” These students identified a common factor (such as a 2 in Q4A) and the crossing out resulted in the “elimination” of the denominator. In this particular question, it is difficult to determine whether the student perceived the cancellation to mean that *nothing* remains (eliminated) or that a *one* remains. However, in Q4D, a number of students stated their final expression as  $\sqrt{2}x$  (Fig. 4). Neglecting “+1” suggests that the students see the cancellation as resulting in zero or “eliminating” the term.

Students who understood reduction in this way were unable to explain their rationale for why cancellation was allowed and seldom had any method to verify their reductions. However, almost all students identified the problem with the understanding they held when it came to reducing integer expressions. Students who held this belief were still able to correctly answer Q10 and Q11. These results differed from the previous group of students and the beliefs they held about reducing rational expressions.

Figure 4: Q4D shows that students who reduce single terms often view the “cancellation” as an “elimination” resulting in no term remaining.

### III. Cancellation occurs when each/all of the numbers in the numerator and the denominator have a common factor

Students who view reductions in this way often described the strategy as dividing each number in the numerator by the denominator. How these students drew their lines for cancelled/reduced terms often gave valuable insight about the way they understood the division. For example, Figure 5 shows the

Figure 5: Q4A shows students reducing **both** terms in the numerator by the denominator.

crossing out relating to both terms in the numerator and only the single term in the denominator. This strategy for reducing expressions appears rather harmless and has likely been taught by many teachers, including myself. The problem, however, exists when the numbers in the numerator represent a single term, such as the case in Q4B (Fig. 6). If the students' strategy is to simply divide each denominator into each numerator, then it is likely they will divide both 12 and 6x by 2, as was the case with almost every student who held this belief (Fig. 6). These students also incorrectly reduced Q4F and Q9. Even more surprising is that these students, without thinking twice, employed the strategy in Q10 instead of simplifying the numerator. While this did not cause any problems in Q10, it certainly caused issues with Q11, which was often incorrectly reduced.

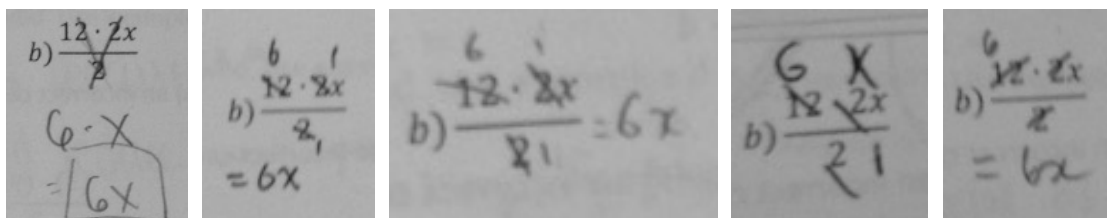


Figure 6: Q4B was incorrectly reduced by almost every student in group II. Students failed to identify the numerator as a single term..

Students in this group often explained that cancelling was allowed when “the numbers on top can be divided by the number on the bottom,” or that you can simply “divide everything on top by the bottom” instead of each “term” in the numerator. These students were frequently unable to recognize a multiplication as a single term, meaning that even if they factor, they may still not be able to correctly reduce. For example, an expression like  $\frac{4(2x+1)}{4}$  may be interpreted as  $\frac{4}{4}$  and  $\frac{(2x+1)}{4}$ , or perhaps even  $\frac{2x}{4}$  and  $\frac{1}{4}$ . This belief about reducing polynomial expressions is the mostly frequently held belief among all of the Grade 10 students surveyed. This shouldn't be surprising, considering that students are stuck trying to reduce a binomial expression without a strong foundation in factoring.

## Discussion

Implications of these three understandings of reduction extend beyond the Grade 10 curriculum and will continually resurface as students repeat common errors year after year. At the root of these beliefs is the students' first experience with reducing fractions. Those who employ the strategy of looking for the greatest common factor to “cancel,” instead of factoring it out, will likely develop the same beliefs as those of the students in group II. This, of course, highlights the importance of early factoring with younger students. This should be done both in the context of a fraction reduction and polynomial division, even if it is simply done with integers. For example, reducing  $\frac{135}{3}$  could be seen as  $\frac{120+15}{3}$ , factored as  $\frac{3(40+5)}{3}$ , and reduced to (40 + 5).

“While shortcuts can offer a temporary method of simplifying expressions, they may create many issues when students are called to extend their understanding in the study of more complex mathematics.”

Perhaps underlying these misconceptions is the problem with the term “cancellation.” By giving the process a name based on *what it looks like* rather than *what it is* causes students to frequently “eliminate” terms rather than

“reduce.” While shortcuts given by teachers often offer a temporary method of simplifying expressions, they may create many issues when students are called to extend their understanding in the study of more complex mathematics.

Investigating student understandings of seemingly simple ideas, such as the one in this study, provides valuable insight for teachers who are looking to interpret student beliefs and redirect them accordingly. However, even more valuable than information regarding *redirection* is the information provided about *misdirection*. That is, the way we introduce students to ideas like “cancellation” often misdirects them and prompts them to extend these incorrect notions to other areas of mathematics. If nothing else, perhaps we will think twice when we tell students to simply “cancel” terms.

## Appendix – Survey Given to Students

### “Cancelling” to Reduce Rational Expressions

Name: \_\_\_\_\_

1. What do you understand about the word, “cancelling” or “cancellation” in mathematics?
2. When is cancelling allowed?
3. Give a situation when cancelling is allowed and show the cancellation/simplification of the expression.
4. If possible, simplify the following expressions using “cancellation”. Put a line through any value that is being cancelled/reduced.

$$a) \frac{6x + 8}{2}$$

$$b) \frac{12 \cdot 2x}{2}$$

$$c) \frac{36x}{3}$$

$$d) \frac{\sqrt{2}x + \sqrt{3}}{\sqrt{3}}$$

$$e) \frac{xy + 2x}{x}$$

$$f) \frac{a \cdot b \cdot 4 \cdot a}{a}$$

5. How would you prove that the cancellations above were allowed (that you cancelled correctly)?
6. Which of the following show(s) an incorrect cancellation of the expression:  $\frac{12x+4}{4}$ . Circle the letter(s) that are **incorrect**.

$$a) \frac{\overset{3}{\cancel{12}}x+4}{\cancel{1} \cancel{4}} \quad b) \frac{\overset{3}{\cancel{12}}x+\cancel{4}}{\cancel{4}} \quad c) \frac{12x+\cancel{4}}{\cancel{4}} \quad d) \frac{12x+\cancel{4}}{\cancel{1} \cancel{4}} \quad e) \frac{\overset{4}{\cancel{12}}(x+1)}{\cancel{4}}$$

$$= 3x+4 \quad = 3x+1 \quad = 12x \quad = 12x+1 \quad = 3x+1$$

7. Which of the following show(s) an incorrect cancellation of the expression:  $\frac{2x+\sqrt{2}}{\sqrt{2}}$ . Circle the letter(s) that are **incorrect**.

$$a) \frac{2x+\overset{\sqrt{2}}{\cancel{\sqrt{2}}}}{\cancel{\sqrt{2}} \cancel{1}} \quad b) \frac{\cancel{2}x+\overset{\sqrt{2}}{\cancel{\sqrt{2}}}}{\cancel{\sqrt{2}}} \quad c) \frac{2x+\overset{\sqrt{2}}{\cancel{\sqrt{2}}}}{\cancel{\sqrt{2}}} \quad d) \frac{\overset{\sqrt{2}}{\cancel{\sqrt{2}}}(x+1)}{\cancel{\sqrt{2}}}$$

$$= 2x+1 \quad = x+1 \quad = 2x \quad = x+1$$

8. Which of the following show(s) an incorrect cancellation of the expression:  $\frac{\sqrt{7}x+\sqrt{28}}{\sqrt{7}}$ . Circle the letter(s) that are **incorrect**.

$$a) \frac{\overset{\sqrt{7}}{\cancel{\sqrt{7}}}x+\sqrt{28}}{\cancel{\sqrt{7}} \cancel{1}} \quad b) \frac{\overset{\sqrt{7}}{\cancel{\sqrt{7}}}x+\overset{\sqrt{28}}{\cancel{\sqrt{28}}}}{\cancel{\sqrt{7}} \cancel{1}} \quad c) \frac{\overset{\sqrt{7}}{\cancel{\sqrt{7}}}x+\overset{\sqrt{28}}{\cancel{\sqrt{28}}}}{\cancel{\sqrt{7}} \cancel{1}} \quad d) \frac{\overset{\sqrt{7}}{\cancel{\sqrt{7}}}x+\overset{\sqrt{28}}{\cancel{\sqrt{28}}}}{\cancel{\sqrt{7}} \cancel{1}} \quad e) \frac{\overset{\sqrt{7}}{\cancel{\sqrt{7}}}(x+\sqrt{4})}{\cancel{\sqrt{7}}}$$

$$= x + \sqrt{28} \quad = \sqrt{7}x + 4 \quad = x + \sqrt{4} \quad = \sqrt{7}x + \sqrt{4} \quad = x + \sqrt{4}$$

9. Which of the following show(s) an incorrect cancellation of the expression. Circle the letter(s) that are **incorrect**.

$$\frac{(3) \cdot (9x)}{3}$$

a)  $\frac{\overset{3}{\cancel{3}} \cdot (9x)}{\cancel{x}^1} = (3) \cdot (3x)$       b)  $\frac{\overset{1}{\cancel{3}} \cdot \overset{3}{\cancel{9x}}}{\cancel{x}^1} = (1) \cdot (3x)$       c)  $\frac{\overset{2}{\cancel{3}} \cdot (9x)}{\cancel{x}^1} = (9x)$       d)  $\frac{\overset{3}{\cancel{3}} \cdot (9x)}{\cancel{x}^1} = (3x)$

10. Which of the following show(s) an incorrect cancellations of the expression. Circle the letter(s) that are **incorrect**.

$$\frac{3+9}{3}$$

a)  $\frac{\overset{1}{\cancel{3+9}}}{\cancel{x}^1} = 10$       b)  $\frac{\overset{3+9}{\cancel{3}}}{\cancel{x}^1} = 6$       c)  $\frac{\overset{1}{\cancel{3+9}} \cdot \overset{3}{\cancel{3}}}{\cancel{x}^1} = 4$       d)  $\frac{\overset{3+9}{\cancel{x}}}{\cancel{x}} = 9$       e)  $\frac{\overset{2(1+3)}{\cancel{3+9}}}{\cancel{x}} = 4$

11. Which of the following show(s) an incorrect cancellations of the expression. Circle the letter(s) that are **incorrect**.

$$\frac{4 \cdot 2}{2}$$

a)  $\frac{\overset{2}{\cancel{4}} \cdot \overset{2}{\cancel{2}}}{\cancel{x}^1} = 4$       b)  $\frac{\overset{4}{\cancel{4}} \cdot \overset{2}{\cancel{2}}}{\cancel{x}^1} = 4$       c)  $\frac{\overset{2}{\cancel{4}} \cdot \overset{2}{\cancel{2}}}{\cancel{x}^1} = 2$       d)  $\frac{\overset{4}{\cancel{4}} \cdot \overset{2}{\cancel{2}}}{\cancel{x}} = 4$

12. Describe **when** you think cancellation is allowed?  
 13. Explain **why** you think cancellation is allowed. Use examples if necessary.



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## What's "New Math"? <sup>2</sup>

Cindy Smith

Since the math curriculum was renewed in Saskatchewan, there has been a great deal of press about the "New Math," whatever that is. Parents are frustrated, while some professors and other critics are saying that our kids lack skills. The Ministry even [launched a review](#) in 2012 (Editorial, 2012), inviting input from parents and teachers. There was a parent group that formed a "[WISE](#)" chapter (Western Initiative for Strengthening Education in Mathematics) in an effort to "take matters into their own hands." Then, we heard that [Manitoba "abandoned" the new curriculum](#) in favour of going back to the "basics" (McDonald, 2013). Is this a step ahead? Who ever said that we would disregard the basics, anyway?

"Who ever said that we would disregard the basics, anyway?"

Our curriculum is based on the WNCP (Western Northern Canadian Protocol), which is based on the [NCTM](#) (National Council of Teachers of Mathematics) *Standards*. The important point here is that while the NCTM advocates for research-based math instruction, which involves a more constructivist approach, nowhere do they say that students shouldn't practice basic math facts. True, it does warn against the memorization of algorithms and procedures in the absence of context, and few could argue that teaching math as a set of memorized steps without fostering understanding is good practice. But once we have taught the concepts, given students the tools (e.g., manipulatives, diagrams), and engaged them in constructing meaning for themselves (e.g., through dialogue, writing, explaining, peer teaching), *then*, of course, we practice. A document that was widely distributed during curriculum change is *Reflections on Research in School Mathematics: Building Capacity in Teaching a Learning* by Florence Glanfield (2007). This guide contains a section on "procedural fluency," citing the National Research Council's advocacy for ensuring rigor and fluency:

Procedural fluency includes knowing the steps and rules for calculating and computing, knowing when and how to use them, and performing them with accuracy, efficiency, and flexibility. Without sufficient procedural fluency, students have trouble deepening their understanding of mathematical ideas or solving mathematical problems. (p. 20)

Glanfield also quotes Sfard (2003, p. 365): "The possibility of learning mathematics without some mastery of basic procedures may be compared to the claim of building a brick house without bricks." While advocating for flexible strategies, the NCTM also recognizes that students need opportunities to practice skills and procedures. Perhaps the greater emphasis on teaching for understanding is "new," but here is how the NCTM advocates for that emphasis: "One of the most robust findings of research is that conceptual understanding is an important component of proficiency, *along with factual knowledge and procedural facility*" (Bransford, Brown, & Cocking, 1999, as cited in NCTM, 2000, p. 20; emphasis mine). Maybe I'm missing something, but I don't read "throw out the baby with the bathwater" here. Our

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<sup>2</sup> A prior version of this article was published on September 16, 2013 on Cindy's blog at <http://blogs.gssd.ca/csmith/2013/09/16/whats-new-math-saskatchewan-curriculum/>. Reprinted with permission.



own Saskatchewan Curriculum document says that math students should be learning skills and that “basic skills are related to Curricular Goals” (Saskatchewan Curriculum, Mathematics 8, p. 2). It advocates for “appropriate” use of technology, like the NCTM Standards, which state that “basic facts, processes, and translations in and among common and decimal fractions, percents, proportions, and integers are important to students’ understanding of computation” (NCTM, 1989, p. 95), does not advocate the use of a calculator until students are performing operations on numbers with *several digits*, and that “students should possess adequate mental arithmetic skills so that they are not dependent on calculators to do simple computations and are able to detect unreasonable answers when

“Procedure and computation has dominated instruction in the past. But this does not produce lasting learning, does not create critical thinkers, and does not foster an appreciation for math as useful in life contexts.”

using calculators” (NCTM, 1989, p. 96). The renewed curriculum warns against meaningless drill and algorithms without understanding of concepts or applications, but it does *not* say that students shouldn’t practice.

Having students perform procedures and algorithms is convenient—not only to teach, but also to assess. Maybe this is why procedure and computation has dominated instruction in the past. But this kind of math instruction does not produce lasting learning or the ability to apply mathematical ideas, does not create global citizens or critical thinkers, and does not foster an appreciation for

mathematics as historically rich, necessary for understanding the world around us, or useful in life contexts. The average adult who studied mathematics under the older, more traditional system, was successful at math if they happened to have innate mathematical ability or were good at memorizing. Many of these adults even now cannot explain *why* certain things work the way they do. For instance, you have a greater probability of finding an eighth grader who could explain *why* eight divided by one half is 16 than any adult over 30. (I was once quoted a poem that was taught in school long ago: “It is not for us to question *why*—just invert and multiply.”) Many adults found math very difficult or inaccessible.

So where is the disconnect? Why has the media (and subsequently some parents) decided that we don’t teach students basic facts anymore? Perhaps the time constraints we work under are partially to blame. Teaching through inquiry and for concept attainment takes time, especially in our differentiated classrooms. Is there time for adequate practice? Or have we misheard the message, focusing so much on constructing meaning through multiple representations, dialogue, and manipulatives, that we have not infused enough practice? Have we felt that it’s wrong to provide practice? The NCTM says that teachers have “pedagogical expertise” and use their professional judgement to deliver appropriate instruction. Perhaps curriculum writers didn’t emphasize skill-building because, traditionally, teachers have been good at that. That’s the easy part (which is perhaps why practice has dominated math instruction in the past).

However, the fact is that we need “Balanced Math”: *both* understanding *and* practice. Yes, we need to teach for deeper understanding, but not at the expense of having students develop automaticity in basic facts and computation. If we do, they truly will be at a deficit as they move up through the grades. Students who lack procedural fluency cannot learn more complex mathematical concepts, because all of their cognitive functioning is tied up in basic computations and is unavailable to grasp and construct new understandings. Conversely, our instruction must not be dominated by “drill,” and must not ever introduce

procedures without first revealing the underlying concepts and giving students the opportunity to connect and construct meaning. In finding the balance between practice and understanding, we need to exercise our professional discretion, knowing that we have “pedagogical expertise.”

We also need to speak up for our profession and our subject. Mathematics is a broad discipline, involving not only numeracy but spatial reasoning, statistical understanding, probability, logic, patterns, predictions, puzzles, technology, and creative thought (to name a few!). Obviously, if we only focus on computation, we are not creating mathematically literate people. Computational fluency alone will not equip our students for life in the 21st century. We need to defend our practices by communicating that we are professionals and have valuable understanding about what is quality instruction. All educational initiatives must be implemented through the filter of professional expertise in the context of the realities of our students, classrooms, and schools, including assessment, Response to Intervention, differentiation, and instruction.

Some more personal thoughts: Practice does not equal “worksheets.” We must not cave in to time-saving through mind-numbing. It is practices like these that fed some people’s hatred of mathematics. Like all things, some practice is good, too much is not. Especially avoid timing students (like “mad minutes”). Though some students enjoy the competition, for others, this adds fuel to a burning fire of mathematics anxiety. We now have at our disposal many creative ways to practice, like using small-group instruction, technology, games, and collaborative learning. Call your math coach or digital learning coach for help with technology-based learning. However, use these with caution, too: they can help, but effect size for web-based learning as an instructional strategy, in general, is not high. As the saying goes, “All things in moderation.”

“We must not cave in to time-saving through mind-numbing. It is practices like these that fed some people’s hatred of mathematics.”

Helping students develop mathematical fluency can be very empowering to them. They may feel more successful and find that the speed with which they can complete math tasks improves. Sometimes, it is the struggling math learners who benefit most through developing basic skills. Without basic skills, it is difficult for them to move ahead. The National Center for Education Evaluation and Regional Assistance (NCEERA), lists spending time on fluency and basic facts as one of the eight recommendations for helping struggling learners (NCEERA, 2009).

Our curriculum warns against the hazards of using rote practice as homework. The fear is that we will spend so much practice it will become “drill and kill” and give students a negative attitude towards math. That being said, if students are open to working on developing basic skills at home, a reasonable amount of practice would be very useful. Often, this should be a “comfortable” kind of home assignment, which helps parents feel like they can contribute to their child’s learning of math. However (again): Use with caution! A *reasonable* amount is key. Let the parent and student have input into how much is too much!

Also, I’m going to say the dreaded “m” word: Memorization. Truthfully, there will always be some memorization in math... but *only* after understanding has been established. Spaced practice is best, bringing concepts up again and again, reviewing, revisiting, connecting. Formative assessments can be very beneficial here and have been proven to be an effective

instructional strategy.

There is really not much that is “new” in the “New Math.” We are advocating for richer teaching, concept development, and authentic learning, that’s all. We still want the kids who work at Canadian Tire after school to be able to figure out our change! We still want to produce mathematical competence. One word summarizes it all:

### Balance.

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Cindy Smith taught high school math at Yorkton Regional High School for 11 years, and in 2008, she was honoured with a Master Teacher award from the Saskatchewan Mathematics Teachers' Society. After completing a Stirling MacDowell research project on Technology-Supported Inquiry in Senior Math and then completing a Master's degree, Cindy took up her current position as Senior Mathematics Coach for Good Spirit School Division. Her focus is on innovative teaching, inquiry, technology, and classroom culture.

## Coding in the Math Classroom

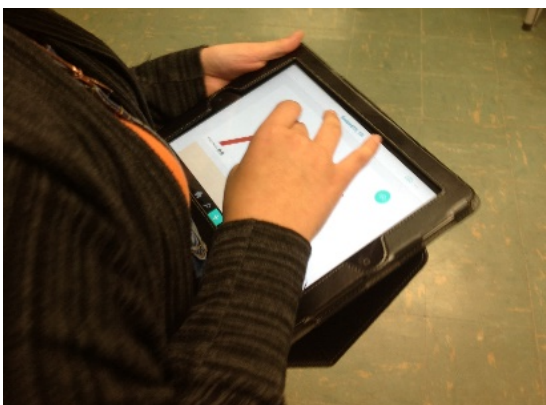
Dean Vendramin

Coding has been an important part of the technology revolution since its inception, but now, more than ever, it is becoming an important skill for *all* students to be familiar with. I have come across many an article arguing that coding is a vital 21st century skill (head to <https://www.kodable.com/infographic> for a great infographic presenting some good reasons to teach kids to code); this research, as well as my own impressions of future employable skills, have convinced me that I need to look at ways of incorporating coding into my classroom.

Coding itself has been around for a while. I still remember getting a good old Commodore 64 (I'm probably dating myself here) and figuring out the code to display a stick man, then using the 'GOTO' command to make that poor stick man do an infinite amount of jumping jacks. This wasn't much, but even the skills I learned way back in the day have served me well. One area is computation, as the thought processes involved in programming are similar to the thought processes involved in mathematical computation (after all, at the most basic level, what a computer does is take commands and execute the math behind them, only at tremendous speeds).

Although Computer Science courses are offered at many schools today (at least in Saskatchewan), the enrollments are not necessarily high, and female students are typically underrepresented. However, many organizations such as [Code.org](http://Code.org) and [Ladies Learning Code](http://Ladies Learning Code) are now trying to change these trends, working to have coding become more widely accessible and more inclusive. And there is little time to lose—the workforce is changing and will continue to change, and it is easy to spot the many trends

“The integration of code into math class makes sense in many ways, as there are many parallels between coding and mathematics.”



that point towards traditional jobs becoming obsolete. Much of the evidence presented in the research that I have explored points to coding, and the skill sets that come with coding, as being a very valuable asset for workers in the future (and not just for making stick men do jumping jacks).

The integration of code into math class makes sense in many ways, as there are many parallels between coding and mathematics. For instance, to succeed in either area, it is important to understand the governing rules, and to be able to sequence, model, and problem solve; moreover, the joy of solving problems and creating new learnings and understandings are available for students in both mathematics and coding, and both offer many opportunities for students to be creative. Additionally, both require an 'I can do this' mindset that allows students to overcome the fear that becomes an obstacle in their minds, as well as grit and rigor to accomplish what they set out for and the ability to see mistakes as an opportunity to grow and learn from.

Coding does have one advantage over mathematics, however, as seeing something that you have constructed work (like a jumping stick man) can be a little more motivating and gratifying than factoring a trinomial. That being said, students should be introduced to the beauty of *both* of these disciplines and acquire an appreciation of what both have to offer in enhancing their understanding of the world around them; they should also understand how skills in both coding and mathematics will serve them well in the future.

So last year, I decided that I needed to start somewhere with regards to bringing coding into my classroom. The first opportunity that I decided to pursue was to participate in the Hour of Code, which is a worldwide promotion of coding sponsored by Code.org. The movement's goal is to get as many students around the world coding for at least one hour over the course of a week. I jumped on board and had students in my class hop on an app called Hopscotch (available for free on the Apple App Store) and follow the built-in tutorials; the students were able to make games like Crossy Roads, Geometry Dash, and Flappy Bird.



The following semester, I took it to another level and included coding as an option for Genius Hour in my Grade 9 math class (Genius Hour is time given to students to pursue personal projects that they are interested in; see—[www.geniushour.com](http://www.geniushour.com) for more information). With both projects, I observed positive results—students were engaged, experimenting, problem solving, and having fun. I particularly enjoyed the Genius Hour projects, as the students who chose coding had more time to explore and work through their own ideas than the students who only participated in the Hour of Code. My next goal is to make coding a part of the daily bell work for the Grade 9 math classes that I teach and find more curricular ties (I will have access to my own set of laptops, as I am part

of a connected educator program offered by my division). I'm looking forward to new adventures, learnings, and outcomes that will come with this.

“In apps like Hopscotch, it is about understanding the process, identifying patterns, and using the tools you have to create... a lot like what you do in math class.”

Do you need a computer science background to get involved in coding? The answer is *no*! The Hopscotch App that I used on the available iPads was all ‘block-based’: this means that you don’t need to know a coding language to write programs in the app—you just drag the blocks with words on them (corresponding to particular commands) and, as one

student suggested, “it is kinda like making a sentence.” In other words, the commands are all set up, and all one has to do is put the ‘sentence’ together and then play the code to see if your ‘sentence’ has the proper sequence, conditions, and logic to make the program work the way that you intended it to work. If it doesn’t, you need to go back to your code and try to figure out what rule or syntax you need to change to get the desired effect. In apps like Hopscotch, it is about understanding the process, identifying patterns, and using the tools you have to create... a lot like what you do in math class.

There are other math concepts embedded into coding, in an app like Hopscotch. Through coding, students learn that computers speak numbers and that programmers have to tell them what to do, but they also learn the process of iteration, that there are a variety of ways to look at and solve problems, and that understanding how each little block works will lead to creating big projects. Students also learn about more specific math concepts: for instance, in learning to move characters on the stage in Hopscotch, students learn about the Cartesian plane,  $x$  and  $y$  coordinates, and slope. An activity like this can therefore help introduce students to linear equations in Grade 9 math. Students also work with concepts like angles, random numbers, conditions, and even with physics as they code pre-set instructions or venture off with their own creations.

My students, especially at the Grade 9 level, enjoyed the simplicity and structure of the Hopscotch app. It had the ability to engage both reluctant and first-time coders, but also enough power to challenge those who were looking for more. The tutorials that are available and the ability to make games that they see on the app store were appreciated by the students, and it was cool to see some students take their learning in a different direction. I should note that Hopscotch isn't the only block-coding app out there—there are many others, such as Scratch, Scratch Jr, and Blockly Block that offer similar experiences. In addition, both Microsoft (Touch Develop) and Apple (Swift) are offering free coding software for people to get started with writing code. If technology is limited in your school setting, there is also “paper programming,” where students put together code blocks cut out from paper to understand and work through the coding process (see the following link for an example: <http://www.fractuslearning.com/2014/11/18/coding-with-paper-printable-game/>).



Dean Vendramin @vendi55 · 11 Dec 2015

@oneilltitans gr 9 math checking out @hopscotch as part of #HourOfCode



I have been pleased with the results that adding coding to my math class has produced. I am excited to take this integration to another level and see where it takes me and my students—I hope to continue to gain new understandings and share what I find with others. I'm not a computer programmer by any means, and my limited programming prowess—although useful—has not been a necessary part of this journey. I also believe that coding can, and should be done in Grades K through 12. As with other ideas, I encourage teachers

to do some research, find a good starting place, and go for it. I feel that both you and your students will be glad that you did.

If you have questions or suggestions, contact me at [d.vendramin@rcsd.ca](mailto:d.vendramin@rcsd.ca) or find me on twitter [@vendi55](https://twitter.com/vendi55).



Dean Vendramin has had a variety of teaching experiences over his 19-year teaching career. Currently, he is a Math/Science Education Leader at Archbishop M.C. O'Neill Catholic High School in Regina, Saskatchewan. He is passionate about technology integration and assessment practices. Dean has been recognized for his work throughout his career, having received among other awards a Certificate of Achievement from Prime Minister's Teaching Excellence Awards and an Innovative Math Teacher Award

from the Saskatchewan Math Teachers' Society, and has been recognized by a number of technology companies for his work in integrating technology in the classroom. He is passionate about improving his craft, sharing ideas, and supporting the needs of those he serves. Contact Dean at [d.vendramin@rcsd.ca](mailto:d.vendramin@rcsd.ca), follow him on Twitter at [@vendi55](https://twitter.com/vendi55), or check out his blog and eportfolio at [deanvendramin.weebly.com](http://deanvendramin.weebly.com).

## Intersections

*In this column, you'll find information about upcoming math education-related workshops, conferences, and other events. Some events fill up fast, so don't delay signing up!*

*For more information about a particular event or to register, follow the link provided below the description. If you know about an event that should be on our list, please contact us at [thevariable@smts.ca](mailto:thevariable@smts.ca).*

### Within Saskatchewan

#### Workshops

##### **Using Tasks in High School Mathematics**

*February 8th, Saskatoon, SK*

*Presented by the Saskatchewan Professional Development Unit*

Using tasks in a high school mathematics classroom can provide rich opportunities for differentiated learning and authentic assessment. How do we choose tasks that meet both curricular outcomes and student needs? Tasks allow students to enter mathematics where they are at and extend their learning. In this workshop we will look at a variety of resources for finding good high school tasks. We will also reflect and discuss what planning and teaching moves can assist in maximizing student learning through mathematics tasks.

See <https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/using-tasks-high-school-mathematics>

##### **Technology in Math Foundations and Pre-Calculus**

*February 9th, Saskatoon, SK*

*Presented by the Saskatchewan Professional Development Unit*

Technology is a tool that allows students to understand senior mathematics in a deeper way. This workshop is designed to have math foundations and pre-calculus teachers experience a variety of technology tools that allow students to represent and visualize mathematics concepts. Tools highlighted are useful for students to explore, learn, communicate, collaborate and practice, in order to enhance their understanding of mathematics in secondary mathematics.

See <https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/technology-math-foundations-and-pre>

##### **Using Tasks in Middle Years Mathematics**

*March 13, Saskatoon, SK*

*Presented by the Saskatchewan Professional Development Unit*

Using tasks in a middle years mathematics classroom can provide rich opportunities for differentiated learning and authentic assessment.



How do we choose tasks that meet both curricular outcomes and student needs? Tasks allow students to enter mathematics where they are at and extend their learning. In this workshop we will look at a variety of resources for finding good middle years tasks. We will also reflect and discuss what planning and teaching moves can assist in maximizing student learning through mathematics tasks.

See <https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/using-tasks-middle-years-mathematics>

### **Structures for Differentiating Middle Years Mathematics**

March 21, Tisdale, SK

*Presented by the Saskatchewan Professional Development Unit*

We know that assessing where students are at in mathematics is essential, but what do we do when we know what they don't know? What do we do when they DO know? Understanding does not change unless there is an instructional response to what we know from that assessment. The question we ask ourselves is how we might respond to individual needs without having to create completely individualized mathematics programs in our classrooms.

See <https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/structures-differentiating-middle-years-1>

### **Beyond the Worksheet: Building Early Numeracy and Automaticity Through Exploration and Authentic Tasks**

April 26, Saskatoon, SK

*Presented by the Saskatchewan Professional Development Unit*

This workshop will focus on the teaching and learning of early number concepts, including place value, addition, subtraction and early multiplication. Participants will consider ways to have their students explore these number concepts in hands-on, meaningful learning experiences that allow them to construct their own understanding. Along with ideas related to content and curriculum, participants will problem solve around structures and transitions amongst whole group, small group and individual learning experiences, allowing for authentic differentiation and rich classroom conversations.

See <https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/beyond-worksheet-building-early-numeric-0>

## **Conferences**

### **Saskatchewan Understands Math (SUM) Conference**



October 23-34, Saskatoon, SK

*Presented by the Saskatchewan Mathematics Teachers' Society (SMTS), the Saskatchewan Educational Leadership Unit (SELU), and the Saskatchewan Professional Development Unit (SPDU)*

This year, the Saskatchewan Mathematics Teachers' Society, the Saskatchewan Educational Leadership Unit and the Saskatchewan Professional Development Unit are partnering to

co-ordinate a province-wide conference to explore and exchange ideas and practices about the teaching and learning of mathematics. The Saskatchewan Understands Math (SUM) conference is for mathematics educators teaching in Grades K-12 and all levels of educational leadership who support curriculum, instruction, number sense, problem-solving, culturally responsive teaching, and technology integration, and will bring together international and local facilitators to work in meaningful ways with participants in a variety of formats. This year, SUM is featuring keynote speakers Steve Leinwand of the American Institutes for Research and Lisa Lunney-Borden of St. Francis Xavier University. See the poster on page 3, and [head to our website](#) for more information.

## Beyond Saskatchewan

### **NCTM Annual Meeting and Exposition**

*April 5-8, 2017, San Antonio, TX*

*Presented by the National Council of Teachers of Mathematics*

Join more than 9,000 of your mathematics education peers at the premier math education event of the year! NCTM's Annual Meeting & Exposition is a great opportunity to expand both your local and national networks and can help you find the information you need to help prepare your pre-K–Grade 12 students for college and career success. Classroom teachers, administrators, math coaches, supervisors, college professors, and preservice teachers can all benefit from the sessions and learning at this event. Improve your knowledge and skills with high quality professional development and hands on activities; gain insights by connecting and sharing with like-minded educators; collect free activities that will keep students engaged and excited to learn; and learn from industry leaders and test the latest educational resources.

*Note:* Did you know that the Saskatchewan Mathematics Teachers' Society is an [NCTM Affiliate](#)? When registering for an NCTM membership, be sure to support the SMTS by noting your affiliation during registration.

See <http://www.nctm.org/Conferences-and-Professional-Development/Annual-Meeting-and-Exposition/>

### **OAME Annual Conference: Now for Something Completely Different**

*May 11-13, Kinston, ON*

*Presented by the Ontario Association for Mathematics Education*

This year's keynote speakers are Dan Meyer, well-known for his work integrating multimedia into an inquiry-based math curriculum, and Gail Vaz Oxlade, host of the Canadian television series *Til Debt Do Us Part*, *Princess* and, most recently, *Money Moron*. Featured speakers are George Gadanidis, Marian Small, Ruth Beatty, and Cathy Bruce.

See <http://oame2017.weebly.com/>; follow [@oame2017](#) on Twitter for updates.

## Online Workshops

### Education Week Math Webinars

*Presented by Education Week*

Once a month, Education Weekly has a webinar focusing on math. They also host their previous webinars on this site. Previous webinars include Formative Assessment, Dynamic vs. Static Assessment, Productive Struggling and Differentiation.

Past webinars: <http://www.edweek.org/ew/webinars/math-webinars.html>

Upcoming webinars:

<http://www.edweek.org/ew/marketplace/webinars/webinars.html>

Did you know that the Saskatchewan Mathematics Teachers' Society is a **National Council of Teachers of Mathematics Affiliate**? When registering for an NCTM membership, be sure to support the SMTS by noting your affiliation during registration.



## Call for Contributions

**D**id you just teach a great lesson? Or maybe it didn't go as planned, but you learned something new about the complexities of teaching and learning mathematics. Maybe you just read a book or attended a workshop that gave you great ideas for presenting a topic your students have always found difficult, or that changed your perspective about some aspect of teaching. **Why not share your ideas with other teachers in the province—and beyond?**

*The Variable* is looking for a wide variety of contributions from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants and coordinators, teacher educators, and researchers. Consider sharing a favorite lesson plan, a reflection, an essay, a book review, or any other article or other work of interest to mathematics teachers in Saskatchewan. If accepted for publication, your piece will be shared, as part of this periodical, with a wide audience of mathematics teachers, consultants, and researchers in Saskatchewan and beyond.

**We are also looking for student contributions** in the form of artwork, stories, poems, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom and their school, and for teachers to recognize their students' efforts during their journey of learning mathematics.

All work will be published under a Creative Commons license. If you are interested in contributing your own or (with permission) your students' work, please contact us at [thevariable@smts.ca](mailto:thevariable@smts.ca).

We look forward to hearing from you!

